

# Time Dilation

R3-1

Einstein and Infeld: In classical physics it was always assumed that all clocks in motion keep the same time and all meter sticks measure the same length. Relativity theory forces us to abandon these truths.

Light clock:

- laser
  - mirror
  - detector
- laser emits pulse
  - mirror reflects pulse
  - detector sees pulse
  - measures time interval.

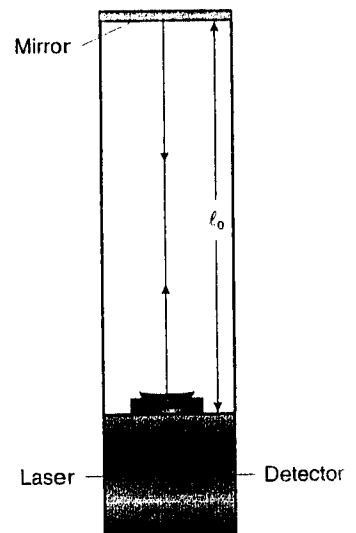


FIGURE 25.9 A special light clock.

System  $S'$  moves with velocity  $V$  along  $x$ -axis of system  $S$ . Origins coincide at  $t = t' = 0$ .

System - S'

• Pulse emitted :

$$\begin{aligned}x_1' &= 0 \\y_1' &= 0 \\z_1' &= 0 \\t_1' &= 0\end{aligned}$$

• Pulse reflected :

$$\begin{aligned}x_2' &= 0 \\y_2' &= L_0 \\z_2' &= 0 \\t_2' &= \frac{L_0}{c}\end{aligned}$$

• Pulse detected :

$$\begin{aligned}x_3' &= 0 \\y_3' &= 0 \\z_3' &= 0 \\t_3' &= \frac{2L_0}{c}\end{aligned}$$

System - S

- observe experiment in  $S'$
- As light pulse travels,  $S'$  is moving to the right. Light pulse moves along triangular path.

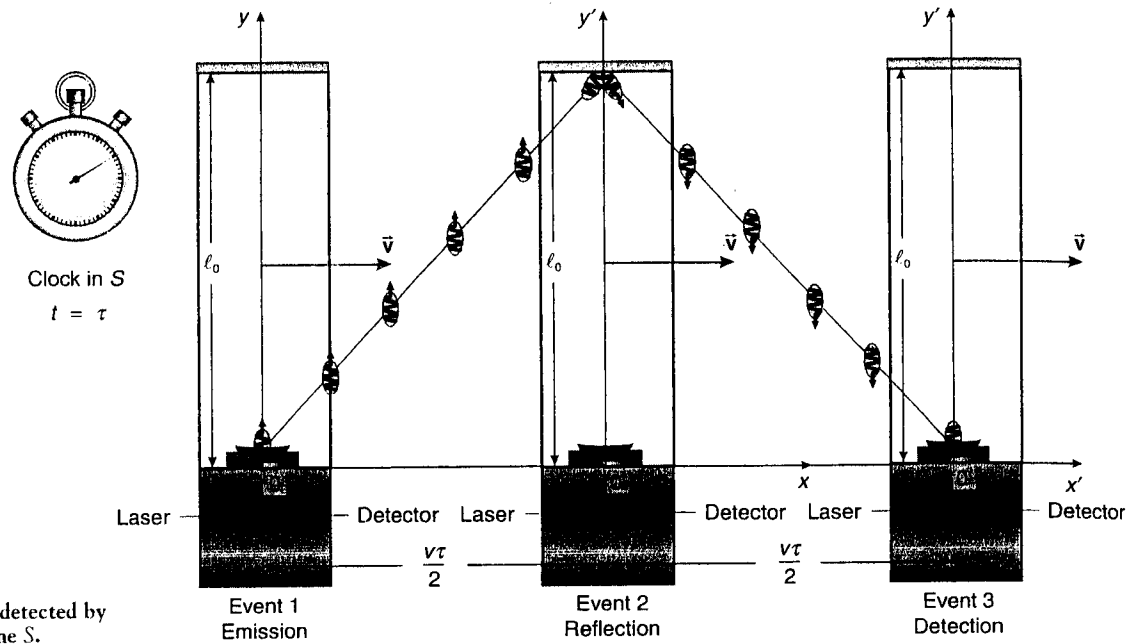


FIGURE 25.13 Light detected by detector, as seen in frame  $S$ .

