MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 Spring 2005

WEEKLY QUIZ 6

Friday, March 11, 2005

FAMILY (LAST) NAME

GIVEN (FIRST) NAME

STUDENT ID NUMBER

Please check (✔) your class ☞

	L01	MTW 10:00	Walter Lewin
	L02	MTW 11:00	Walter Lewin
	L03	MTW 2:00	Min Chen
	L04	MTW 3:00	Min Chen

INSTRUCTIONS:

1. The FORMULA SHEET is in the back of this exam. You may tear it off. There is also an extra BLANK PAGE in case you need it.
2. This is a closed book exam. CALCULATORS, BOOKS, and NOTES are NOT ALLOWED.
3. Unless otherwise stated, to earn full credit you must show a valid DERIVATION and/or EXPLANATION of your answer, and you must express it in terms of the GIVEN VARIABLES.

Problem	**Maximum**	**Score**	**Grader**
1	25		
2	40		
3	35		
TOTAL	100		

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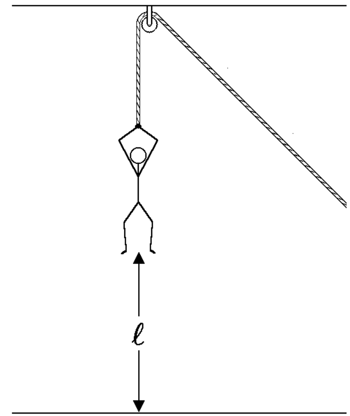


Problem 1: Basic concepts about energy (*25 points*)

Ali, who has mass M, grabs a rope that is wrapped around a pulley attached to the ceiling. He holds the rope tight as several friends pull on the other end, lifting him a distance ℓ above the floor. The acceleration of gravity is denoted by g. Assume that the mass of the rope and all frictional effects can be neglected.

(a) (*3 points*) From the time he starts at rest on the ground to the time he is hanging at rest at a height ℓ, what is the total work done on Ali by all forces, including gravity?

(i) 0 (ii) $Mg\ell$ (iii) $-Mg\ell$ (iv) It depends on how fast his friends pulled.

(b) (*3 points*) During this period, by how much has Ali's gravitational potential energy changed?

(i) no change (ii) increase by $Mg\ell$ (iii) decrease by $Mg\ell$

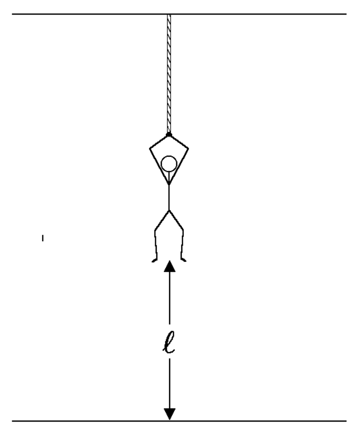
(iv) It depends on how fast his friends pulled.

(c) (*3 points*) During this period, how much work was done on Ali by the rope?

(i) 0 (ii) $Mg\ell$ (iii) $-Mg\ell$ (iv) It depends on how fast his friends pulled.

(d) (*3 points*) During this period, how much work has Ali done on himself?

(i) 0 (ii) $Mg\ell$ (iii) $-Mg\ell$ (iv) It depends on how fast his friends pulled.



Suppose now that the rope is tied directly to the ceiling. Ali starts at rest on the ground, and then climbs the rope hand over hand to the same height ℓ as before.

(e) (*3 points*) From the time he starts at rest on the ground to the time he is hanging at rest at a height ℓ, how much work was done on Ali by the rope?

(i) 0 (ii) $Mg\ell$ (iii) $-Mg\ell$

(iv) It depends on how fast Ali pulled.

(f) (*3 points*) During this period, how much work has Ali done on himself?

(i) 0 (ii) $Mg\ell$ (iii) $-Mg\ell$ (iv) It depends on how fast Ali pulled.

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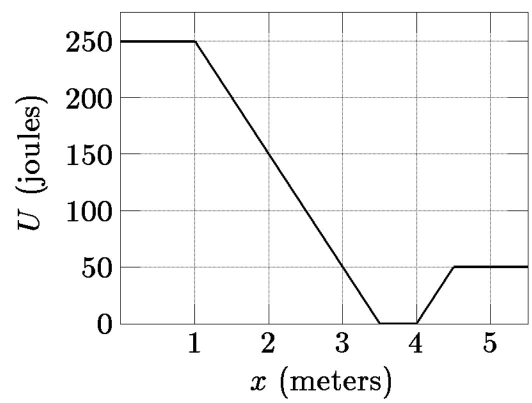
A particle with mass 2 kg moves in one dimension, in the presence of a force that is described by the following potential energy graph.



(g) (*2 points*) If the particle is located at $x = 0.5$ m, what is F_x, the x-component of the force acting on the particle?

