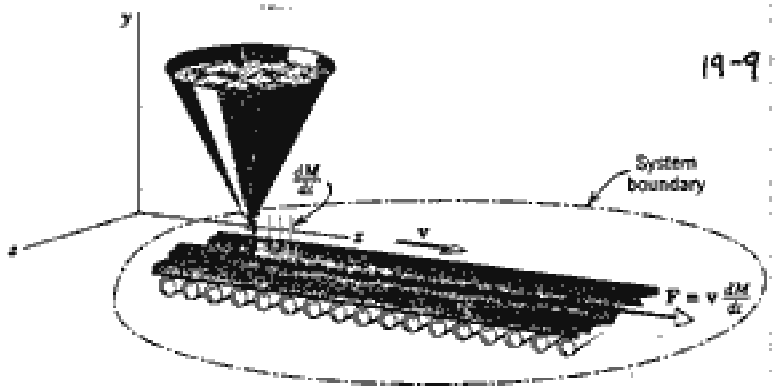
Example: Conveyor Belt



Gravel dropped on belt at rate of 75.0 kg/s.
Belt speed is $v = 2.2$ m/s
What force required to keep belt moving?

Hopper is at rest so $u = 0$

$$\frac{dM}{dt} = 75.0 \text{ kg/s}$$

Belt at constant speed

$$\therefore \frac{dv}{dt} = 0$$

$$F_{ext} = \underbrace{M\frac{dv}{dt}}_{=0} - (\underbrace{u}_{u=0} - v)\frac{dM}{dt}$$

$$F_{ext} = v\frac{dM}{dt} = 2.2 \times 75 = 165 \text{ N}$$

Collisions/Impulse

When two objects collide, the forces they exert on each other usually act only for a short time. Such forces are called impulsive forces.

During the collision the impulsive force produces a large change in the motion of the object while any other forces present produce only small changes usually neglected.

From Newton's 2ND Law

$$\frac{d\vec{p}}{dt} = \vec{F}$$

During a time interval dt, the momentum changes by

$$d\vec{p} = \vec{F}\,dt$$

Integrating over the time of collision

$$\vec{P_f} - \vec{P_i} = \int_{P_i}^{P_f} d\vec{p} = \underbrace{\int_{t_i}^{t_f} \vec{F}\,dt}$$

Area under curve represents Impulse.

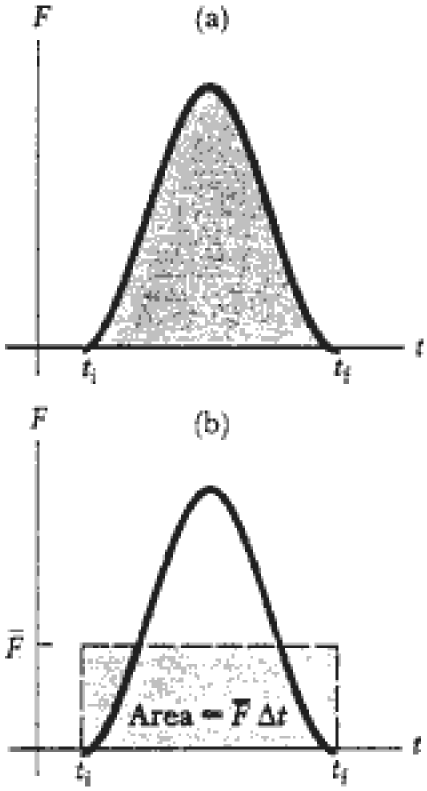
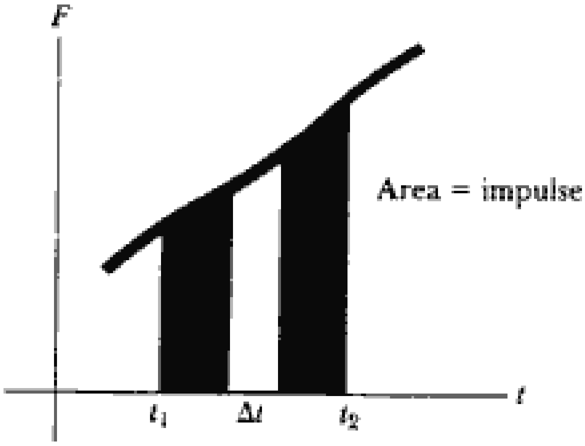


Figure (a) A force acting on a particle may vary in time. The impulse is the area under the force versus time curve. (b) The average force (horizontal line) would give the same impulse to the particle in the time Δt as the real time-varying force described in (a).

$$\vec{I} = \int_{t_i}^{t_f} \vec{F}(t)\, dt$$

[Impulse]

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = \vec{I}$$



The area of the small rectangle is approximately equal to the total change in momentum in the interval Δt.