• Pulse emitted:

$$\begin{aligned} x_1 &= 0 \\ y_1 &= 0 \\ z_1 &= 0 \\ t_1 &= 0 \end{aligned}$$

• clock moves to right a distance

$$v\tilde{t}/2$$

$\tilde{t}$ : total travel time.

- Pulse reflected:

$$x_2 = \frac{v\tau}{2}$$

$$y_2 = L_0$$

$$z_2 = 0$$

$$t_2 = \frac{v\tau}{2}$$

- Clock continues to move to right
- Pulse detected

$$x_3 = v\tau$$

$$y_3 = 0$$

$$z_3 = 0$$

$$t_3 = \tau$$

Path length of light in S is:

$$\sqrt{L_0^2 + \left(\frac{v\tau}{2}\right)^2}$$

Light travels this distance at a constant speed  $c$ :

time to travel each path is

$$\frac{v\tau}{2} = \frac{\sqrt{L_0^2 + \left(\frac{v\tau}{2}\right)^2}}{c}$$

$$\frac{\tilde{c}^2}{4} = \frac{L_0^2 + \left(\frac{v\tilde{c}}{2}\right)^2}{c^2}$$

Solving for  $\tilde{c}$ :

$$\tilde{c}^2 = \frac{4L_0^2/c^2}{1 - v^2/c^2}$$

$$\tilde{c} = \frac{2L_0/c}{\sqrt{1 - v^2/c^2}} = \gamma \tilde{c}_0$$

Let  $\frac{2L_0}{c} = \tilde{c}_0$ ; travel time measured in  $S'$  with one clock which is at rest.

$\tilde{c}_0 \equiv$  Proper Time

$\tilde{c}$  is measured in frame  $S$  with clocks at different locations on  $x$ -axis.

Time interval  $\tilde{c}$  measured in  $S$  is longer than the time interval  $\tilde{c}_0$  measured in  $S'$  where the clock is at rest.

A moving clock ticks more slowly than a clock at rest. A moving clock runs slow. Effect called time dilation

## Time Dilation: Lov.-Fitz.

R3-6

- Clock at rest in  $S'(x', t')$
  - Two events A and B
- A:  $x'_0$   $t'_A$   
B:  $x'_0$   $t'_B$

$\tilde{\tau}_0 = t'_B - t'_A$  ; Proper time interval between events in rest system

Use:  $t = \gamma(t' + vx'/c^2)$

$$t_A = \gamma \left( t'_A + \frac{v x'_0}{c^2} \right)$$

$$t_B = \gamma \left( t'_B + \frac{v x'_0}{c^2} \right)$$

$$T = t_B - t_A$$

$$= \gamma (t'_B - t'_A)$$

$$T = \gamma \tilde{\tau}_0$$

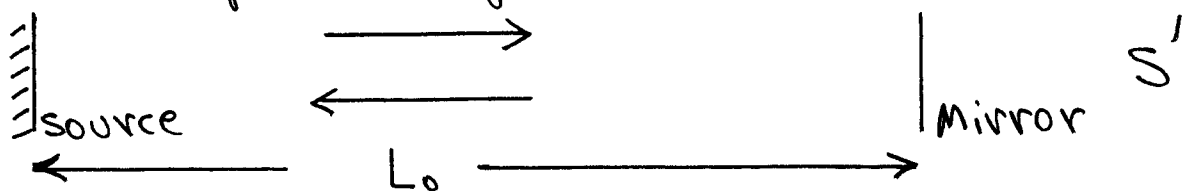
Time interval  $T > \tilde{\tau}_0$

Moving clocks run slow.

Time Dilation!

