

Newton's Third Law

9-1

From observations, Newton concluded that all forces in nature act in pairs.

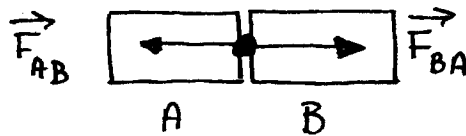
- Impossible to have a single isolated force.

Fig. (a) Tugboat pushes on barge (action). (b) Barge pushes on tugboat (reaction).



Action \longleftrightarrow Reaction

Newton: "To every action (force) there is always opposed an equal reaction (force); or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts."



$$\vec{F}_{AB} = -\vec{F}_{BA} \quad [\text{Action-Reaction Pair}]$$

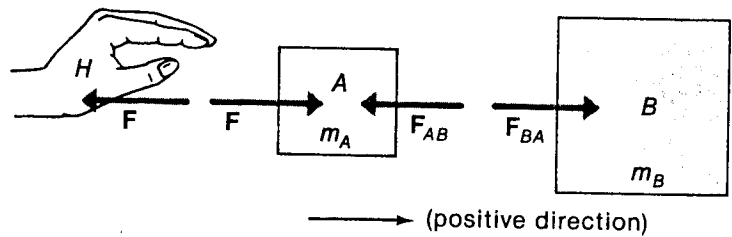
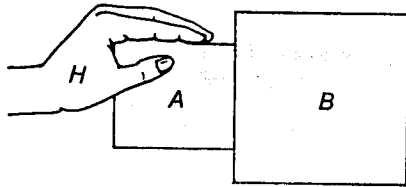
- Note: - The two forces in an action-reaction pair act on different objects.
- A body is accelerated only by the forces acting on it. Forces it exerts on another body do not affect its motion.

Example

9-2

$F = 2\text{ N}$ is applied to block A which is in contact with B.

Identify all action-reaction pairs:



$$m_A = 1\text{ kg}$$
$$m_B = 2\text{ kg}$$

$$\vec{a} = ? \quad [\text{Acceleration}]$$

$$\vec{F}_{AB} = ? \quad [\text{Contact Force}]$$

Choose coordinate system, x-axis positive to right.
Apply Newton's 2nd law for motion of B:

$$F_{BA} = m_B a_B \quad (1)$$

Forces acting on A :

$$F - F_{AB} = m_A a_A \quad (2)$$

Since both blocks stay in contact,
 $\therefore a_A = a_B = a$ [Constraint in problem]

Add Eq. ① + ②

$$F - F_{AB} + F_{BA} = (m_A + m_B) a$$

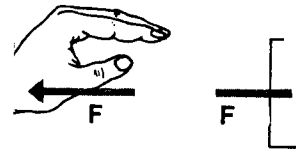
$$|F_{AB}| = |F_{BA}|$$

magnitudes equal due to
 Newton's 3rd Law.

$$\therefore F = \underbrace{(m_A + m_B)}_M a$$

Single mass $M = m_A + m_B$ on which
 only external force F is important.

$$a = \frac{F}{m_A + m_B} = \frac{2\text{N}}{(1+2)\text{kg}} = \frac{2}{3} \text{ m/s}^2$$



Contact Force

$$F_{BA} = m_B a_B = m_B a$$

$$= 2 \times \frac{2}{3} = \frac{4}{3} \text{ N}$$

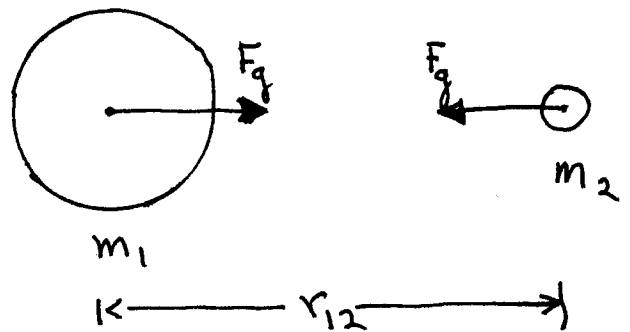
$$\neq F$$

Gravitational Force

10-1

Newton's Postulate: Every pair of particles in the universe exerts on one another a mutual gravitational force of attraction. This force is proportional to the product of the masses and inversely proportional to the square of the distance between them.

$$F_g = \frac{G m_1 m_2}{r_{12}^2}$$



$G = 6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2$
Universal Gravitation Constant.

