MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 Spring 2005

FINAL EXAM

Tuesday, May 17, 2005

FAMILY (LAST) NAME

GIVEN (FIRST) NAME

STUDENT ID NUMBER

Please check (✔) your class ☞

	L01	MTW 10:00	Walter Lewin
	L02	MTW 11:00	Walter Lewin
	L03	MTW 2:00	Min Chen
	L04	MTW 3:00	Min Chen

INSTRUCTIONS:

1. The FORMULA SHEET is in the back of this exam. You may tear it off. There is also an extra BLANK PAGE in case you need it.
2. This is a closed book exam. CALCULATORS, BOOKS, and NOTES are NOT ALLOWED.
3. Unless otherwise stated, to earn full credit you must show a valid DERIVATION and/or EXPLANATION of your answer, and you must express it in terms of the GIVEN VARIABLES.

Part I: Quiz 13

Problem	**Maximum**	**Score**	**Grader**
1	20		
2	30		
3	35		
4	15		
TOTAL	100		

Part II: Make-Up Exam

Problem	**Maximum**	**Score**	**Grader**
5	20		
6	35		
7	30		
8	35		
9	30		
10	35		
11	15		
TOTAL	200		

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FINAL EXAM CORRECTIONS

Tuesday, May 17, 2005

Problem 8a:

Express your answer in terms of P_E, M_E, and R_E.

Problem 9:

Assume that the mass of the spring is negligible.

Problem 10c:

The skater is twirling about a **fixed** axis, and L denotes the angular momentum about that axis.

Note on Procedure:

If you decide that you do not want us to grade the make-up part of the exam, put an X over the grade box for the Make-Up Exam on the first sheet of the exam. Remember, however, that if you want us to grade what you have written, there is no penalty. The Make-Up part of the final will count only if it brings your average up.

Name ____________________

PART I: QUIZ 13

Part I is the first of two parts. It replaces Quiz 13, and will count toward your final grade the same as any other weekly quiz. The only difference is that lowest of the first 12 quizzes will be dropped, while Quiz 13 will not be dropped. Part I is intended to be one third of the exam, but you are free to divide your time as you wish. Part II, which is optional, starts on p. 12.

Problem 1: Basic concepts about special relativity (*20 points*)

(a) (*5 points*) Which of the following statements best describes the relativistic definition of momentum and its relationship to the Newtonian definition, $\vec{\mathbf{p}} \equiv m\vec{\mathbf{v}}$?

(i) The relativistic definition of momentum is the same as the Newtonian definition of the momentum.

(ii) The Newtonian definition is consistent with all the principles of relativity, but experimentally it has been found to fail at velocities near the speed of light. It therefore must be replaced by the relativistic definition.

(iii) The Newtonian definition of momentum is inconsistent with the principles of relativity, because the relativistic transformation equations (i.e., the Lorentz transformations) imply that if the Newtonian momentum were conserved in one inertial frame of reference, it would not be conserved in other inertial frames of reference.

(iv) The relativistic definition of momentum is different from the Newtonian definition because in Newtonian mechanics the total momentum of a system is always conserved when no external forces are acting, while in relativity it is not.

(v) The relativistic definition of momentum differs from the Newtonian definition very significantly for low velocities, but the two definitions are approximately equal for velocities approaching that of light.

— Problem 1 Continues —

(b) (*5 points*) Which of the following statements best describes the relativistic definition of velocity and its relationship to the Newtonian definition, $\vec{\mathbf{v}} \equiv \mathrm{d}\vec{\mathbf{x}}/\mathrm{d}t$?

(i) The relativistic definition of velocity is the same as the Newtonian definition of the velocity.

(ii) The Newtonian definition is consistent with all the principles of relativity, but experimentally it has been found to fail at velocities near the speed of light. It therefore must be replaced by the relativistic definition.

(iii) The relativistic definition of velocity differs from the Newtonian definition very significantly for low velocities, but the two definitions are approximately equal for velocities approaching that of light.

(iv) The relativistic definition of velocity differs from the Newtonian definition very significantly for high velocities, comparable to that of light, but the two definitions are approximately equal for velocities very small compared to that of light.

(c) (*5 points*) Which of the following statements best describes the behavior of the velocity and energy of a relativistic particle (of nonzero rest mass) as it is accelerated to very high velocities?

(i) As in Newtonian physics, both the velocity and energy can in principle increase without limit. The velocity and energy are limited only by the capacity of the device that is causing the acceleration.

(ii) The velocity of the particle is limited by the speed of light, which can never be reached, while the energy can grow arbitrarily large, limited only by the capacity of the accelerating device.

(iii) Both the velocity and the energy of the particle have finite limits, which are the speed of light and the corresponding energy. No matter what device is used to accelerate the particle, the speed can never quite reach the (finite) speed of light, and the energy can never quite reach the (finite) energy corresponding to travel at light speed.

(iv) The energy of the particle is limited by a certain maximal value, which can never be reached, while the speed can grow arbitrarily large, limited only by the capacity of the accelerating device.

— *Problem 1 Continues* —

(d) (*5 points*) A muon is a particle like an electron, except that its mass is about 207 times larger. A high energy electron (e^-) can collide with a high energy positron (e^+), resulting in the annihilation (disappearance) of the two particles, producing a muon (μ^-) and an antimuon (μ^+). If the process happens in empty space, with no external forces acting, which of the following statements correctly described the conservation laws that apply? Circle as many answers as you believe to be valid. *Hint: For a relativistic particle there are three kinds of energy that can be defined: the rest energy, the kinetic energy, and the total energy, where the total energy is the sum of the rest energy and the kinetic energy.*

(i) The total momentum and the total kinetic energy are conserved.

(ii) The total momentum and the total energy are conserved.

(iii) The total momentum and the total rest energy are conserved.

(iv) The total momentum is not conserved, but the total kinetic energy is.

(v) The total momentum is not conserved, but the total energy is.

(vi) The total momentum is not conserved, but the total rest energy is.

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Problem 2: Relativistic Momentum vs. Newtonian Momentum (*30 points*)

For the following questions, you are not expected to numerically simplify your answers. For example, an expression such as

$$c\sqrt{1+\left(\frac{5}{183}\right)^{7/2}+\left(\frac{17}{365}\right)^{4/11}+\frac{\pi}{2}}$$

would be a perfectly acceptable, albeit implausible, answer.

(a) (*10 points*) At what speed is the momentum of a particle three times as great as the result obtained from the nonrelativistic expression mv? Express your answer in terms of the speed of light.

(b) (*10 points*) A force is applied to a particle along its direction of motion. At what speed is the magnitude of force required to produce a given acceleration three times as great as the force required to produce the same acceleration when the particle is at rest? Express your answer in terms of the speed of light.

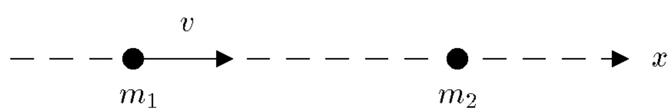
(c) (*10 points*) A force is applied to a particle perpendicular to its direction of motion. At what speed is the magnitude of force required to produce a given acceleration three times as great as the force required to produce the same acceleration when the particle is at rest? Express your answer in terms of the speed of light.

Name

Name

Problem 3: A Totally Inelastic Particle Collision (*35 points*)

A particle of rest mass m_1 moves with relativistic speed v along the x-axis, in the positive direction. It collides with a particle of rest mass m_2, which is at rest. The two stick together, and continue to move as one particle.



In the following sequence of questions, you may express the answers to each part in terms of the original given variables and/or the answers to any previous part, whether or not you correctly answered the previous part. You need not carry out the algebra to express each answer in terms of the original given variables.