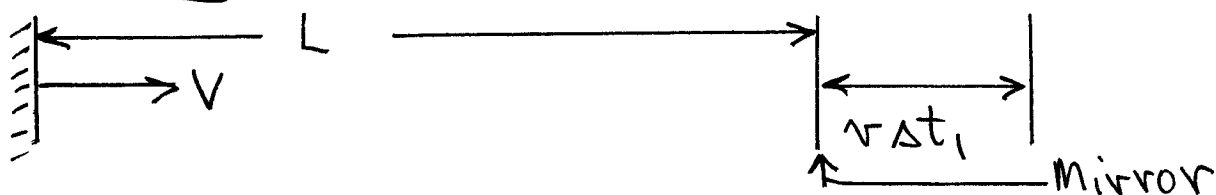
## Relativity of length: Parallel

R3-7



- Meter stick at rest in  $S'$
  - Light pulse makes a round trip
- $$\Delta t_0 = \frac{2L_0}{c}$$
- Proper Time

S-Frame: Ruler moves to right with velocity  $v$ . Length of ruler is  $L$ .



Time to travel distance in  $S$  is  $\Delta t_1$ .  
 During this time mirror moves a distance  $v\Delta t_1$ ,  
 Total path in  $S$  is

$$d = L + v\Delta t_1$$

Light pulse travels with speed  $c$ ;

$$\therefore d = c\Delta t_1$$

$$c\Delta t_1 = L + v\Delta t_1$$

$$\therefore \Delta t_1 = \frac{L}{c-v}$$

R3-8

Note: Light does not travel with a speed  $c-v$ ; it travels a distance longer than  $L$ .

Return trip:

$$\Delta t_2 = \frac{L}{c+v}$$

Total Time:

$$\Delta t = \Delta t_1 + \Delta t_2 = \frac{L}{c-v} + \frac{L}{c+v} = \frac{2L}{c \left[ 1 - \frac{v^2}{c^2} \right]}$$

Time dilation relates  $\Delta t$  and  $\Delta t_0$

$$\frac{\Delta t}{\gamma} = \Delta t_0 = \frac{2L_0}{c}$$

Eliminate  $\Delta t$ :

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}} = \frac{L_0}{\gamma}$$

Length  $L$  measured in  $S$  is shorter than the proper length  $L_0$  in  $S'$ .

## Relativity of length: Parallel [Lor-Fitz] R3-9

- Ruler at rest in  $S'$  ( $x'_A, x'_B$ )
- Length  $L_0 = x'_B - x'_A$  Proper Length
- $S'$  moves to right with velocity  $v$  wrt system -  $S$

To measure length in system -  $S$  we must mark both ends at a common time  $t$ .

$$x'_B = \gamma(x_B - vt)$$

$$x'_A = \gamma(x_A - vt)$$

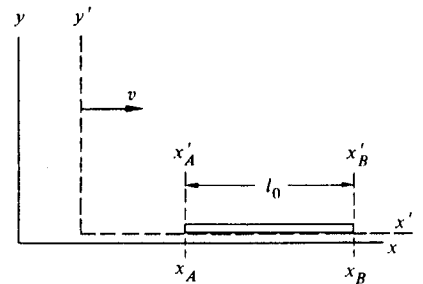
$$x'_B - x'_A = \gamma(x_B - x_A)$$

$$L_0 = \gamma L$$

$$L = L_0 / \gamma$$

$L < L_0$ , meter stick is contracted

$\Rightarrow$  Lorentz contraction



## Relativity of Length : Transverse

R3-10

- In time dilation used  $L_0$  the same in both frames. Was that correct?
- Take two identical meter sticks: length  $L_0$
- Attach along  $y$ -axis in  $S$  and  $S'$
- $S'$  moves relative to  $S$  with  $v$ .
- Attach sharp nail to top of each stick.