Consider a mass m at the surface of the earth which has a mass = M_E and radius = R_E .

$$F_g = m \left(\underbrace{\frac{M_E G}{R_E^2}}_g \right) = m g$$

Gravitation constant G can be determined by measuring the force with known masses that are separated by a known distance. [Cavendish Expt.]

Gravitational acceleration g is measured by observing the fall of objects.

$$M_E = \frac{g R_E^2}{G}$$

"weighing the earth"

Object m resting on surface of the earth.

Object:

Two forces acting.

Earth's attractive gravitational force \vec{F}_g . Contact force \vec{N} with ground.

Since object is at rest:

$$\vec{N} + \vec{F}_g = 0$$

$$\vec{N} = -\vec{F}_g$$

Earth:

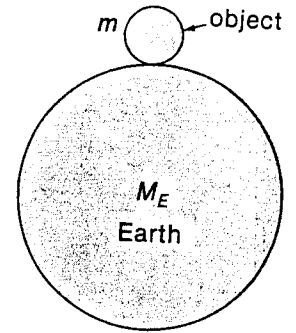
Two forces acting.

Attractive gravitational force \vec{f}_g due to object.

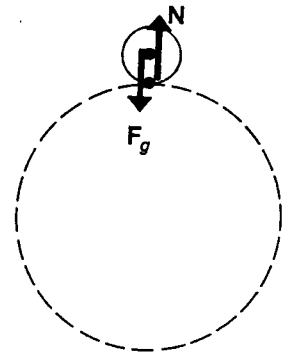
Contact force \vec{w} with object.

No relative motion:

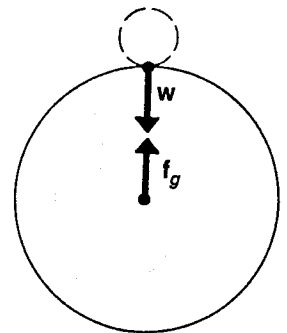
$$\vec{w} = -\vec{f}_g$$



(a)



(b)



(c)

Fig. (a) An object at rest on the Earth's surface. (b) The forces acting on the object. (c) The forces acting on the Earth. The forces shown together in (b) and (c) are not action-reaction pairs.

Action-Reaction Force Pairs :

$$\vec{F}_g = -\vec{f}_g$$

$$\vec{N} = -\vec{w}$$

All four forces have equal magnitudes
 $F_g = f_g = N = w = mg$

Weight

The contact force \vec{w} that an object exerts on whatever is supporting it is called the weight of the object.

For above example \vec{F}_g acts on the object and \vec{w} acts on the earth. For this case of zero acceleration

$$\vec{w} = m\vec{g}$$

Accelerating Elevator

10-4

- Object on floor of elevator that is accelerating upward with acceleration \vec{a} .

For forces acting on m :

$$N - F_g = ma$$

$$N = F_g + ma = mg + ma = m(g+a)$$

N : Force elevator exerts on object

F_g : Force earth exerts on object

$$\vec{N} = -\vec{w} \quad [\text{Action - Reaction}]$$

$$\therefore w = m(g+a) \quad [\text{Magnitude}]$$

- Weight is increased by the amount ma over weight at rest.

For elevator moving downward with acceleration a' :

$$w = m(g - a')$$

- weight is decreased by ma' .

Free Fall: $a' = g \Rightarrow w = 0$

Object is weightless - exerts no contact force on support.

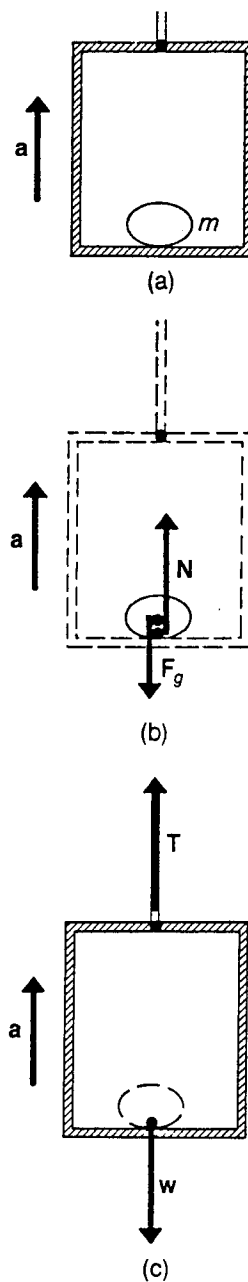


Fig. 10-4 (a) An object with a mass m is on the floor of an elevator that is accelerating upward. (b) Forces acting on the object. (c) Forces acting on the elevator.