(a) (*5 points*) What is $\vec{\mathbf{P}}_i$, the total momentum of the two-particle system, before the collision? Express your answer in terms of some or all of the variables m_1, m_2, and v. Here "i" stands for "initial".

(b) (*5 points*) What is the total energy E_i of the two-particle system before the collision? Again express your answer in terms of some or all of the variables m_1, m_2, and v.

(c) (*5 points*) After the collision, what is the energy E_f and the momentum $\vec{\mathbf{P}}_f$ of the single particle. Here "f" stands for "final". You may express your answer in terms of some or all of the variables m_1, m_2, v, $\vec{\mathbf{P}}_i$, and E_i.

(d) (*5 points*) What is the x-velocity u ($u \equiv dx/dt$) of the final particle? You may express your answer in terms of some or all of the variables m_1, m_2, v, $\vec{\mathbf{P}}_i$, E_i, $\vec{\mathbf{P}}_f$, E_f.

(e) (*5 points*) What is the rest mass M of the final particle?

(f) (*5 points*) Is the rest mass M of the final particle larger than, smaller than, or equal to $m_1 + m_2$. Explain briefly how you know this.

(g) (*5 points*) Suppose the collision is observed from a frame of reference that moves to the right at speed u (the same u found in part (d)). Let $\vec{\mathbf{k}}_1$ and $\vec{\mathbf{k}}_2$ denote the momenta of the two initial particles, as seen in this frame. State whether $\left|\vec{\mathbf{k}}_1\right| > \left|\vec{\mathbf{k}}_2\right|$, $\left|\vec{\mathbf{k}}_1\right| < \left|\vec{\mathbf{k}}_2\right|$, or $\left|\vec{\mathbf{k}}_1\right| = \left|\vec{\mathbf{k}}_2\right|$, and explain briefly how you know this.

Name ______________________________

Problem 4: Doppler Shift with Two Spaceships and a Station (*15 points*)

Two spaceships, the Excelsior and the Reliant, are each approaching Star Base 12 from exactly opposite directions. Each spaceship is moving at speed v relative to Star Base 12.

(a) (*7 points*) The Excelsior sends a radio signal with wavelength λ. What is the wavelength λ' of the radio signal as it is received by Star Base 12?

(b) (*8 points*) The radio signal continues past Star Base 12, and is soon received by the Reliant. What is the wavelength λ'' of the radio signal as it is received by the Reliant? *Hint: when the radio signal passes Star Base 12, you can imagine that it is received at wavelength λ' and then retransmitted at the same wavelength.*

— End of Part I of Final Exam —

Name

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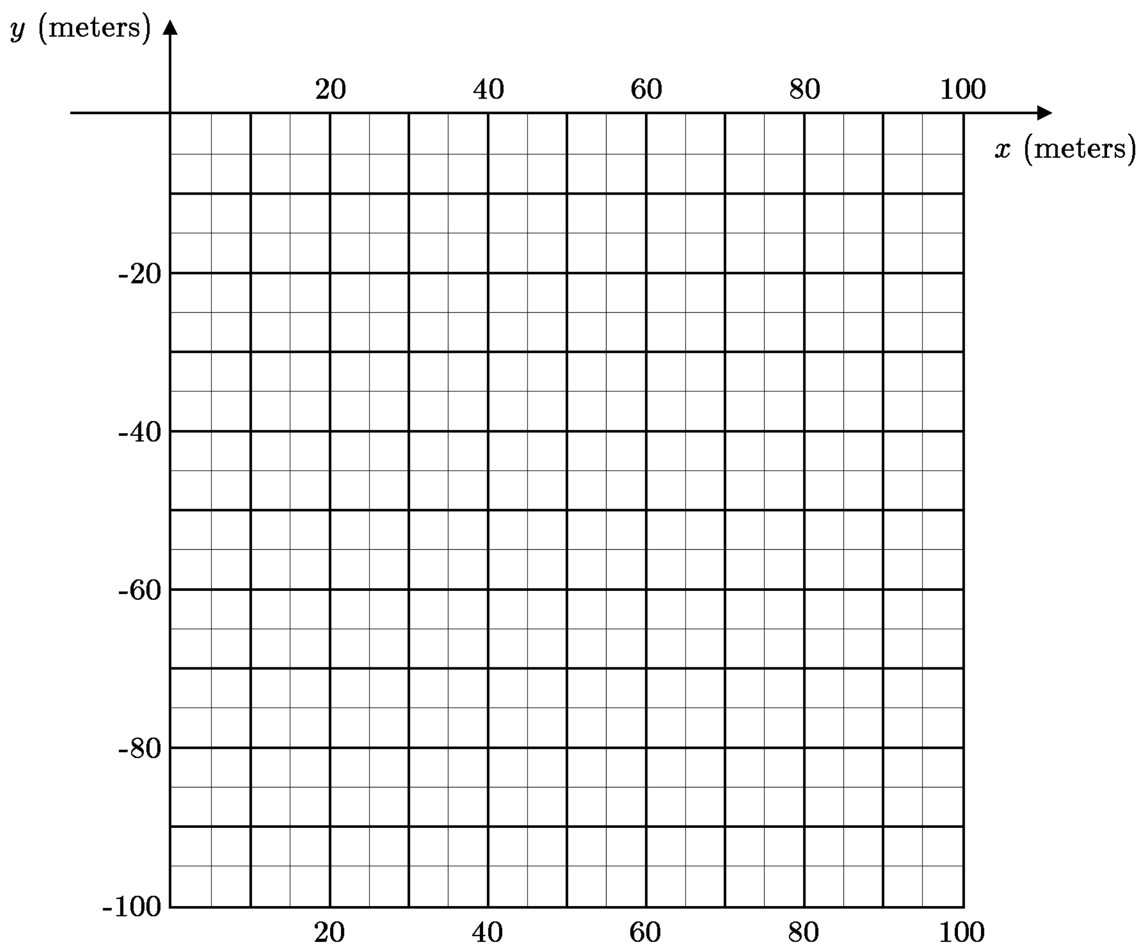
PART II: MAKE-UP

Part II is the second of two parts. It is OPTIONAL, and serves as a make-up quiz for the 12 weekly quizzes that took place during the term. This part can only raise your grade, and cannot lower it. If your average for this make-up is higher than your average for the first 12 weekly quizzes, after dropping one of them, then your average for the first 12 weekly quizzes will be replaced by 0.35 × (Make-up grade) + 0.65 × (Original grade), where all the grades have been scaled to a maximum of 100.

Problem 5: A Simple Trajectory (*20 points*)

A ball is thrown from a cliff, with an initial velocity that is exactly horizontal, with a magnitude of 20 meter/second. It travels under the influence of gravity, and for numerical simplicity we use the approximate value $g = 10$ meter/second^2 for the acceleration of gravity. Use a coordinate system in which x axis is horizontal and the y axis is directed upward. The ball is thrown from [0, 0, 0], and travels initially in the positive x-direction. Ignoring all frictional effects, indicate the trajectory followed by the ball by putting a dot on the following graph at the location of the ball at $t = 1$, 2, 3, and 4 seconds.