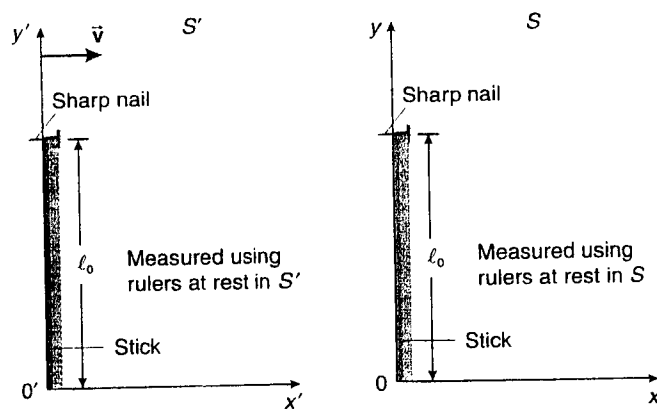


FIGURE 25.16 Two sticks of equal length, each measured in their respective reference frames. Note that  $S'$  is here on the left, the origins have not yet coincided, so both  $t$  and  $t'$  are  $< 0$  s, but we are not concerned with time here, only length.

- Two sticks pass each other.
- Assume moving stick  $\perp$  motion is longer than stick at rest.

### S-Frame

- Moving stick in  $S'$  is longer
- When sticks pass, nail in  $S$ -stick makes a mark in  $S'$
- Nail in  $S'$  misses  $S$ .

- Stick in  $S'$  has a scratch on it.
- Stick in  $S$  has no scratch.

$S'$ -Frame :

- Stick in  $S$  is moving and is longer
- When sticks pass, nail in  $S'$ -stick makes a mark in  $S$ -stick
- Nail on  $S$  misses  $S'$ .
- Stick in  $S$  has a mark.
- Stick in  $S'$  has no mark.

Results of measurement communicated.

Contradictory  
 Conclusion: Lengths measured  $\perp$  direction of motion are unaffected by the motion.

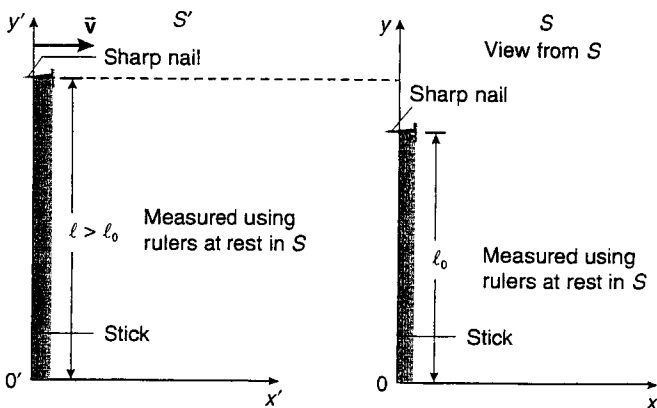


FIGURE 25.17 The view from  $S$ : frame  $S'$  is moving. Hypothesis: Frame  $S$  measures the moving stick (in  $S'$ ) to be longer than the stick at rest in  $S$ .

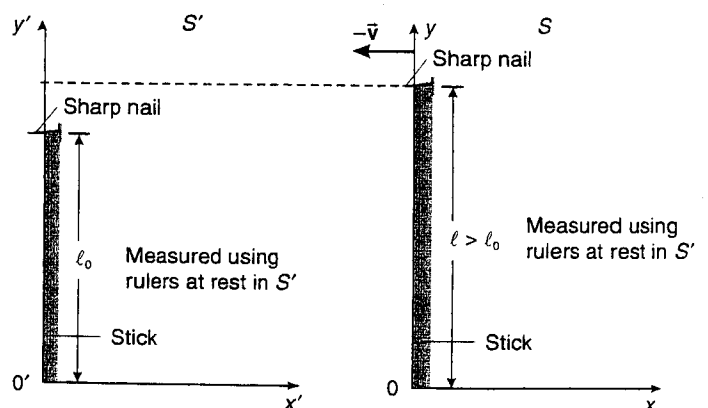


FIGURE 25.18 The view from  $S'$ : frame  $S$  is moving. Hypothesis: Frame  $S'$  measures the moving stick (in  $S$ ) to be longer than the stick at rest in  $S'$ .

