MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 Spring 2005

WEEKLY QUIZ 7

Friday, March 18, 2005

FAMILY (LAST) NAME

GIVEN (FIRST) NAME

STUDENT ID NUMBER

Please check (✔) your class ☞

	L01	MTW 10:00	Walter Lewin
	L02	MTW 11:00	Walter Lewin
	L03	MTW 2:00	Min Chen
	L04	MTW 3:00	Min Chen

INSTRUCTIONS:

1. The FORMULA SHEET is in the back of this exam. You may tear it off. There is also an extra BLANK PAGE in case you need it.
2. This is a closed book exam. CALCULATORS, BOOKS, and NOTES are NOT ALLOWED.
3. Unless otherwise stated, to earn full credit you must show a valid DERIVATION and/or EXPLANATION of your answer, and you must express it in terms of the GIVEN VARIABLES.

Problem	**Maximum**	**Score**	**Grader**
1	30		
2	20		
3	25		
4	25		
TOTAL	100		

Name ______________________

Problem 1: Basic concepts about momentum and systems of particles (*30 points*)

The following questions are to be answered either YES or NO. Circle the correct answer. You need not explain your answer.

(a) (*3 points*) If I stand motionless on the ground, the force that my feet exert on the ground is equal in magnitude but opposite in direction to the force that the ground exerts on my feet. Are these two forces required to be equal and opposite by Newton's third law? Yes or No ?

(b) (*3 points*) Two clay balls moving in empty space, with no forces acting on them, collide and stick together. Is kinetic energy conserved in the collision? Yes or No ?

(c) (*3 points*) For the collision described in part (b), is the momentum of the balls conserved in the collision? That is, does the total momentum of the two particles after the collision have the same value as it had before the collision? Yes or No ?

(d) (*2 points*) A tennis ball is thrown against the wall in Room 26-100, and it bounces back. Is the momentum of the ball conserved in the collision? That is, does the momentum of the ball after the collision equal the value it had before the collision? Yes or No ?

A baseball bat and a soccer ball are tied together by a massless and inextensible string. The combination is thrown through the air so that the baseball bat tumbles in a complicated pattern. As the bat tumbles it sometimes pulls the string taut, while at other times the string is slack. Ignore air friction, but take into account the pull of gravity, in the negative y-direction.

(e) (*3 points*) Is the x-component of the total momentum of the baseball bat and soccer ball conserved? Yes or No ?

(f) (*3 points*) Is the y-component of the total momentum of the baseball bat and soccer ball conserved? Yes or No ?

(g) (*3 points*) Is the total kinetic energy plus the gravitational potential energy of the baseball bat and soccer ball conserved? Yes or No ?

(h) (*3 points*) Is the total momentum of the baseball bat and soccer ball equal to $M_{\text{tot}}\vec{\mathbf{v}}_{\text{cm}}$, where M_{tot} is the total mass of the two objects, and $\vec{\mathbf{v}}_{\text{cm}}$ is the velocity of the center of mass of the two objects? Yes or No ?

(i) (*3 points*) Is the kinetic energy of the baseball bat and soccer ball equal to $\frac{1}{2}M_{\text{tot}}v_{\text{cm}}^2$, where M_{tot} is again the total mass of the system, and v_{cm} is the speed of the center of mass of the system? Yes or No ?

(j) (*3 points*) Does the center of mass of the baseball bat / soccer ball system travel on a parabolic trajectory, as a single particle would? Yes or No ?

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Problem 2: Force and potential energy in one dimension (reprise) (*20 points*)

A particle of mass m is constrained to move in one dimension, described by a coordinate y. The force on the particle depends on its position, and is given by

$$F_y = \begin{cases} 0 & \text{if } y \geq 0 \\ \frac{1}{2}ky^2 & \text{if } -a < y < 0 \\ \frac{1}{2}ka^2 & \text{if } y \leq -a \ , \end{cases}$$

where k and a are positive constants.

(a) (*5 points*) Is this force conservative? Give a brief explanation of your answer.

(b) (*5 points*) Choose the zero of potential energy $U(y)$ so that $U(0) = U_0$, where U_0 is a constant. What is $U(y)$ for $y > 0$?

(c) (*5 points*) What is $U(y)$ in the range $-a < y < 0$?

(d) (*5 points*) What is $U(y)$ in the range $y \leq -a$?