Example: Astronaut/Satellite

10-5

An artificial satellite is in a circular orbit around the Earth. At position of orbit acceleration due to gravity is g' .

$$g' = \frac{v^2}{R}$$

[Free-fall motion]

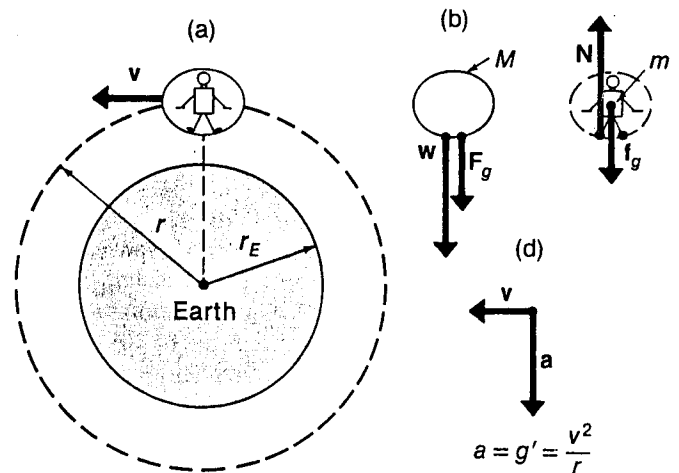


Fig. An astronaut in an orbiting artificial satellite is weightless.

Force acting on satellite:

$$F_s = F_g + w = Mg'$$

$$[F = ma]$$

Forces acting on astronaut:

$$f_a = f_g - N = mg'$$

$$[F = ma]$$

$$\left. \begin{aligned} F_g &= Mg' \\ f_g &= mg' \end{aligned} \right\} \text{Gravitational Force on each object by definition.}$$

$$\therefore w = N \equiv 0 \quad [\text{No contact force/No weight}]$$

Astronaut is weightless - can exert no net force on satellite.

Note: Weight of an object is always equal to the vector force exerted on the supporting object or system.

Gravitational / Inertial Mass

10-6

Mass plays essentially two different roles in mechanics.

1. Inertial Mass:

In Newton's Laws the quantity m connects the force applied to an object with its acceleration.

$$\vec{F} = m_I \vec{a}$$

The mass measured in this way is called the inertial mass, M_I .

2. Gravitational Mass:

Mass is also the property of matter that causes bodies to exert attractive gravitational forces on each other. eg. measure force on object due to the earth.

$$\vec{F} = \frac{G M_E M_G}{R_E^2} = \vec{g} M_G$$

This mass is called the gravitational mass.

Question: Is $M_I = M_G$ for all bodies?

Experiment: Must provide the ultimate answer.
Do all bodies fall due to gravity with the same acceleration?

Body #1: $M_I(1) a(1) = \frac{G M_E}{R_E^2} M_G(1) = g M_G(1)$ ①

Body #2: $M_I(2) a(2) = g M_G(2)$ ②

①/② $\frac{M_I(1) a(1)}{M_I(2) a(2)} = \frac{M_G(1)}{M_G(2)}$

$\frac{M_I(1)}{M_G(1)} = \frac{M_I(2)}{M_G(2)} \times \frac{a(2)}{a(1)}$

Experiment finds that $a(2)/a(1) = \text{constant}$. Tested for many different materials.

Galileo

Eötvös } $< 1 \times 10^{-12}$ difference.
Dicke }

Ratio $\frac{M_I}{M_G} = 1$ by a proper choice of G.

Newtonian Theory: $M_I \propto M_G$ not required

General Relativity: $M_I \propto M_G$ absolutely a must

↳ 'Principle of Equivalence'
-gravitation is equivalent to acceleration.

Principle of Equivalence

- A fundamental postulate of Einstein's General Theory of Relativity. In this theory gravity is interpreted as a curvature in space and time

- Equivalence

• Physicist in a box cannot tell the difference between gravity and acceleration.