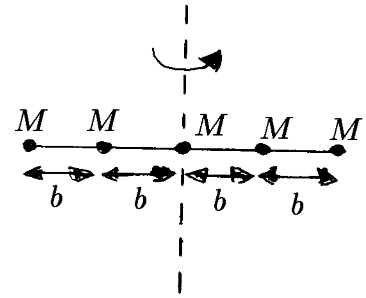


Name

Problem 6: Assorted Multiple Choice Problems (*35 points*)

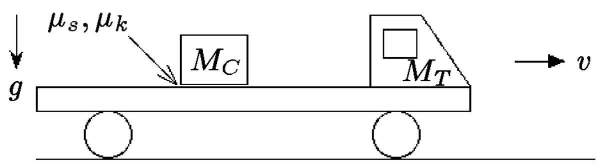
Mark your answer by circling it. For these multiple choice questions you need not show your work, and there will be no partial credit, except as indicated in part (e). *Hint: the right answer is not a leafy vegetable. (Don't say that we don't try to help you out.)*

(a) (*6 points*) Five small balls, each of mass M, are attached to a massless rigid rod of length $4b$. One ball is at the center, one ball is at each end, and one ball is a distance b from the center in each direction, as shown. What is the moment of inertia of this object for rotation about an axis through the center of the rod and perpendicular to it?



(i) $2Mb^2$ (ii) $5Mb^2$ (iii) $8Mb^2$ (iv) $10Mb^2$ (v) $12Mb^2$ (vi) asparagus

(b) (*6 points*) A crate of mass M_C rests on a flatbed truck of mass M_T. The coefficients of static and kinetic friction between the crate and the truck are μ_s and μ_k, respectively. The acceleration of gravity is g. If the truck is moving in a straight horizontal line at constant speed v, what is the magnitude of the frictional force that the truck exerts on the crate?

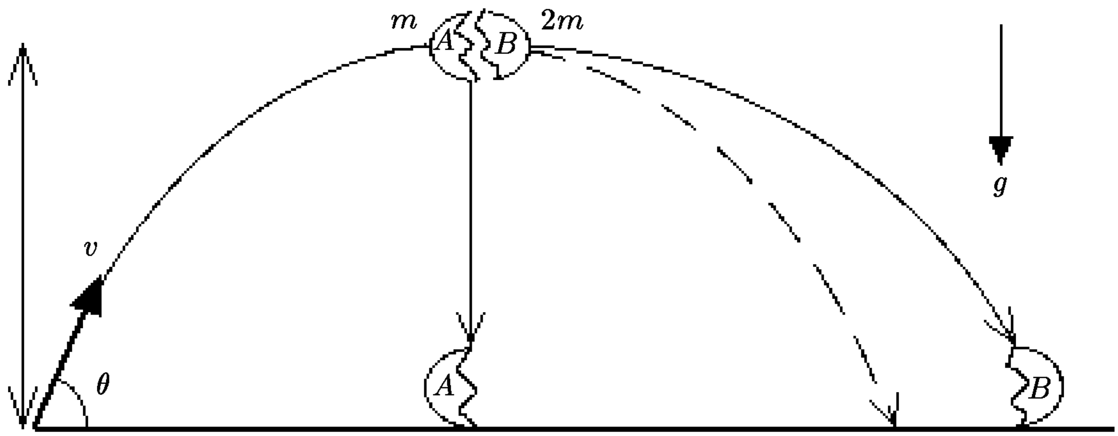


(i) $\mu_k M_C g$ (ii) $\mu_s M_C g$ (iii) $\mu_k M_T g$ (iv) $\mu_s M_T g$ (v) zero (vi) broccoli

— *Problem 6 continues on the next page* —

Name

Problem 6 Continued:



(c) (*6 points*) A projectile is launched from the origin with speed v at an angle θ from the horizontal. At the highest point in the trajectory, the projectile breaks into two pieces, A and B, of masses m and $2m$, respectively. Immediately after the breakup piece A is at rest relative to the ground. Neglect air resistance. Which of the following sentences most accurately describes what happens next?

(i) Piece B will hit the ground first, since it is more massive.

(ii) Both pieces have zero vertical velocity immediately after the breakup, and therefore they hit the ground at the same time.

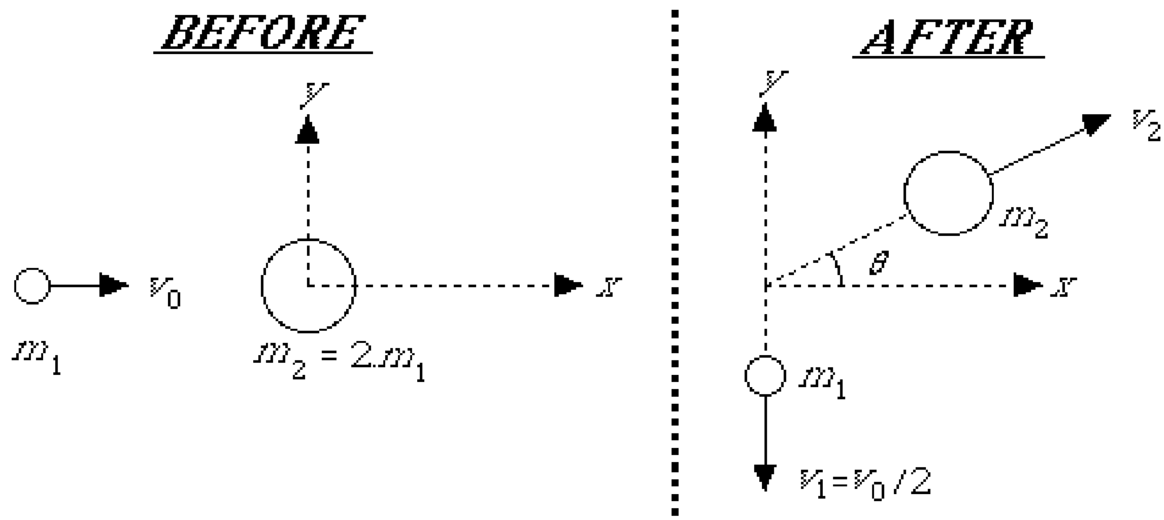
(iii) Piece A will hit the ground first, because it will have a downward velocity immediately after the breakup.

(iv) There is no way of knowing which piece will hit the ground first, because not enough information is given about the breakup.

(v) Spinach.

— *Problem 6 continues on the next page* —

Problem 6 Continued:



(d) (*7 points*) A particle of mass m_1 with initial speed v_0 in the positive x-direction collides with a particle of mass $m_2 = 2m_1$, which is initially at rest at the origin. After the collision the first particle (m_1) moves off with speed $v_1 = \frac{1}{2}v_0$ in the negative y-direction and the second particle (m_2) moves off with speed v_2 at an angle θ, as shown. Which one of the following equations is a valid conservation law?

(i) $m_2 v_2 \sin\theta = \frac{1}{2} m_1 v_0$

(ii) $m_2 v_2 \cos\theta = \frac{1}{2} m_1 v_0$

(iii) $m_1 v_0 = m_2 v_2 + \frac{1}{2} m_1 v_0$

(iv) $m_1 v_0 = m_2 v_2 - \frac{1}{2} m_1 v_0$

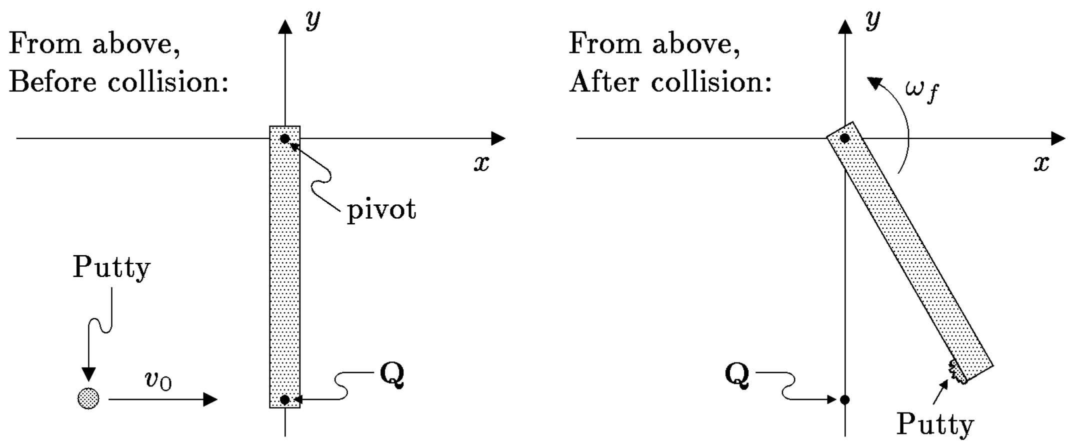
(v) $\frac{1}{2} m_1 v_0^2 = \frac{1}{2} m_1 \left(v_1^2 + v_2^2\right)$