The change in momentum of an object is equal to the impulse acting on it.

Sometimes useful to talk about an average force $\bar{F}$ which acting over the same time interval

$$\Delta t = t_f - t_i$$

produces the same impulse and consequently the same momentum change.

$$\bar{F} = \frac{1}{\Delta t} \int_{t_i}^{t_f} F(t)\, dt$$

$$\bar{F}\Delta t = (\vec{p}_f - \vec{p}_i) = \vec{I}$$

Example

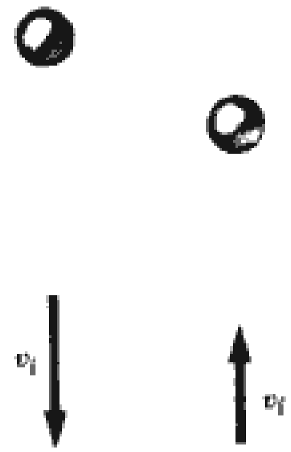
A ball of mass 100g dropped from height $h = 2m$ to the floor. It rebounds to a height $h' = 1.5m$. Assume time-of-collision is .01s (typical)

To find velocity just before it hits the floor:

$$\frac{1}{2} m v_i^2 = mgh \qquad v_i = \sqrt{2gh} = 6.26 \text{ m/s}$$

After rebounding it has a velocity:

$$\frac{1}{2} m v_f^2 = mgh' \qquad v_f = \sqrt{2gh'} = 5.42 \text{m/s}$$



$$\vec{p_i} = m\vec{v_i} = -0.63\,\hat{z} \text{ kg}\cdot\text{m/s}$$

$$\vec{p_f} = m\vec{v_f} = 0.54\,\hat{z} \text{ kg}\cdot\text{m/s.}$$

$$\text{Impulse} \quad \vec{I} = \Delta\vec{p} = \vec{p_f} - \vec{p_i} = \bar{F}\Delta t$$

$$\Delta\vec{p} = [0.54 - (-0.63)]\,\hat{z}$$
$$= 1.17\,\hat{z} \text{ kg}\cdot\text{m/s}$$

$$\bar{F} = \frac{\Delta p}{\Delta t} = \frac{1.17\,\hat{z}}{.01} = 117\text{N}\,\hat{z}$$

Much larger than force of gravity (~ 1N).

Collisions

We want to study the collision of objects — how they move (velocities) after colliding. In special cases momentum conservation is sufficient. In general it is not enough. Can categorize collisions in terms of two types:

Elastic Collisions

- Interaction forces are all conservative
- Total kinetic energy is the same before and after
- Momentum is conserved

$$K_i = K_f \quad ; \quad \vec{P}_i = \vec{P}_f$$

Inelastic Collisions

- Momentum is conserved
- Total kinetic energy after collision is less than before.

$$\vec{P}_i = \vec{P}_f$$

$$K_i \neq K_f$$

A. Inelastic Collisions

Collisions in which KE is not conserved are called inelastic collisions. Some of the energy is absorbed and converted to other forms.

If the amount of KE absorbed is a maximum that is allowed by momentum conservation, the collision is said to be perfectly inelastic

⇒ No internal KE left only KE of cm.

$$K_{INT} \equiv 0$$

For collisions involving isolated objects, momentum is always conserved.

In practise almost all collisions are inelastic to some degree. To solve inelastic problems is not easy. Need to know how much energy is lost.

The easy problem involves a perfectly inelastic collision (– sticking collision –). Then conservation of momentum gives the answer.

i) Sticking Collision - One Dimension

Two particles messes m_1 and m_2 move with velocities v_{1i} and v_{2i} along a st. line. They collide and stick, moving as a unit with velocity v_f after the collision.