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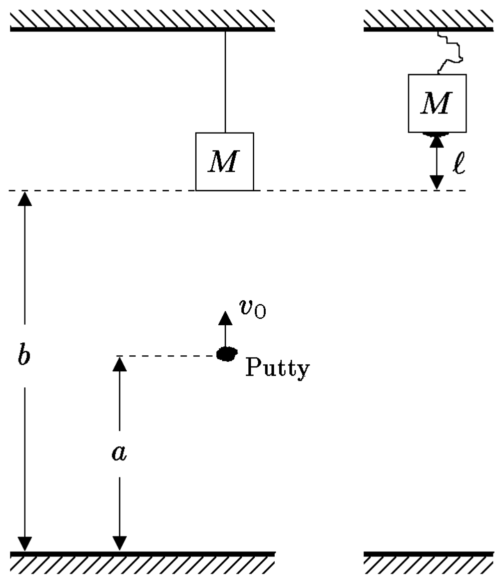
Problem 3: The vertical ballistic pendulum (*25 points*):

A block of mass M hangs from the ceiling on a string, at a height b above the floor. Tommy stands directly below the block, and throws a blob of putty directly upward at it. The putty has mass m, and starts at height a above the floor with an initial upward speed v_0.



(a) (*8 points*) What is the upward speed v_1 of the putty when it reaches height b, but just before it hits the block?

(b) (*9 points*) The putty sticks to the block, so after the collision the putty and block move upward together. (You may assume that the deformation of the putty takes negligible time.) What is the upward speed v_2 of the putty-block system immediately after the two stick together? Your answer may depend on the variable v_1, the answer to part (a), whether or not you solved part (a).

(c) (*8 points*) Measured from the initial position of the block, by what distance ℓ does the putty-block system move upward before gravity pulls it back down? Your answer may depend on v_1, v_2, or any of the given variables, whether or not v_1 and v_2 have been found.

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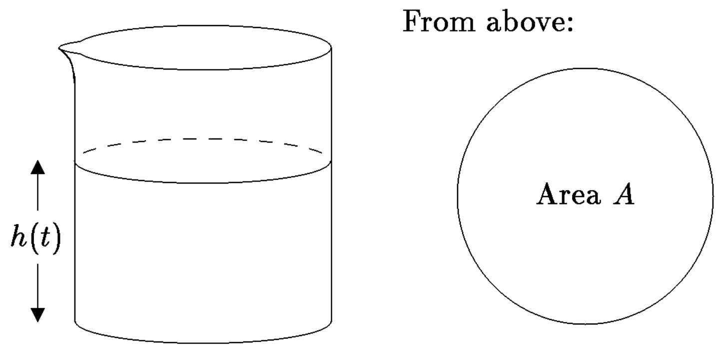
Problem 4: A beaker in the rain (*25 points*)

An initially empty beaker, in the shape of a cylinder with cross sectional area A, is left out in the rain. The raindrops hit the beaker vertically downward with speed v. The rain continues at a constant rate, so the height of the water in the beaker $h(t)$ increases with time t at a rate $\mathrm{d}h/\mathrm{d}t = w$, where w is negligible compared to v. The raindrops quickly come to rest inside the beaker, so we can neglect any kinetic energy of the water that has collected in the beaker. Let ρ denote the density of water (i.e., the mass per unit volume).



(a) (*5 points*) What is the rate at which the mass of the water in the beaker increases with time?

(b) (*5 points*) Let $y_{\text{cm}}(t)$ denote the height of the center of mass of all the water that has collected in the beaker by time t. What is the rate $\mathrm{d}y_{\text{cm}}/\mathrm{d}t$ at which this height increases?

(c) (*5 points*) The total momentum of any system of particles $\vec{\mathbf{P}}_{\text{tot}}$ is equal to the total mass M_{tot} times the velocity $\vec{\mathbf{v}}_{\text{cm}}$ of the center of mass. Should we conclude, therefore, that the water in the beaker has a vertical momentum equal to its mass times the value of $\mathrm{d}y_{\text{cm}}/\mathrm{d}t$ as described in part (b)? Explain your answer in one or two sentences.

(d) (*10 points*) If the beaker is placed on a scale, while the beaker is still in the rain, the impact of the raindrops on the beaker will cause the reading on the scale to be larger than the weight of the beaker and the water it contains. By how much is the reading on the scale increased by the impact of the raindrops? (Neglect the effect of raindrops that hit the scale directly.)