(vi) Cabbage

— *Problem 6 continues on the next page* —

Problem 6 Continued:

(e) (*10 points*) On a frictionless horizontal table a slender rigid rod is attached at one end to a fixed, frictionless pivot. A nonrotating disk of putty is moving with speed v_0 perpendicular to the line of the rod, and collides with it at point Q, at the opposite end from the pivot. The putty sticks to the end of the rod, so that after the collision they move together. Assume that the putty disk is small enough to be treated as a point particle. Circle each of the quantities that are conserved throughout the entire process, from the initial linear motion of the putty through the rotational motion of the rod/putty system at the end. *Partial credit on this question: 4 points off for one error, 8 points off for two errors, no credit for more than two errors.*



(i) Linear momentum of the putty and rod in the x-direction

(ii) Linear momentum of the putty and rod in the y-direction

(iii) Kinetic energy of the putty and rod

(iv) Angular momentum of the putty and rod about the point Q

(v) Angular momentum of the putty and rod about the pivot

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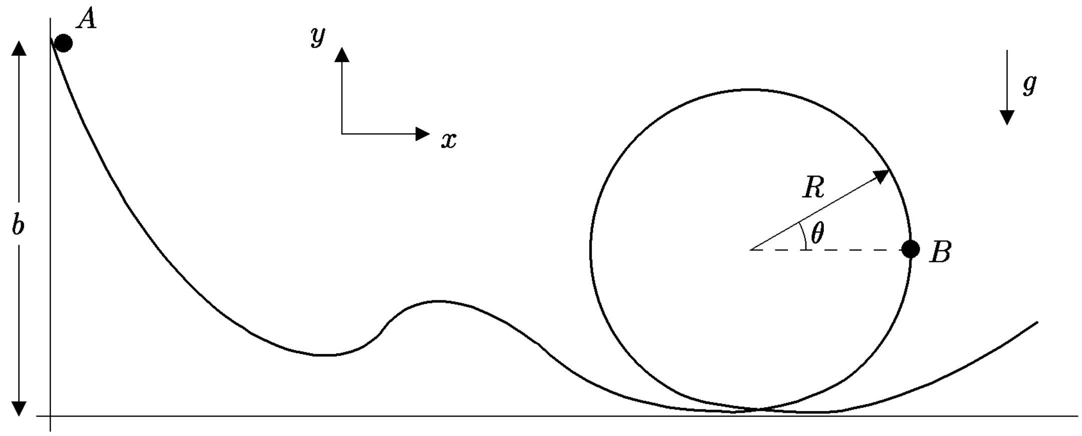
Problem 7: A Puck Sliding on a Frictionless Track (*30 points*)

A small puck of mass m slides along a frictionless track of the following shape:



It starts from rest at point A, at a height b above the ground level. Let g denote the acceleration of gravity. The track includes a circular loop of radius R, tangential to the ground. The point B lies at the same height as the center of the circle.

(a) (*6 points*) What is the minimum initial height b_B that is needed to allow the puck to reach the point B?

(b) (*6 points*) Assuming that $b > b_B$, what is the speed v_B of the puck as it passes point B? Express your answer in terms of some or all of the variables m, b, R, and g.

(c) (*6 points*) What is the acceleration vector $\vec{\mathbf{a}}_B$ of the puck as it passes point B? You may express your answer in terms of v_B and the given variables.

(d) (*6 points*) If b is large enough, then it will be possible for the puck to complete the full circle without falling off. What is the minimum initial height b_{circle} that would allow this to happen?

(e) (*6 points*) If $b_B < b < b_{\text{circle}}$, then the puck will lose contact with the track at some angle θ, measured from the horizontal, as shown. What is this angle θ? Express your answer in terms of some or all of the variables m, b, R, and g.

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Problem 8: Orbit of the Earth and the Lagrange Point L2 (*35 points*)

The Earth orbits the Sun with a period P_E. (Of course we know that P_E is one year, but please express your answers in terms of P_E, and not its explicit value.) The orbit is actually slightly elliptical, but we will approximate it here as a circle of radius R_E.

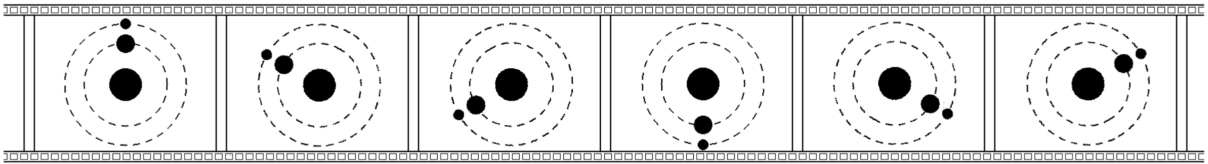
(a) (*7 points*) If the mass of the Earth is M_E, what is the magnitude and direction of the force necessary to hold the Earth in its orbit?

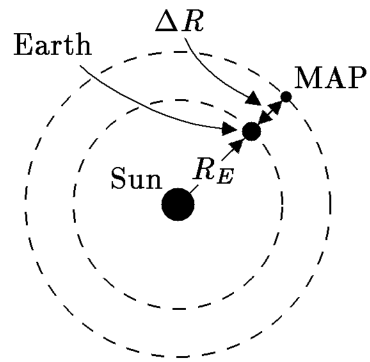
(b) (*7 points*) By applying Newton's laws of motion to the Earth's orbit, express the period P_E of the Earth's orbit in terms of the radius R_E, the mass of the Sun M_S, and Newton's gravitational constant G. (No credit will be given for simply writing down the result.)

— *Problem 8 Continues* —

Name

Problem 8 Continued:





Joseph Lagrange showed in the 1700s that it is possible for a satellite to follow the Earth in its orbit around the Sun, at a slightly larger radius, as shown in the filmstrip above, and in the larger diagram to the right. The satellite would remain a fixed distance ΔR from the center of the Earth, and would always be located along the extension of a radial line from the Sun to the Earth, as shown. This orbit is called L2, and since October 1, 2001, it has been the home of the Wilkinson Microwave Anisotropy Probe (WMAP), a NASA satellite dedicated to high precision measurements of the cosmic microwave background radiation. Note that a satellite at L2 must undergo a slightly larger acceleration than the Earth, but that the extra force can be supplied by the gravitational pull caused by the Earth.

(c) (*7 points*) Using only the variables P_E, R_E, and ΔR, write an expression for the magnitude of the acceleration of the WMAP satellite at L2. What is the direction of the acceleration?

(d) (*7 points*) If WMAP has a mass M_{WMAP}, what is the magnitude and direction of the total force acting on the satellite? Your answer may depend only on the variables R_E, ΔR, M_E, M_S, M_{WMAP}, and G. Note that P_E is **NOT** on this list.