(i) 0 (ii) 25 N (iii) 50 N (iv) 75 N (v) 100 N (vi) 200 N (vii) 400 N

(h) (*2 points*) If the particle is located at $x = 2$ m, what is F_x, the x-component of the force acting on the particle?

(i) 0 (ii) 25 N (iii) 50 N (iv) 75 N (v) 100 N (vi) 200 N (vii) 400 N

(i) (*3 points*) If the particle is released from rest at $x = 2$ m, what will be its speed when it crosses $x = 5$ m?

(i) 0 (ii) $\sqrt{50}$ m/s (iii) 10 m/s (iv) $\sqrt{150}$ m/s (v) $\sqrt{200}$ m/s (vi) $\sqrt{250}$ m/s

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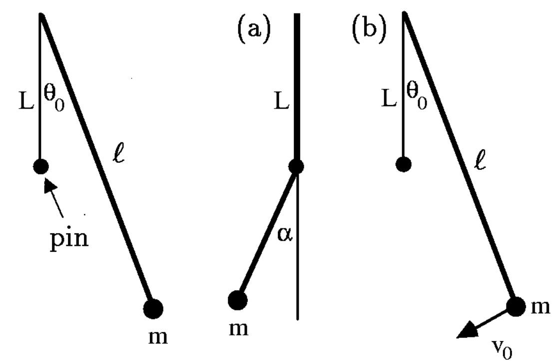
Problem 2: The pin and the pendulum (*40 points*)

A simple pendulum consisting of a mass m attached to a string of length ℓ is released from rest at an angle θ_0. A pin is located at a distance L below the pivot point. When the pendulum swings down, the string hits the pin as shown. Assume that gravity acts downward, with acceleration $g > 0$, and that the mass of the string and any frictional effects are negligible.



(a) (*20 points*) What is the maximum angle α that the string makes with the vertical after hitting the pin?

(b) (*7 points*) If the bob had been released with an initial velocity v_0 as shown, what would be the maximum value of α?

(c) (*3 points*) How would this be affected if v_0 were in the opposite direction?

(d) (*5 points*) At what point in the motion will the tension in the string reach its maximum value? Be sure that your answer is clear enough to specify whether the maximum tension is reached before or after the string makes contact with the pin.

(e) (*5 points*) If the particle is released from rest at an angle θ_0, as in part (a), what is the maximum tension T that the string experiences during this motion?

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Problem 3: Force and potential in one dimension (*35 points*)

A particle of mass m is constrained to move in one dimension, described by a coordinate x. The force on the particle depends on its position, and is given by

$$F_x = \begin{cases} 0 & \text{if } x \leq 0 \\ -Ax^2 & \text{if } 0 < x < b \\ 0 & \text{if } x \geq b \ , \end{cases}$$

where A is a positive constant.

(a) (*5 points*) Is this force conservative? Give a brief explanation of your answer.

(b) (*5 points*) Choose the zero of potential energy $U(x)$ so that $U(0) = 0$. What is $U(x)$ for $x < 0$?

(c) (*10 points*) What is $U(x)$ in the range $0 < x < b$?

(d) (*5 points*) What is $U(x)$ in the range $x \geq b$?

(e) (*5 points*) Suppose that the particle begins in the region $x < 0$ with a speed v_0 to the right (i.e., to larger values of x). If v_0 is larger than a certain value v_1, then the particle will continue moving to the right forever. Find an expression for v_1 either in terms of m, A, and b, or in terms of m and $U(x)$. *[Note that you can express your answer in terms of the function $U(x)$ even if you have not calculated it. Remember, however, that the symbol $U(x)$ does not represent any particular number, since x can have any value. Your answer should involve something like $U(0)$, $U(b)$, $U(b/2)$, $U(2b)$, etc.]*

(f) (*5 points*) If $v_0 < v_1$, then the particle will reach a maximum value of x, which we will call $x_{\max}$, and then it will move back to the left. Find an expression for $x_{\max}$ either in terms of m, v_0, A, and b or in terms of m, v_0, and the function $U(x)$.

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QUIZ 6
FORMULA SHEET
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For motion in one dimension:

$$v_{\rm av} = \frac{\Delta x}{\Delta t} \qquad \text{Average velocity;}$$

$$v = \frac{{\rm d}x}{{\rm d}t} \qquad \text{Instantaneous velocity;}$$

For motion in three dimensions:

$$\vec{\mathbf{v}} = \frac{{\rm d}\vec{\mathbf{r}}}{{\rm d}t}\,; \qquad \vec{\mathbf{a}} = \frac{{\rm d}\vec{\mathbf{v}}}{{\rm d}t} = \frac{{\rm d}^2\vec{\mathbf{r}}}{{\rm d}t^2}\,; \qquad \vec{\mathbf{r}}(t_1) = \vec{\mathbf{r}}_0 + \int_0^{t_1} \vec{\mathbf{v}}\,{\rm d}t\,; \qquad \vec{\mathbf{v}}(t_1) = \vec{\mathbf{v}}_0 + \int_0^{t_1} \vec{\mathbf{a}}\,{\rm d}t\ .$$