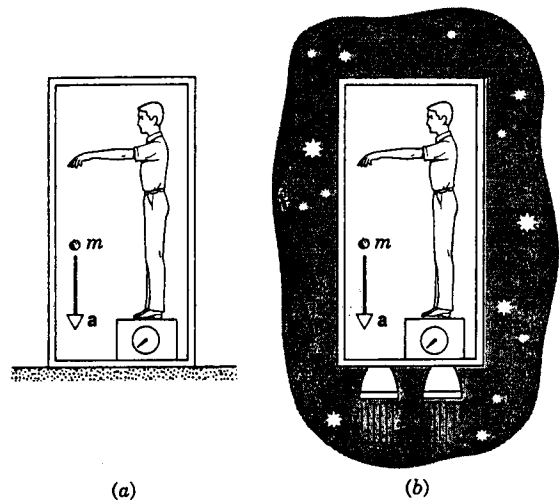


Figure (a) A physicist-in-a-box, resting on the earth. (b) The same, accelerating in deep space at 9.8 m/s^2 . It is not possible, by doing experiments within the box, for the physicist to tell which box he is in.

Ex: Suppose $\left(\frac{Mg}{M_I}\right)_A = 2\left(\frac{Mg}{M_I}\right)_B$. Release both objects in an Einstein elevator. If they fall with same acceleration then elevator must be accelerating upwards. If A falls with twice acceleration of B then it must be due to gravitational field. Violation of Principle of Equivalence!! Predicted bending of light by a massive object.

Hooke's Law - Springs

13-1

All bodies are elastic to varying degrees. When a stretching or compressing force is applied they deform.

- steel balls
- rubber bands
- springs

A body resists deformation by means of a restoring force.

Pull on a spring - it pulls back.

In many cases the relationship between the resulting restoring force and the deformation obeys a simple empirical law known as Hooke's law.

"The magnitude of the restoring force is directly proportional to the deformation"

- approximate
- empirical description
- good for small deformations

Coil Spring

+x - spring stretched
-x - spring compressed

Hooke's law:

$$F = -kx$$

spring constant

Negative sign means the restoring force opposes the deformation.

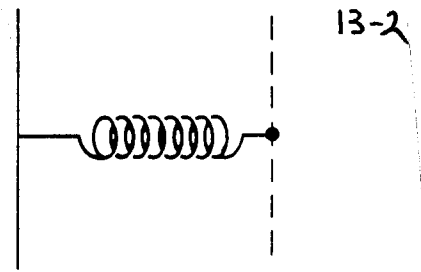


Fig. Spring, relaxed.

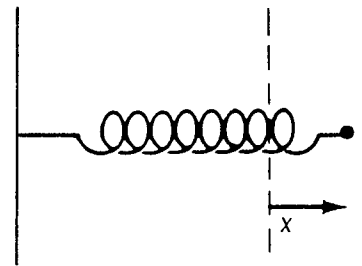
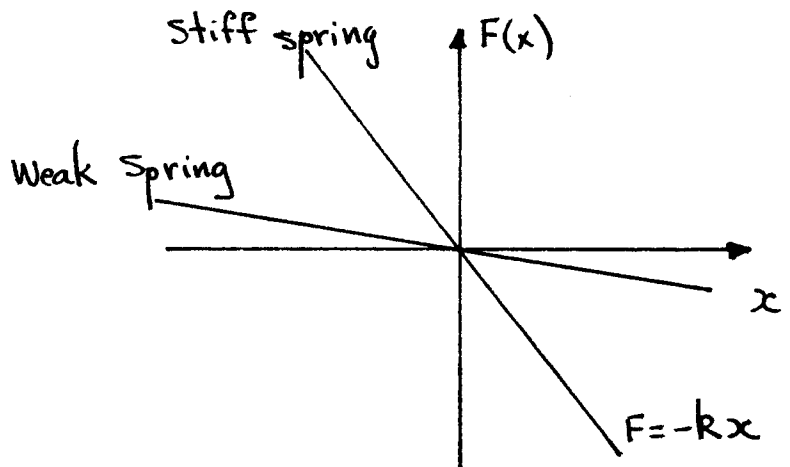


Fig. Spring, stretched by a length x.