(e) (*7 points*) By applying Newton's laws of motion to the orbit of WMAP, write an equation that must be satisfied by ΔR. The equation might also involve any of the quantities R_E, P_E, M_E, M_S, and G. Do not try to solve this equation for ΔR.

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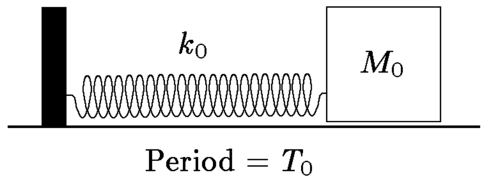
Problem 9: Blocks and Springs (*30 points*)

A block of mass M_0 slides on a frictionless horizontal plane. It is attached to a spring of spring constant k_0, which is attached at its other end to a fixed support. Assume that the spring obeys Hooke's law. The block is observed to slide back and forth along a line with a period T_0.

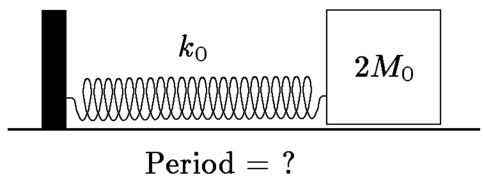


(a) (*7 points*) Suppose the block of mass M_0 is replaced by another block of mass $M = 2M_0$. What is the new period?

(i) $4T_0$ (ii) $2\sqrt{2}T_0$ (iii) $2T_0$

(iv) $\sqrt{2}T_0$ (v) T_0 (vi) $\frac{T_0}{\sqrt{2}}$

(vii) $\frac{T_0}{2}$ (viii) $\frac{T_0}{2\sqrt{2}}$ (ix) $\frac{T_0}{4}$

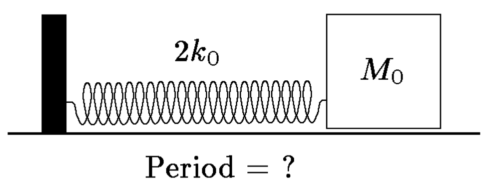


(b) (*7 points*) Suppose that the original block of mass M_0 is restored, but the spring is replaced by one of spring constant $2k_0$. What is the new period in this case?

(i) $4T_0$ (ii) $2\sqrt{2}T_0$ (iii) $2T_0$

(iv) $\sqrt{2}T_0$ (v) T_0 (vi) $\frac{T_0}{\sqrt{2}}$

(vii) $\frac{T_0}{2}$ (viii) $\frac{T_0}{2\sqrt{2}}$ (ix) $\frac{T_0}{4}$



— *Problem 9 Continues* —

Name

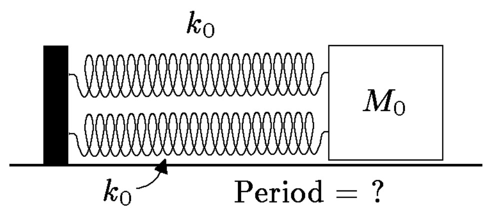
Problem 9 Continued:

(c) (*7 points*) Suppose now that the original spring of spring constant k_0 is replaced, but an additional spring of identical spring constant is attached between the block and the fixed support. What is the new period in this case?

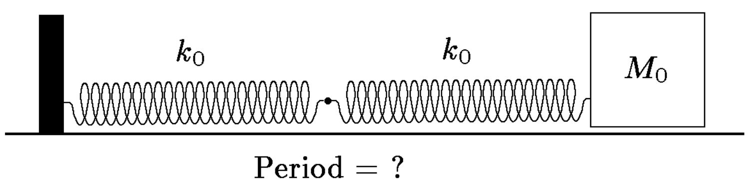
(i) $4T_0$ (ii) $2\sqrt{2}T_0$ (iii) $2T_0$

(iv) $\sqrt{2}T_0$ (v) T_0 (vi) $\frac{T_0}{\sqrt{2}}$

(vii) $\frac{T_0}{2}$ (viii) $\frac{T_0}{2\sqrt{2}}$ (ix) $\frac{T_0}{4}$



(d) (*9 points*) Suppose now that the additional spring is disconnected, but is then placed between the first spring and the block, as shown. What is the new period in this case?



(i) $4T_0$ (ii) $2\sqrt{2}T_0$ (iii) $2T_0$

(iv) $\sqrt{2}T_0$ (v) T_0 (vi) $\frac{T_0}{\sqrt{2}}$

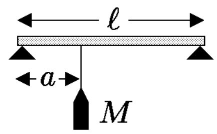
(vii) $\frac{T_0}{2}$ (viii) $\frac{T_0}{2\sqrt{2}}$ (ix) $\frac{T_0}{4}$

Name

Problem 10: Assorted Multiple Choice Questions on Rotational Physics(*35 points*)

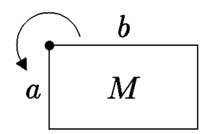
Please mark your answers by circling them. For these multiple choice questions you need not show your work, and there will be no partial credit.

(a) (*7 points*) A horizontal bar of length ℓ and negligible mass is supported at its two ends. A mass M is hung from the bar at a distance a from the left end, as shown. What is the magnitude of the force that the support on the right applies to the bar?



(i) $\frac{1}{2}Mg$ (ii) Mg (iii) $Mg\,\dfrac{\ell}{a}$ (iv) $Mg\,\dfrac{a}{\ell}$ (v) $Mg\,\dfrac{a}{\ell + a}$ (vi) $Mg\,\dfrac{\ell}{\ell + a}$

(b) (*7 points*) A rectangular slab of mass M has uniform thickness and density, and has height a and width b, as shown. The slab is pivoted about its upper left corner, and rotated in the plane of the paper. What is its moment of inertia about this axis?



(i) $\dfrac{1}{3}Ma^2$ (ii) $\dfrac{1}{3}Mb^2$ (iii) $\dfrac{1}{3}M(a^2 + b^2)$ (iv) $\dfrac{2}{3}M(a^2 + b^2)$ (v) $\frac{1}{3}Mab$ (vi) $\frac{2}{3}Mab$

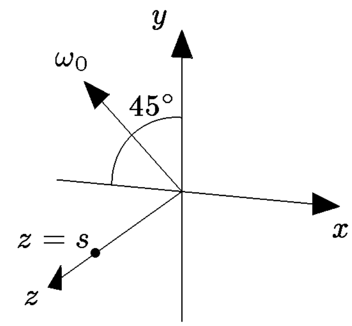
(c) (*7 points*) A skater twirling about a vertical axis pulls her arms inward. Ignoring all frictional effects, which of the following statements are true? You may circle as many as apply. Denote the magnitude of her angular velocity by ω, the magnitude of her angular momentum by L, and her kinetic energy by E_k

(i) L and E_k both increase;
(ii) L increases, E_k is constant;
(iii) L is constant, E_k increases;
(iv) L and E_k are both constant;
(v) L and ω both increase
(vi) L increases, ω is constant;
(vii) L is constant, ω increases;
(viii) L and ω are both constant;

— *Problem 10 Continues* —

Problem 10 Continued:

(d) (*7 points*) An object which is pivoted at the origin rotates with an angular velocity of magnitude ω_0 directed in the x-y plane at 45° from both the y-axis and the negative x-axis, as shown. What is the velocity $\vec{\mathbf{v}}$ of a point on the rotating object that is located along the positive z-axis at a distance s from the origin?