## Orientation of Moving Rod

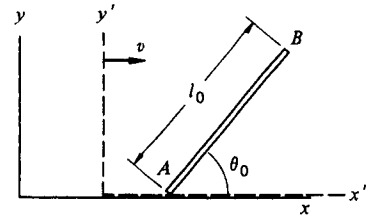
R3-12

S'-Frame : Velocity  $v$

- Rod of length  $L_0$
- Angle  $\theta_0$  wrt  $x'$ -axis
- Ends of rod:

$$A: \quad x'_A = 0 \quad y'_A = 0$$

$$B: \quad x'_B = L_0 \cos \theta_0 \quad y'_B = L_0 \sin \theta_0$$



S-Frame

$$A: \quad x_A = 0 = \gamma(x'_A - vt) \quad y_A = 0 = y'_A$$

$$x_B = L_0 \cos \theta_0 = \gamma(x'_B - vt)$$

$$y_B = L_0 \sin \theta_0 = y'_B$$

$$x_B - x_A = \frac{L_0 \cos \theta_0}{\gamma}$$

$$y_B - y_A = L_0 \sin \theta_0$$

The length is

$$\begin{aligned} L &= \left[ (x_B - x_A)^2 + (y_B - y_A)^2 \right]^{1/2} \\ &= L_0 \left[ \left(1 - \frac{v^2}{c^2}\right) \cos^2 \theta_0 + \sin^2 \theta_0 \right]^{1/2} \\ &= L_0 \left[ 1 - \frac{v^2}{c^2} \cos^2 \theta_0 \right]^{1/2} \end{aligned}$$

The angle the rod makes to the x-axis

$$\theta = \arctan \frac{y_B - y_A}{x_B - x_A}$$

$$= \arctan \gamma \left( \frac{\sin \theta_0}{\cos \theta_0} \right)$$

$$= \arctan (\gamma \tan \theta_0)$$

$\therefore$  moving rod is contracted and rotated

$$\theta > \theta_0 !!$$

# Pole-Vaulter and the Barn

R3-14

Frame - S : Barn Length:  $L_{ob} = 10\text{ m}$   
 Frame - S' : Pole-Vaulter Length:  $L_{op} = 20\text{ m}$

$$u = \frac{\sqrt{15}}{4} c ; \quad \gamma = 4$$

- Farmer sees a pole of length  $\frac{L_{op}}{\gamma} = 5\text{ m}$ .
- Plan is to close left door then open right door.  
Trapping pole.
- Pole-vaulter sees barn length  $\frac{L_{ob}}{4} = 2.5\text{ m}$ .
- Pole does not fit!
- Paradox !!

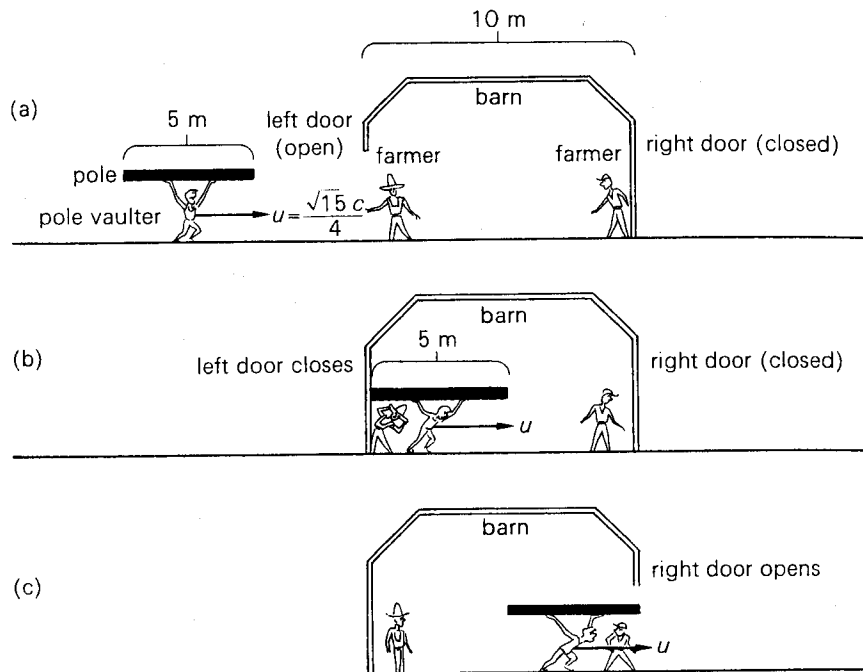


Figure 14-11. The pole passing through the barn, as described by the farmers. The left door closes before the right door opens. (a) Before the pole reaches the barn, the left door is open and the right door is closed. (b) When the pole is entirely in the barn, the left door is closed. (c) The right door is opened, allowing the pole to emerge.