Total momentum is conserved.

$$\vec{P}_i = \vec{P}_f$$

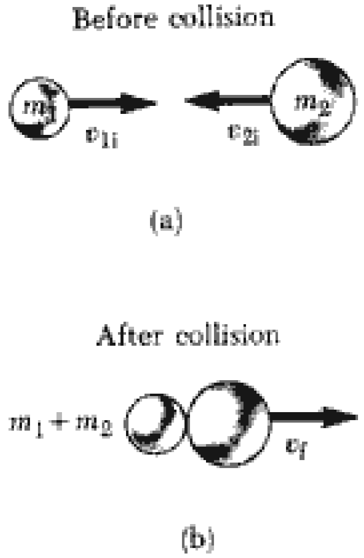


Figure Schematic representation of a perfectly inelastic head-on collision between two particles: (a) before the collision and (b) after the collision.

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$$

$$\vec{v}_f = \frac{m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i}}{m_1 + m_2}$$

<u>Example:</u>

Body-2 is at rest, $\vec{v}_{2i} = 0$.

Then $\vec{v}_f = \dfrac{m_1 \vec{v}_{1i}}{m_1 + m_2}$

The corresponding Kinetic Energies are:

$$K_i = \frac{1}{2} m_1 v_{1i}^2$$

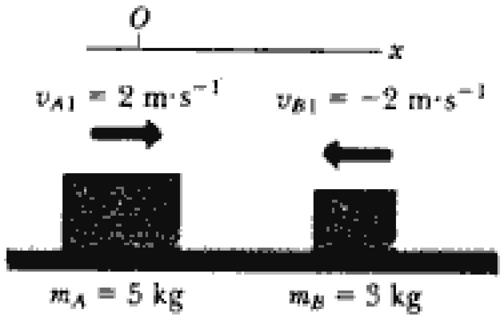
$$K_f = \frac{1}{2}(m_1 + m_2)\frac{m_1^2 v_{1i}^2}{(m_1+m_2)^2}$$

$$\frac{K_f}{K_i} = \frac{m_1}{m_1 + m_2} < 1$$

Final KE is always less than initial KE in such collisions.

<u>Example</u>

Two objects collide and stick.



$$\vec{P_i} = \vec{P_f}$$

$$(5\,\mathrm{kg})(2\,\mathrm{m/s}) - (3\,\mathrm{kg})(2\,\mathrm{m/s}) = (5+3)\,v_f$$

$$v_f = \frac{10-6}{8} = 0.5\,\mathrm{m/s}$$ [Positive moving to right].

$$K_i = \frac{1}{2} m_A v_{A1}^2 + \frac{1}{2} m_B v_{B1}^2$$

$$= \frac{1}{2}\times 5\times 2^2 + \frac{1}{2}\times 3\times 2^2 = 16\ \mathrm{J}$$

$$K_f = \frac{1}{2}(m_A + m_B)\, v_f^2$$

$$= \frac{1}{2}\times 8\times\left(\frac{1}{2}\right)^2 = 1\ \mathrm{J}$$

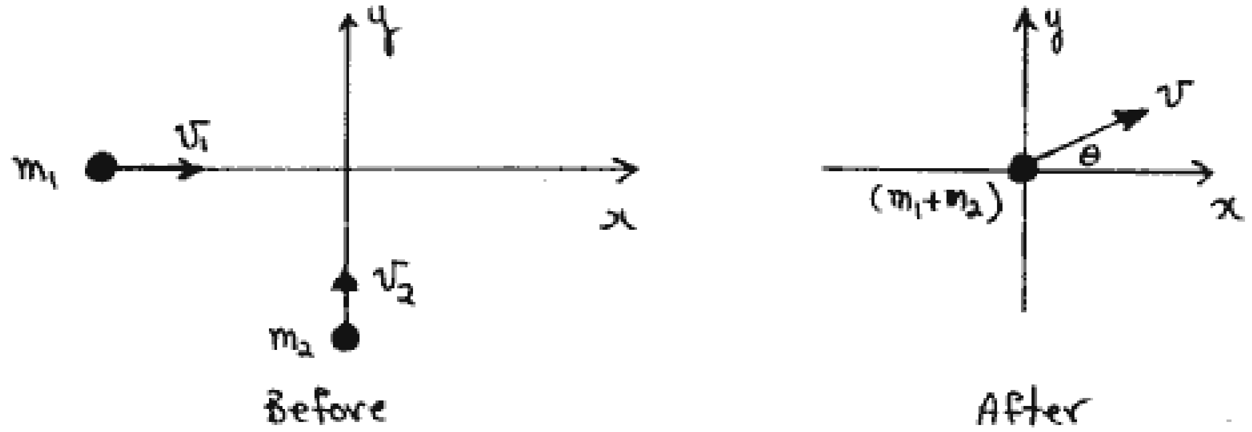
$$\frac{K_f}{K_i} = \left(\frac{1}{16}\right)$$

Most of the KE was lost!!

ii) Sticking Collision - Two Dimensions

- Linear momentum conserved.
- Two equations, one for each component.
- Simplest problem is a sticking collision.