(i) $\omega_0\, s\,[1, 1, 0]$ (ii) $\omega_0\, s\,[1, -1, 0]$

(iii) $\omega_0\, s\,[-1, 1, 0]$ (iv) $\omega_0\, s\,[-1, -1, 0]$

(v) $\dfrac{\omega_0\, s}{\sqrt{2}}[1, 1, 0]$ (vi) $\dfrac{\omega_0\, s}{\sqrt{2}}[-1, 1, 0]$

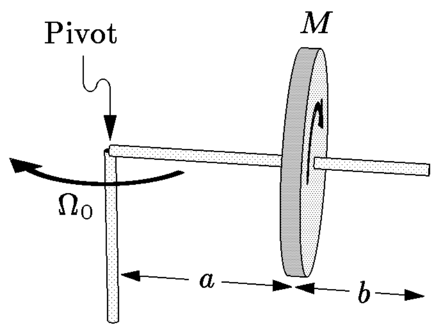
(e) (*7 points*) An unknown comet has a highly elliptical orbit about the Sun; its maximum distance from the center of the Sun is 100 times larger than its minimum distance. If its speed at perihelion (the point of closest approach to the Sun) is 20 km/s, what is its speed at aphelion (the farthest point in its orbit from the Sun)?

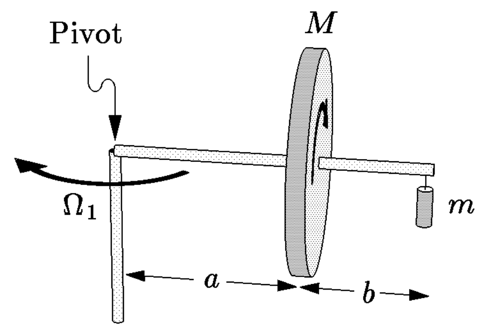
(i) 2,000 km/s (ii) 200 km/s (iii) 20 km/s (iv) 2 km/s (v) 0.2 km/s (vi) 0.02 km/s

Problem 11: A Spinning Wheel Gyroscope (*15 points*)

A gyroscope consists of a spinning wheel, rotating freely about a horizontal axle at a distance a from a pivot, as shown. The axle extends a distance b beyond the wheel. Assume that the wheel has a mass M, and the mass of the axle is negligible. The axle is free to rotate freely about the pivot in the horizontal plane. Take the acceleration of gravity as g, with $g > 0$.

The gyroscope is spinning rapidly about the axle, and the axle is observed to precess about the vertical axis with an angular velocity Ω_0. If a small weight of mass m is hung from the tip of the axle, without disturbing the rotational speed of the wheel about its axis, what will be the new angular velocity Ω_1 of the precession?





— End of Final Exam —

Name

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 Spring 2005

FINAL EXAM
FORMULA SHEET

Final Exam Date: Tuesday, May 17, 2005

For motion in one dimension:

$$v_{\rm av} \equiv \frac{\Delta x}{\Delta t} \qquad \text{Average velocity;}$$

$$v \equiv \frac{{\rm d}x}{{\rm d}t} \qquad \text{Instantaneous velocity;}$$

For motion in three dimensions:

$$\vec{\mathbf{v}} \equiv \frac{{\rm d}\vec{\mathbf{r}}}{{\rm d}t}\,; \qquad \vec{\mathbf{a}} \equiv \frac{{\rm d}\vec{\mathbf{v}}}{{\rm d}t} = \frac{{\rm d}^2\vec{\mathbf{r}}}{{\rm d}t^2}\,; \qquad \vec{\mathbf{r}}(t_1) = \vec{\mathbf{r}}_0 + \int_0^{t_1} \vec{\mathbf{v}}\,{\rm d}t\,; \qquad \vec{\mathbf{v}}(t_1) = \vec{\mathbf{v}}_0 + \int_0^{t_1} \vec{\mathbf{a}}\,{\rm d}t\ .$$

For *constant* acceleration $\vec{\mathbf{a}}$, if $\vec{\mathbf{r}} = \vec{\mathbf{r}}_0$ and $\vec{\mathbf{v}} = \vec{\mathbf{v}}_0$ at time $t = 0$, then

$$\vec{\mathbf{v}}(t) = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t$$

$$\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \frac{1}{2}\vec{\mathbf{a}}t^2\ .$$

For one-dimensional motion with constant acceleration a:

$$v^2 = v_0^2 + 2a(x - x_0)\ .$$

For circular motion at constant speed v:

$$a = \frac{v^2}{r}\ ,$$

where r is the radius of the circle, and the acceleration is directed towards the center of the circle.

If an object has position $\vec{\mathbf{r}}$ and velocity $\vec{\mathbf{v}}$, its position and velocity relative to an observer with position $\vec{\mathbf{r}}_0$ and velocity $\vec{\mathbf{v}}_0$ are given respectively by

$$\vec{\mathbf{r}}' = \vec{\mathbf{r}} - \vec{\mathbf{r}}_0\ , \qquad \vec{\mathbf{v}}' = \vec{\mathbf{v}} - \vec{\mathbf{v}}_0\ .$$

Average velocity and acceleration are given by

$$\vec{\mathbf{v}}_{\rm average} \equiv \frac{\Delta\vec{\mathbf{r}}}{\Delta t}\ , \qquad \vec{\mathbf{a}}_{\rm average} \equiv \frac{\Delta\vec{\mathbf{v}}}{\Delta t}\ .$$

Mass, Acceleration, and Force:

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} \qquad \text{(Newton's second law);}$$

$$\vec{\mathbf{F}} = -\frac{GMm}{r^2}\hat{\boldsymbol{r}} \qquad \text{(the gravitational force between two particles);}$$

$$\vec{\mathbf{F}} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}\hat{\boldsymbol{r}} \qquad \text{(the electrostatic force between two particles);}$$

$$F_x = -kx \qquad \text{(Hooke's law);}$$

where $\hat{\boldsymbol{r}}$ is a unit vector pointing from the particle which is the source of the force, toward the particle on which the force is acting.

Friction:

$$\left|\vec{\mathbf{F}}_k\right| = \mu_k \left|\vec{\mathbf{N}}\right| \qquad \text{(kinetic friction);}$$

$$\left|\vec{\mathbf{F}}_s\right| \le \mu_s \left|\vec{\mathbf{N}}\right| \qquad \text{(static friction).}$$

Kinetic Energy, Work, and Potential Energy:

Description	**1 Dimension**	**3 Dimensions**
Work done by a constant force $\vec{\mathbf{F}}$	$W \equiv F\Delta x$	$W \equiv \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{\Delta r}}$
Work done by a varying force $\vec{\mathbf{F}}$	$W \equiv \int F(x)\,\mathrm{d}x$	$W \equiv \int_{\vec{\mathbf{r}}_1}^{\vec{\mathbf{r}}_2} \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{dr}}$
Potential energy derived from force $\vec{\mathbf{F}}$	$U(x_p) \equiv U_0 - \int_{x_0}^{x_p} F\,\mathrm{d}x$	$U(\vec{\mathbf{r}}_p) \equiv U_0 - \int_{\vec{\mathbf{r}}_0}^{\vec{\mathbf{r}}_p} \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{dr}}$
Force derived from potential energy	$F = -\frac{\mathrm{d}U}{\mathrm{d}x}$	$\vec{\mathbf{F}} = \left[-\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y}, -\frac{\partial U}{\partial z}\right]$

$$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \equiv |\vec{\mathbf{a}}||\vec{\mathbf{b}}| \cos\theta = a_x b_x + a_y b_y + a_z b_z$$ (scalar (or dot) product of two vectors);

$$E_k \equiv \frac{1}{2}mv^2$$ (kinetic energy of a particle);

$$W_{\rm tot} = E_{k,f} - E_{k,i}$$ (work-energy theorem: always true if $W_{\rm tot}$ includes work due to all forces; a non-rigid object can do work on itself!);

$$E_{k,i} + U_i + W_{\rm other} = E_{k,f} + U_f$$ (generalized work-energy theorem: always true if $W_{\rm other}$ includes work due to all forces not included in U);

$$\frac{1}{2}mv^2 + U(x) = \text{constant}$$ (conservation of mechanical energy: true in the absence of dissipative forces);

$$\frac{1}{2}mv^2 + mgh = \frac{1}{2}mv_0^2$$ (conservation of mechanical energy for a projectile: true in the absence of dissipative forces);

$$W = \frac{1}{2}kx^2$$ (work to compress a spring);

$$U = \frac{1}{2}kx^2$$ (potential energy for spring force);

$$W = mgh$$ (work to lift a body near the surface of the Earth);

$$U = mgh$$ (gravitational potential energy, near the surface of the Earth);

$$U = -\frac{GMm}{r}$$ (gravitational potential energy, spherical bodies);

$$U = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r}$$ (electrostatic potential energy, spherical charges).