Assume  $t=t'=0$  when  $x=x'=0$

Barn: Frame - S

Event-1: Left door closes when left end of pole enters barn.

$$t_1 = 0, \quad x_1 = 0$$

At  $t_1$ , right end of pole at  $x = 5\text{m}$ .

Event-2: Right end door opens when front of pole reaches there moving with speed  $u$ .

$$t_2 = \frac{5\text{m}}{u} = \frac{20}{\sqrt{15}} \frac{1}{c} \quad x_2 = 10\text{m}$$

Frame - S'

Event-1: Left door closes;  $x'_1 = 0, t'_1 = 0$

Event-2: Right door opens;  $x'_2 = 20\text{m}, t'_2 = ?$

Event-1: left door closes

$$x_1 = 0 \quad x'_1 = 0$$

$$t_1 = 0 \quad t'_1 = 0$$

Event-2: right door opens

$$x_2 = 10\text{m} \quad x'_2 = 20\text{m}$$

$$t_2 = \frac{20}{\sqrt{15}} \frac{1}{c} \quad t'_2 = ?$$

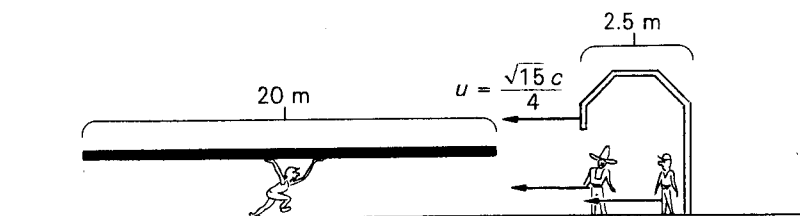


Figure 14-12. The situation before the barn reaches the pole, as described by the pole vaulter.

Lorentz transformation;

$$t'_2 = \gamma \left( t_2 - \frac{u}{c^2} x_2 \right) = 4 \left( \frac{20m}{\sqrt{15}c} - \frac{\sqrt{15}c}{4c^2} \times 10m \right)$$

$$= \frac{4}{c} \left[ \frac{20m}{\sqrt{15}} - \frac{\sqrt{15}}{4} \times 10m \right]$$

$$= \frac{4}{3 \times 10^8} (5.16 - 9.69) = -6.0 \times 10^{-8} \text{ sec}$$

$t'_2 < 0 !!$

In  $S'$  Event -2 occurs before Event -1.  
 Right door opens before left door closes.  
 Paradox resolved!!