Example



- Assume no external forces acting

$$\vec{P}_i = \vec{P}_f$$

Conservation of Momentum : x-component

$$m_1 v_1 = V(m_1+m_2)\cos\theta \qquad (1)$$

Conservation of Momentum y-component.

$$m_2 v_2 = V(m_1+m_2)\sin\theta \qquad (2)$$

(2)/(1) $$\tan\theta = \frac{m_2 v_2}{m_1 v_1}$$

from (2) $$v = \frac{m_2}{m_1+m_2}\,\frac{v_2}{\sin\theta}$$

$m_1 = 70\text{kg}$ $\quad v_1 = 2\text{m/s}$

$m_2 = 50\text{kg}$ $\quad v_2 = 3\text{m/s}$

$$\tan\theta = \frac{50}{70} \times \frac{3}{2}$$

$$\theta = 47^\circ$$

$$v = \frac{50}{50+70} \times \frac{3}{\sin 47^\circ} = 1.71\text{ m/s}$$

Initial KE:

$$K_i = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

$$= \frac{1}{2}\times 70\times 2^2 + \frac{1}{2}\times 50\times 3^2$$

$$= 365\text{ J}$$

Final KE:

$$K_f = \frac{1}{2}(m_1+m_2)V^2$$

$$= \frac{1}{2}(50+70)\,1.71^2$$

$$= 175.5\text{ J}$$

KE is lost in this inelastic collision.

3. Elastic Collisions

- Total energy is conserved.
- Total linear momentum is conserved.

i) Elastic Collision – One Dimension.

- Assume particles moving with velocities $\vec{v}_1$ and $\vec{v}_2$ before the collision.
- Particles move with velocities $\vec{v}_1{}'$ and $\vec{v}_2{}'$ after the collision.

$v > 0$ if particle moves to the right.
$v < 0$ if particle moves to the left

Cons. of Momentum: $\vec{P}_i = \vec{P}_f$

$$① \quad m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1{}' + m_2\vec{v}_2{}'$$

Cons. of Energy: $K_i = K_f$

$$② \quad \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 = \tfrac{1}{2}m_1v_1'^2 + \tfrac{1}{2}m_2v_2'^2$$

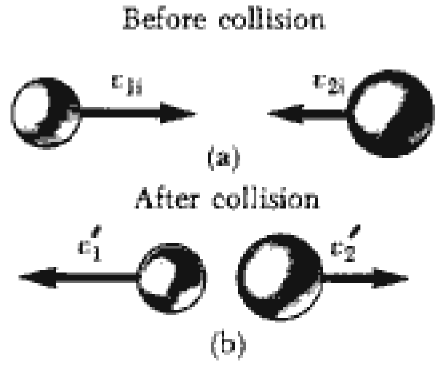


Figure Schematic representation of an elastic head-on collision between two particles; (a) before the collision and (b) after the collision.

We have two equations, can solve for two unknowns.
Given the masses:

$$(v_1, v_2) \Longrightarrow (v_1', v_2')$$

Rewrite Eq. ①: $$m_1(\vec{v}_1 - \vec{v}_1{}') = m_2(\vec{v}_2{}' - \vec{v}_2) \qquad ③$$

Rewrite Eq ②: $$m_1(v_1^2 - v_1'^2) = m_2(v_2'^2 - v_2^2) \qquad ④$$

④/③ $$v_1 + v_1' = v_2' + v_2$$

or $$\boxed{\vec{v}_1 - \vec{v}_2 = \vec{v}_2{}' - \vec{v}_1{}'}$$

The relative velocities of the two particles after the collision is the negative of the relative velocities before the collision for any elastic head-on collision, no matter what the masses.
This can be combined with eq for cons. of momentum to solve all problems.