Momentum, Center of Mass, and Systems of Particles:

$$\vec{\mathbf{F}}_{\rm AB} = -\vec{\mathbf{F}}_{\rm BA}$$ (Newton's third law);

$$\vec{\mathbf{p}} \equiv m\vec{\mathbf{v}}$$ (momentum);

$$\frac{\mathrm{d}\vec{\mathbf{P}}_{\rm tot}}{\mathrm{d}t} = 0$$ (conservation of momentum in absence of external force)

$$\vec{\mathbf{F}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}$$ (Newton's second law in terms of momentum);

$$\vec{\mathbf{r}}_{\rm cm} \equiv \frac{1}{M_{\rm tot}}\sum_i m_i\vec{\mathbf{r}}_i$$ (position of center of mass);

$$\vec{\mathbf{v}}_{\rm cm} \equiv \frac{\mathrm{d}\vec{\mathbf{r}}_{\rm cm}}{\mathrm{d}t} = \frac{1}{M_{\rm tot}} \sum_i m_i \vec{\mathbf{v}}_i \qquad \text{(velocity of center of mass);}$$

$$\vec{\mathbf{F}}^{\rm ext}_{\rm tot} = M_{\rm tot}\vec{\mathbf{a}}_{\rm cm} = \frac{\mathrm{d}\vec{\mathbf{P}}_{\rm tot}}{\mathrm{d}t} \qquad \text{(acceleration of a system of particles);}$$

$$\vec{\mathbf{P}}_{\rm tot} = \sum_i m_i \vec{\mathbf{v}}_i = M_{\rm tot}\vec{\mathbf{v}}_{\rm cm} \qquad \text{(momentum of a system of particles);}$$

$$K_{\rm tot} = \frac{1}{2} M_{\rm tot} v_{\rm cm}^2 + \sum_i \frac{1}{2} m_i \left(\vec{\mathbf{v}}_i - \vec{\mathbf{v}}_{\rm cm}\right)^2 \qquad \text{(K.E. of a system of particles);}$$

$$\vec{\mathbf{J}} = \int_{t_1}^{t_2} \vec{\mathbf{F}}\,\mathrm{d}t = \int_{t_1}^{t_2} \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}\,\mathrm{d}t = \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1 \qquad \text{(impulse-momentum theorem).}$$

Rotation in Two Dimensions:

Most of the equations for this topic are most easily remembered in the context of the analogous equations for linear motion in one dimension:

TRANSLATION (one dimension)		ROTATION (about fixed axis)	
Name	Symbol	Name	Symbol
Position	x	Orientation	θ
Velocity	$v = \frac{\mathrm{d}x}{\mathrm{d}t}$	Angular velocity	$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}$
Acceleration	$a = \frac{\mathrm{d}v}{\mathrm{d}t}$	Angular acceleration	$\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t}$
Mass	$M = \sum_i m_i$	Moment of inertia	$I = \sum_i m_i R_i^2$
Force	F	Torque	$\boldsymbol{\tau} = F_\perp R$ $= \pm\lvert\vec{\mathbf{F}}\rvert R_\perp$
Force equation	$\sum_i \vec{\mathbf{F}}^{\rm ext} = M\vec{\mathbf{a}}_{\rm cm}$	Torque equation	$\sum_i \boldsymbol{\tau}^{\rm ext} = I\alpha$
Momentum	$p = Mv$	Angular momentum	$L = I\omega$
Kinetic energy	$\frac{1}{2}Mv^2$	Kinetic energy	$\frac{1}{2}I\omega^2$
Work done	$\vec{\mathbf{F}} \cdot \overrightarrow{\boldsymbol{\Delta r}}$	Work done	$\boldsymbol{\tau}\,\Delta\theta$

Other equations about rotation in two dimensions:

$v_r = 0$; $v_\perp = R\omega$	(velocity of point on rotating body);
$a_r = -\dfrac{v^2}{R} = -R\omega^2$; $a_\perp = R\alpha$	(acceleration of point on rotating body);
$v = \pm R\vert\omega\vert$	(rolling without slipping);
$\left.\begin{aligned}\sum \vec{\mathbf{F}}^{\text{ext}} &= M\vec{\mathbf{a}}_{\text{cm}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} \\ \sum \boldsymbol{\tau}^{\text{ext}} &= I_{\text{cm}}\alpha = \frac{\mathrm{d}L}{\mathrm{d}t}\end{aligned}\right\}$	(combined translational and rotational motion);
$K_{\text{tot}} = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2$	(kinetic energy for combined translational and rotational motion);
$I_{\Vert} = I_{\text{cm}} + Md^2$	(parallel-axis theorem);
$I_z = I_x + I_y$	(perpendicular-axis theorem).

TABLE OF STANDARD MOMENTS OF INERTIA:

Slender uniform rod of length ℓ, axis through center and perpendicular to axis of rod	$\frac{1}{12}m\ell^2$
Rectangular plate with dimensions $a \times b$, axis along one of the b edges	$\frac{1}{3}ma^2$
Thin-walled hollow cylinder of radius R, axis along axis of cylinder	mR^2
Uniform solid cylinder of radius R, axis along axis of cylinder	$\frac{1}{2}mR^2$
Thin-walled hollow sphere of radius R, axis through center	$\frac{2}{3}mR^2$
Solid uniform sphere of radius R, axis through center	$\frac{2}{5}mR^2$

Rotations in Vector Notation:

$$c_x = a_y b_z - a_z b_y \ ; \quad c_y = a_z b_x - a_x b_z \ ; \quad c_z = a_x b_y - a_y b_x \ .$$
(vector cross product, component form);

$$|\vec{\mathbf{c}}| = |\vec{\mathbf{a}}||\vec{\mathbf{b}}| \sin\theta$$
(magnitude of vector cross product);

$$\vec{\mathbf{v}} = \vec{\boldsymbol{\omega}} \times \vec{\mathbf{r}}$$
(velocity of atom in rotating body with a fixed point);

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_P + \vec{\boldsymbol{\omega}} \times \left(\vec{\mathbf{r}} - \vec{\mathbf{r}}_P\right)$$
(velocity of atom in rotating body, general case);

$$\vec{\mathbf{L}} = \sum_i \vec{\mathbf{r}}_i \times \vec{\mathbf{p}}_i$$
(angular momentum, as vector product);

$$\vec{\boldsymbol{\tau}} = \sum_i \vec{\mathbf{r}}_i \times \vec{\mathbf{F}}_i$$
(vector torque, as vector product);

$$\vec{\boldsymbol{\tau}} = \frac{\mathrm{d}\vec{\mathbf{L}}}{\mathrm{d}t}$$
(torque equation);

$$\left.\begin{aligned} \sum \vec{\mathbf{F}}^{\text{ext}} &= M\vec{\mathbf{a}}_{\text{cm}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} \\ \sum \vec{\boldsymbol{\tau}}^{\text{ext}} &= \frac{\mathrm{d}\vec{\mathbf{L}}_{\text{cm}}}{\mathrm{d}t} \end{aligned}\right\}$$
(combined translational and rotational motion);

$$\vec{\mathbf{L}} = \vec{\mathbf{r}}_{\text{cm}} \times \vec{\mathbf{p}}_{\text{tot}} + \sum_i \vec{\mathbf{r}}_{\text{rel},i} \times m_i \vec{\mathbf{v}}_{\text{rel},i}$$
(angular momentum decomposition);

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}}_{\text{cm}} \times \vec{\mathbf{F}}_{\text{tot}} + \sum_i \vec{\mathbf{r}}_{\text{rel},i} \times \vec{\mathbf{F}}_i$$
(torque decomposition).