For *constant* acceleration $\vec{\mathbf{a}}$, if $\vec{\mathbf{r}} = \vec{\mathbf{r}}_0$ and $\vec{\mathbf{v}} = \vec{\mathbf{v}}_0$ at time $t = 0$, then

$$\vec{\mathbf{v}}(t) = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t$$

$$\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \frac{1}{2}\vec{\mathbf{a}}t^2\ .$$

For one-dimensional motion with constant acceleration a:

$$v^2 = v_0^2 + 2a(x - x_0)\ .$$

For circular motion at constant speed v:

$$a = \frac{v^2}{r}\ ,$$

where r is the radius of the circle, and the acceleration is directed towards the center of the circle.

If an object has position $\vec{\mathbf{r}}$ and velocity $\vec{\mathbf{v}}$, its position and velocity relative to an observer with position $\vec{\mathbf{r}}_0$ and velocity $\vec{\mathbf{v}}_0$ are given respectively by

$$\vec{\mathbf{r}}' = \vec{\mathbf{r}} - \vec{\mathbf{r}}_0\ , \qquad \vec{\mathbf{v}}' = \vec{\mathbf{v}} - \vec{\mathbf{v}}_0\ .$$

Average velocity and acceleration are given by

$$\vec{\mathbf{v}}_{\rm average} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}\ , \qquad \vec{\mathbf{a}}_{\rm average} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}\ .$$

Mass, Acceleration, and Force:

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} \qquad \text{(Newton's second law);}$$

$$\vec{\mathbf{F}} = -\frac{GMm}{r^2}\hat{\boldsymbol{r}} \qquad \text{(the gravitational force between two particles);}$$

$$\vec{\mathbf{F}} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}\hat{\boldsymbol{r}} \qquad \text{(the electrostatic force between two particles);}$$

$$F_x = -kx \qquad \text{(Hooke's law);}$$

where $\hat{\boldsymbol{r}}$ is a unit vector pointing from the particle which is the source of the force, toward the particle on which the force is acting.

Friction:

$$\left|\vec{\mathbf{F}}_k\right| = \mu_k \left|\vec{\mathbf{N}}\right| \qquad \text{(kinetic friction);}$$

$$\left|\vec{\mathbf{F}}_s\right| \le \mu_s \left|\vec{\mathbf{N}}\right| \qquad \text{(static friction).}$$

Kinetic Energy, Work, and Potential Energy:

1 Dimension	**3 Dimensions**	**Description**
$W \equiv F\Delta x$	$W \equiv \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{\Delta r}}$	Work done by a constant force $\vec{\mathbf{F}}$
$W \equiv \int F(x)\,\mathrm{d}x$	$W \equiv \int_{\vec{\mathbf{r}}_1}^{\vec{\mathbf{r}}_2} \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{dr}}$	Work done by a varying force $\vec{\mathbf{F}}$
$U(x_p) \equiv U_0 - \int_{x_0}^{x_p} F\,\mathrm{d}x$	$U(\vec{\mathbf{r}}_p) \equiv U_0 - \int_{\vec{\mathbf{r}}_0}^{\vec{\mathbf{r}}_p} \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{dr}}$	Potential energy derived from force $\vec{\mathbf{F}}$
$F = -\frac{\mathrm{d}U}{\mathrm{d}x}$	$\vec{\mathbf{F}} = \left[-\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y}, -\frac{\partial U}{\partial z}\right]$	Force derived from potential energy

$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = \lvert\vec{\mathbf{a}}\rvert\lvert\vec{\mathbf{b}}\rvert \cos\theta$	(scalar (or dot) product of two vectors);
$\quad = a_x b_x + a_y b_y + a_z b_z$	
$E_k = \frac{1}{2}mv^2$	(kinetic energy of a particle);
$W = E_{k,f} - E_{k,i}$	(work-energy theorem);
$\frac{1}{2}mv^2 + U(x) = \text{constant}$	(energy conservation);
$\frac{1}{2}mv^2 + mgh = \frac{1}{2}mv_0^2$	(kinetic & potential energy for projectile);
$W = \frac{1}{2}kx^2$	(work to compress a spring);
$U = \frac{1}{2}kx^2$	(potential energy for spring force);
$W = mgh$	(work to lift a body near the surface of the Earth);
$U = mgh$	(gravitational potential energy, near the surface of the Earth);
$U = -\frac{GMm}{r}$	(gravitational potential energy, spherical bodies);
$U = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r}$	(electrostatic potential energy, spherical charges).