Special Cases

- m_1, m_2, v_1 and v_2 are known.
- v_1' and v_2' to be determined

a) Equal Masses : $m_1 = m_2$

Momentum : $\vec{v}_1 + \vec{v}_2 = \vec{v}_1{}' + \vec{v}_2{}'$ ①

Rel Velocities : $\vec{v}_1 - \vec{v}_2 = \vec{v}_2{}' - \vec{v}_1{}'$ ②

① + ② $\vec{v}_2{}' = \vec{v}_1$

① − ② $\vec{v}_1{}' = \vec{v}_2$

- Particles exchange velocities during collision
  - Particle-2 acquires velocity of particle-1
  - Particle-1 acquires velocity of particle-2

If $\vec{v}_2 = 0$ (Particle-2 initially at rest)

$v_2{}' = v_1$

$v_1{}' = 0$

- Particle-1 stops
- Particle-2 moves forward with velocity of particle-1

b) Particle-2 at rest initially, $v_2 = 0$

Momentum: $$m_1 v_1 = m_1 v_1' + m_2 v_2'$$

Rel. Velocities: $$v_1 = -v_1' + v_2'$$

$$v_2' = v_1 \left(\frac{2m_1}{m_1 + m_2} \right) \quad (5)$$

$$v_1' = v_1 \left(\frac{m_1 - m_2}{m_1 + m_2} \right) \quad (6)$$

i) $v_2 = 0$, $m_1 = m_2$

$$\left. \begin{aligned} v_2' &= v_1 \\ v_1' &= 0 \end{aligned} \right\} \text{ Same result as Case (a)}$$

ii) $v_2 = 0$, $m_1 \gg m_2$

– A heavy object strikes a light object at rest.

From (5) and (6)
$$v_2' \simeq 2v_1$$
$$v_1' \simeq v_1$$

- Velocity of incoming particle unchanged
- light particle moves forward with twice the velocity of the heavy object.

iii) $v_2 = 0$, $m_1 \ll m_2$

- A moving light particle strikes a stationary heavy object.

$$v_2' \sim 0$$
$$v_1' \sim -v_1$$

- Heavy object essentially remains at rest
- Light particle reverses direction and moves of with insident speed.

c) General Solution

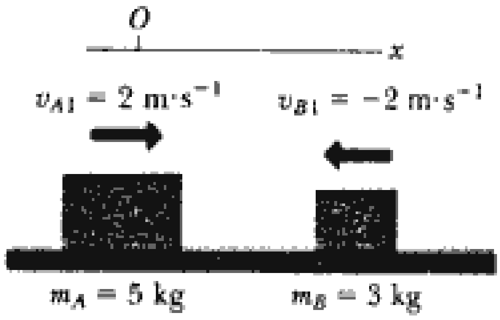
- Can solve original equations for velocities of particles after the collision.
- Usually it is best to start from momentum and energy conservation laws instead of memorizing formulas.

$$v_2' = v_1\left(\frac{2m_1}{m_1+m_2}\right) + v_2\left(\frac{m_2-m_1}{m_1+m_2}\right)$$

$$v_1' = v_1\left(\frac{m_1-m_2}{m_1+m_2}\right) + v_2\left(\frac{2m_2}{m_1+m_2}\right)$$

Example: Elastic 1-D

Conservation of Momentum:



$$(5kg)(2m/s) + (3kg)(-2m/s) = 5\,v_{A2} + 3\,v_{B2}$$

$$5v_{A2} + 3v_{B2} = 4 \quad (m/s)$$

Since collision is completely elastic:

$$v_{B2} - v_{A2} = -(v_{B1} - v_{A1})$$

$$= -(-2-2) = 4\ m/s.$$

Solving: $v_{A2} = -1\,m/s$
$v_{B2} = 3\,m/s$

Bodies reverse their directions of motion. A moves to left at 1m/s and B to right at 3m/s.

$$KE_i = \frac{1}{2} m_A v_{A1}^2 + \frac{1}{2} m_B v_{B1}^2 = \frac{1}{2} \times 5 \times 2^2 + \frac{1}{2} \times 3 \times 2^2$$

$$= 16\,J$$

$$KE_f = \frac{1}{2} m_A v_{A2}^2 + \frac{1}{2} m_B v_{B2}^2 = \frac{1}{2} \times 5 \times 1^2 + \frac{1}{2} \times 3 \times 3^2$$

$$= 16\,J.$$

$$KE_i = KE_f$$