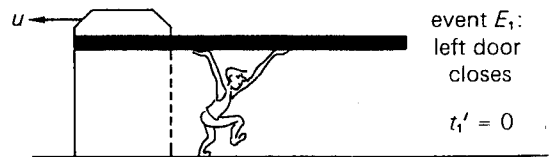
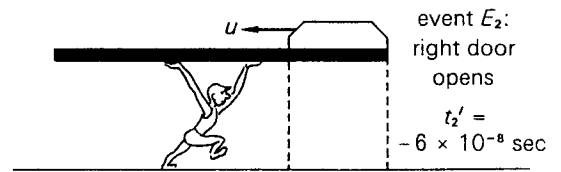
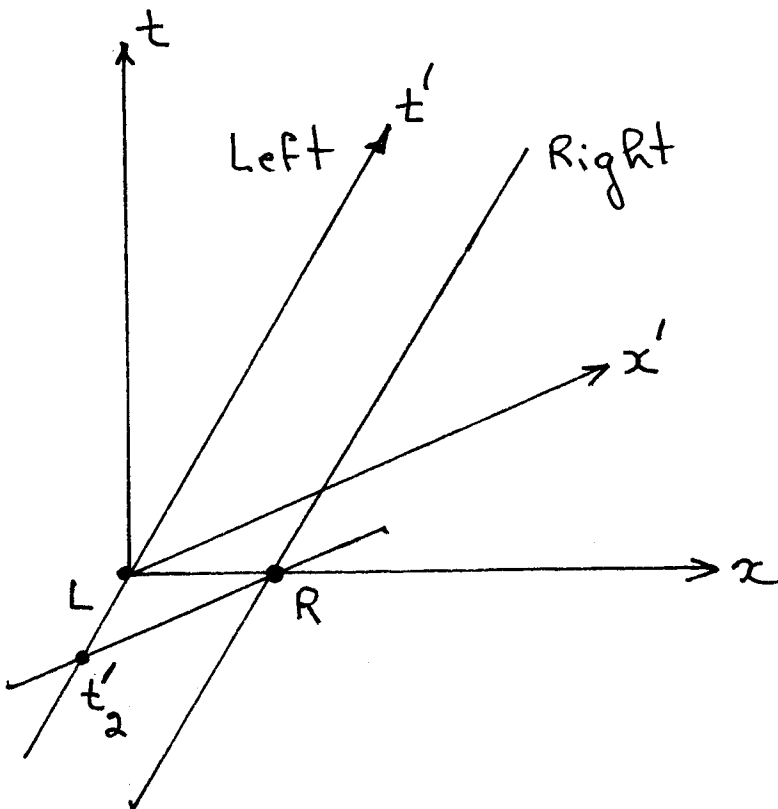


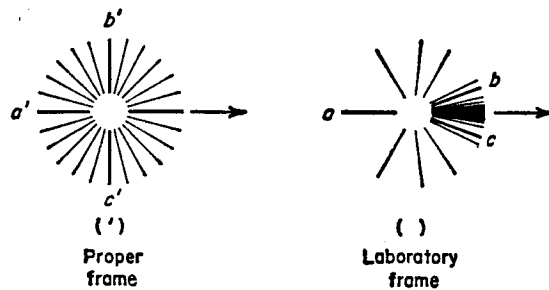
Figure 14-15. The sequence of events as observed by the pole vaulter. The left door closes after the right door opens.



# Headlight Effect

R3-16

- Light beam in rest frame  $S'$  is emitted at angle  $\theta^\circ$  wrt  $x'$ .
- What is angle  $\theta$  in frame  $S$  wrt which  $S'$  is moving with  $v$ .



- In  $S'$  after 1-second

$$x' = c \cos \theta_0$$

$$y' = c \sin \theta_0$$

- The corresponding coordinates in Frame-S are;

$$x = \gamma(x' + vt') = \gamma(c \cos \theta_0 + v)$$

$$y = y'$$

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right) = \gamma\left(1 + \frac{v}{c} \cos \theta_0\right)$$

$$\cos \theta = \frac{x}{ct} = \frac{\gamma(c \cos \theta_0 + v)}{\gamma(c + v \cos \theta_0)} = \frac{\cos \theta_0 + v/c}{1 + \frac{v}{c} \cos \theta_0} \quad \text{R3-17}$$

- Assume in  $S'$  rays are emitted uniformly in all directions.
- Half-rays contained within  $\theta_0 = \pm \frac{\pi}{2}$

For  $\theta_0 = \pm \pi/2$

$$\cos \theta = v/c$$

As  $v \rightarrow c$   $\cos \theta \sim 1 \quad \therefore \theta \sim 0^\circ$   
 Light is very forward peaked!  
 Synchrotron radiation from relativistic electrons

Example:

$$\beta = 0.9$$

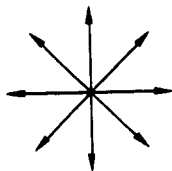
$$\cos \theta = 0.9$$

$$\theta = 25.8^\circ$$

$$\beta = 0.99$$

$$\cos \theta = 0.99$$

$$\theta = 8.1^\circ$$



Rest frame