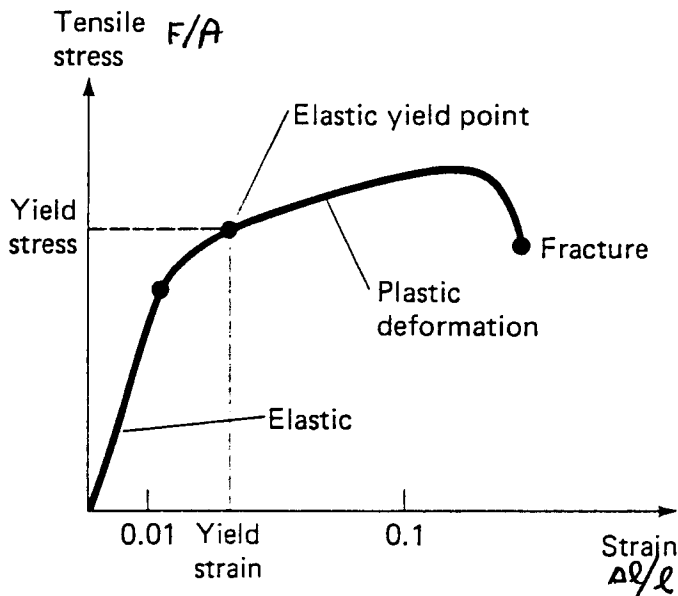
$$[k] = \text{N/m}$$

k large: stiff spring
large force/unit displacement

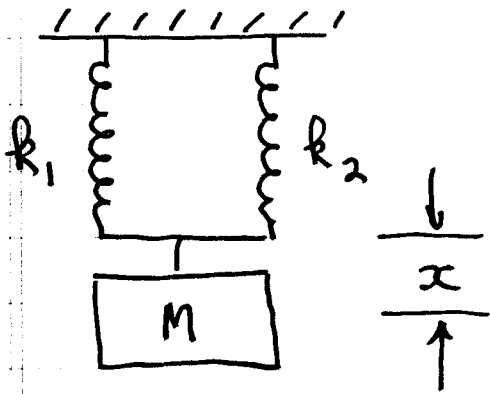
k small: soft spring
small force/unit displacement



+x -elongation; Force negative opposes stretching
 -x -compression; Force positive opposes compression



Parallel Springs



$$-k_1 x - k_2 x + Mg = 0$$

$$x(k_1 + k_2) = Mg$$

$$x = \frac{Mg}{(k_1 + k_2)} = \frac{Mg}{k_{\text{eff}}}$$

$$k_{\text{eff}} = k_1 + k_2$$

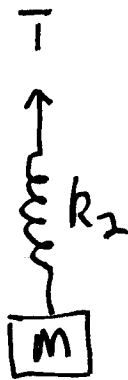
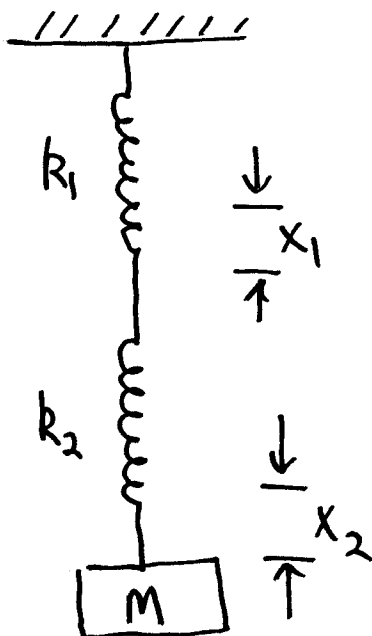
$k_{\text{eff}} \Rightarrow$ Stiffer Spring

$$\text{If } k_1 = k_2 = k$$

$$k_{\text{eff}} = 2k$$

Twice as stiff

Series Springs



$$T = Mg$$

$$\therefore k_2 x_2 = Mg$$

Also $k_1 x_1 = Mg$

$$x = x_1 + x_2 = \frac{Mg}{k_1} + \frac{Mg}{k_2}$$

$$x = Mg \left[\frac{1}{k_1} + \frac{1}{k_2} \right]$$

Let $\frac{1}{k_{\text{eff}}} = \frac{1}{k_1} + \frac{1}{k_2}$ Weaker Spring

If $k_1 = k_2 = k$

$k_{\text{eff}} = \frac{1}{2} k$ Weaker