For Static Bodies:

$$\sum \vec{\mathbf{F}}^{\text{ext}} = 0$$
(total external force vanishes);

$$\sum \vec{\boldsymbol{\tau}}^{\text{ext}} = 0$$
(total external torque about ANY point vanishes).

Gravitation:

$$F_g = \frac{Gm_1 m_2}{r^2} \qquad \text{(Newton's law of gravity);}$$

$$g = \frac{Gm_E}{R_E^2} \qquad \text{(Acceleration due to gravity at Earth's surface);}$$

$$U = -\frac{Gm_E m}{r} \qquad \text{(Gravitational potential energy);}$$

$$v = \sqrt{\frac{Gm_E}{r}} \qquad \text{(Speed in cicular orbit);}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}} \qquad \text{(Period of a circular orbit);}$$

$$R_S = \frac{2GM}{c^2} \qquad \text{(Schwarzschild radius).}$$

Periodic Motion:

$$f = \frac{1}{T} \qquad \text{(Frequency and period);}$$

$$\omega = 2\pi f = \frac{2\pi}{T} \qquad \text{(angular frequency);}$$

$$F_x = -kx \qquad \text{(simple harmonic oscillator);}$$

$$\omega = \sqrt{\frac{k}{m}} \qquad \text{(angular frequency of simple harmonic oscillator);}$$

$$x = A\cos(\omega t + \phi) \qquad \text{(motion of simple harmonic oscillator);}$$

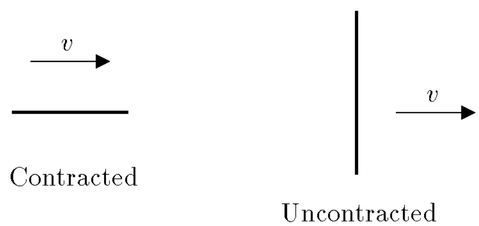
$$\omega = \sqrt{\frac{\kappa}{I}}$$ (angular frequency ω in terms of torsion constant κ and moment of inertia I)

Special Relativity:

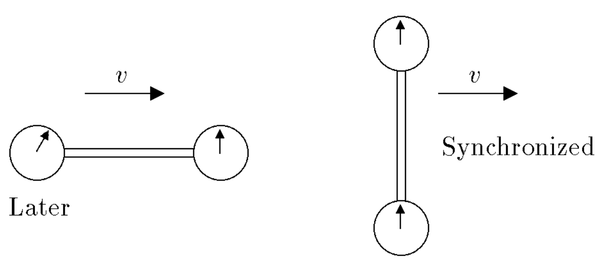
(1) TIME DILATION: Any clock which is moving at speed v relative to a given reference frame will appear (to an observer using that reference frame) to run slower than normal by a factor denoted by the Greek letter γ (gamma), and given by

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}} \; , \qquad \beta \equiv v/c \; .$$

(2) LORENTZ-FITZGERALD CONTRACTION: Any rod which is moving at a speed v along its length relative to a given reference frame will appear (to an observer using that reference frame) to be shorter than its normal length by the same factor γ. A rod which is moving perpendicular to its length does not undergo a change in apparent length.



(3) RELATIVITY OF SIMULTANEITY: Suppose a rod which has rest length ℓ_0 is equipped with a clock at each end. The clocks can be synchronized in the rest frame of the system by using light pulses. If the system moves at speed v along its length, then the trailing clock will appear to read a time which is later than the leading clock by an amount $\beta\ell_0/c$. If, on the other hand, the system moves perpendicular to its length, then the synchronization of the clocks is not disturbed.



There are a few minor qualifications that must be appended to the above statements. First, they hold only for inertial reference frames— they do not hold for rotating or accelerating reference frames. Any reference frame which moves at a uniform velocity relative to an inertial reference frame is also an inertial reference frame. Second, one must define the word "appear" which occurs in each of the three statements.

In plain English, the word “appear” normally refers to the perception of the human eyes. However, in these situations the perception of the human eyes would be very complicated. The complication is that one sees with light, and light travels with a less-than-infinite speed. Thus, when an object is moving toward you, the light which you see coming from the front of the object has left the object later than the light which you see coming from the back of the object. Effects of this kind lead to complicated distortions, which are not taken into account in the statements above. For purposes of interpreting these statements, one can imagine that each reference frame is covered by an infinite number of local observers, each of which observes only events so close that the time delay for light travel is negligible. Each observer carries a clock which has been synchronized with the others by light pulses. The “appearance” is then the description which is assembled after the fact by combining the reports of these local observers.

The Lorentz Transformation:

If an (x', y', z', t') inertial coordinate system is moving to the right (positive x direction) with speed v relative to an (x, y, z, t) inertial coordinate system, then the coordinates are related by

$$\begin{aligned} x' &= \gamma(x - vt) \\ t' &= \gamma\left(t - \frac{vx}{c^2}\right) \\ y' &= y \\ z' &= z \ . \end{aligned}$$

and

$$\begin{aligned} x &= \gamma(x' + vt') \\ t &= \gamma\left(t' + \frac{vx'}{c^2}\right) \\ y &= y' \\ z &= z' \ . \end{aligned}$$

Relativistic velocity addition and subtraction:

$$\begin{aligned} v'_x &= \frac{v_x - v}{1 - \frac{vv_x}{c^2}} \\ v_x &= \frac{v'_x + v}{1 + \frac{vv'_x}{c^2}} \ . \end{aligned}$$

Relativistic Doppler Shift (Electromagnetic Radiation):

$$f_{\text{received}} = \sqrt{\frac{c+u}{c-u}}\, f_{\text{emitted}}$$ (where u is the speed of approach between source and observer).

Relativistic Energy and Momentum:

$\vec{\mathbf{p}} = \dfrac{m\vec{\mathbf{v}}}{\sqrt{1 - v^2/c^2}} = \gamma m\vec{\mathbf{v}}$	(relativistic momentum, where m is the rest mass);
$E = \dfrac{mc^2}{\sqrt{1 - v^2/c^2}} = \gamma mc^2$	(relativistic energy);
$K = E - mc^2 = (\gamma - 1)mc^2$	(relativistic kinetic energy);
$E^2 - (pc)^2 = \left(mc^2\right)^2$	(relation between energy, momentum, and mass);
$E = pc$	(relation between energy and momentum of massless particles, such as photons);
$\vec{\mathbf{F}} = \dfrac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}$	(force is the rate of change of momentum, even in relativity)
$F = \gamma^3 ma$	(when $\vec{\mathbf{F}}$ and $\vec{\mathbf{v}}$ are parallel);
$F = \gamma ma$	(when $\vec{\mathbf{F}}$ and $\vec{\mathbf{v}}$ are perpendicular).