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QUIZ 7
FORMULA SHEET
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For motion in one dimension:

$$v_{\rm av} \equiv \frac{\Delta x}{\Delta t} \qquad \text{Average velocity;}$$

$$v \equiv \frac{\mathrm{d}x}{\mathrm{d}t} \qquad \text{Instantaneous velocity;}$$

For motion in three dimensions:

$$\vec{\mathbf{v}} \equiv \frac{\mathrm{d}\vec{\mathbf{r}}}{\mathrm{d}t}\,; \qquad \vec{\mathbf{a}} \equiv \frac{\mathrm{d}\vec{\mathbf{v}}}{\mathrm{d}t} = \frac{\mathrm{d}^2\vec{\mathbf{r}}}{\mathrm{d}t^2}\,; \qquad \vec{\mathbf{r}}(t_1) = \vec{\mathbf{r}}_0 + \int_0^{t_1} \vec{\mathbf{v}}\,\mathrm{d}t\,; \qquad \vec{\mathbf{v}}(t_1) = \vec{\mathbf{v}}_0 + \int_0^{t_1} \vec{\mathbf{a}}\,\mathrm{d}t\ .$$

For *constant* acceleration $\vec{\mathbf{a}}$, if $\vec{\mathbf{r}} = \vec{\mathbf{r}}_0$ and $\vec{\mathbf{v}} = \vec{\mathbf{v}}_0$ at time $t = 0$, then

$$\vec{\mathbf{v}}(t) = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t$$

$$\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \frac{1}{2}\vec{\mathbf{a}}t^2\ .$$

For one-dimensional motion with constant acceleration a:

$$v^2 = v_0^2 + 2a(x - x_0)\ .$$

For circular motion at constant speed v:

$$a = \frac{v^2}{r}\ ,$$

where r is the radius of the circle, and the acceleration is directed towards the center of the circle.

If an object has position $\vec{\mathbf{r}}$ and velocity $\vec{\mathbf{v}}$, its position and velocity relative to an observer with position $\vec{\mathbf{r}}_0$ and velocity $\vec{\mathbf{v}}_0$ are given respectively by

$$\vec{\mathbf{r}}\,' = \vec{\mathbf{r}} - \vec{\mathbf{r}}_0\ , \qquad \vec{\mathbf{v}}\,' = \vec{\mathbf{v}} - \vec{\mathbf{v}}_0\ .$$

Average velocity and acceleration are given by

$$\vec{\mathbf{v}}_{\rm average} \equiv \frac{\Delta\vec{\mathbf{r}}}{\Delta t}\ , \qquad \vec{\mathbf{a}}_{\rm average} \equiv \frac{\Delta\vec{\mathbf{v}}}{\Delta t}\ .$$

Mass, Acceleration, and Force:

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} \qquad \text{(Newton's second law);}$$

$$\vec{\mathbf{F}} = -\frac{GMm}{r^2}\hat{\boldsymbol{r}} \qquad \text{(the gravitational force between two particles);}$$

$$\vec{\mathbf{F}} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}\hat{\boldsymbol{r}} \qquad \text{(the electrostatic force between two particles);}$$

$$F_x = -kx \qquad \text{(Hooke's law);}$$

where $\hat{\boldsymbol{r}}$ is a unit vector pointing from the particle which is the source of the force, toward the particle on which the force is acting.

Friction:

$$\left|\vec{\mathbf{F}}_k\right| = \mu_k \left|\vec{\mathbf{N}}\right| \qquad \text{(kinetic friction);}$$

$$\left|\vec{\mathbf{F}}_s\right| \leq \mu_s \left|\vec{\mathbf{N}}\right| \qquad \text{(static friction).}$$

Kinetic Energy, Work, and Potential Energy:

Description	**1 Dimension**	**3 Dimensions**
Work done by a constant force $\vec{\mathbf{F}}$	$W \equiv F\Delta x$	$W \equiv \vec{\mathbf{F}} \cdot \overrightarrow{\boldsymbol{\Delta}\mathbf{r}}$
Work done by a varying force $\vec{\mathbf{F}}$	$W \equiv \int F(x)\,\mathrm{d}x$	$W \equiv \int_{\vec{\mathbf{r}}_1}^{\vec{\mathbf{r}}_2} \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{dr}}$
Potential energy derived from force $\vec{\mathbf{F}}$	$U(x_p) \equiv U_0 - \int_{x_0}^{x_p} F\,\mathrm{d}x$	$U(\vec{\mathbf{r}}_p) \equiv U_0 - \int_{\vec{\mathbf{r}}_0}^{\vec{\mathbf{r}}_p} \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{dr}}$
Force derived from potential energy	$F = -\frac{\mathrm{d}U}{\mathrm{d}x}$	$\vec{\mathbf{F}} = \left[-\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y}, -\frac{\partial U}{\partial z}\right]$

$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \equiv \lvert\vec{\mathbf{a}}\rvert\lvert\vec{\mathbf{b}}\rvert \cos\theta$ $= a_x b_x + a_y b_y + a_z b_z$	(scalar (or dot) product of two vectors);
$E_k \equiv \frac{1}{2} m v^2$	(kinetic energy of a particle);
$W_{\text{tot}} = E_{k,f} - E_{k,i}$	(work-energy theorem: always true if W_{tot} includes work due to all forces; a non-rigid object can do work on itself!);
$E_{k,i} + U_i + W_{\text{other}} = E_{k,f} + U_f$	(generalized work-energy theorem: always true if W_{other} includes work due to all forces not included in U);
$\frac{1}{2} m v^2 + U(x) = \text{constant}$	(conservation of mechanical energy: true in the absence of dissipative forces);
$\frac{1}{2} m v^2 + mgh = \frac{1}{2} m v_0^2$	(conservation of mechanical energy for a projectile: true in the absence of dissipative forces);
$W = \frac{1}{2} k x^2$	(work to compress a spring);
$U = \frac{1}{2} k x^2$	(potential energy for spring force);
$W = mgh$	(work to lift a body near the surface of the Earth);
$U = mgh$	(gravitational potential energy, near the surface of the Earth);
$U = -\frac{GMm}{r}$	(gravitational potential energy, spherical bodies);
$U = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r}$	(electrostatic potential energy, spherical charges).

Momentum, Center of Mass, and Systems of Particles:

$\vec{\mathbf{F}}_{\text{AB}} = -\vec{\mathbf{F}}_{\text{BA}}$	(Newton's third law);
$\vec{\mathbf{p}} \equiv m\vec{\mathbf{v}}$	(momentum);
$\frac{\mathrm{d}\vec{\mathbf{P}}_{\text{tot}}}{\mathrm{d}t} = 0$	(conservation of momentum in absence of external force)
$\vec{\mathbf{F}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}$	(Newton's second law in terms of momentum);
$\vec{\mathbf{r}}_{\text{cm}} \equiv \frac{1}{M_{\text{tot}}} \sum_i m_i \vec{\mathbf{r}}_i$	(position of center of mass);

$$\vec{\mathbf{v}}_{\rm cm} \equiv \frac{{\rm d}\vec{\mathbf{r}}_{\rm cm}}{{\rm d}t} = \frac{1}{M_{\rm tot}}\sum_i m_i \vec{\mathbf{v}}_i$$ (velocity of center of mass);

$$\vec{\mathbf{F}}^{\rm ext}_{\rm tot} = M_{\rm tot}\vec{\mathbf{a}}_{\rm cm} = \frac{{\rm d}\vec{\mathbf{P}}_{\rm tot}}{{\rm d}t}$$ (acceleration of a system of particles);

$$\vec{\mathbf{P}}_{\rm tot} = \sum_i m_i \vec{\mathbf{v}}_i = M_{\rm tot}\vec{\mathbf{v}}_{\rm cm}$$ (momentum of a system of particles);

$$K_{\rm tot} = \frac{1}{2}M_{\rm tot}v_{\rm cm}^2 + \sum_i \frac{1}{2}m_i\left(\vec{\mathbf{v}}_i - \vec{\mathbf{v}}_{\rm cm}\right)^2$$ (K.E. of a system of particles);

$$\vec{\mathbf{J}} = \int_{t_1}^{t_2}\vec{\mathbf{F}}\,{\rm d}t = \int_{t_1}^{t_2}\frac{{\rm d}\vec{\mathbf{p}}}{{\rm d}t}\,{\rm d}t = \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1$$ (impulse-momentum theorem).