MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 Spring 2005

WEEKLY QUIZ 8

Friday, April 1, 2005

FAMILY (LAST) NAME

GIVEN (FIRST) NAME

STUDENT ID NUMBER

Please check (✔) your class ☞

	L01	MTW 10:00	Walter Lewin
	L02	MTW 11:00	Walter Lewin
	L03	MTW 2:00	Min Chen
	L04	MTW 3:00	Min Chen

INSTRUCTIONS:

1. The FORMULA SHEET is in the back of this exam. You may tear it off. There is also an extra BLANK PAGE in case you need it.
2. This is a closed book exam. CALCULATORS, BOOKS, and NOTES are NOT ALLOWED.
3. Unless otherwise stated, to earn full credit you must show a valid DERIVATION and/or EXPLANATION of your answer, and you must express it in terms of the GIVEN VARIABLES.

Problem	**Maximum**	**Score**	**Grader**
1	40		
2	30		
3	30		
TOTAL	100		

Name ______________________

Problem 1: Basic concepts about the rotation of rigid bodies in two dimensions (*40 points*)

Mark your answer by circling it. For these multiple choice questions you need not show your work, and there will be no partial credit. *Warning: some problems may contain irrelevant information.*

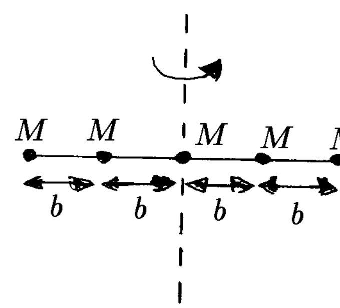
(a) (*5 points*) A wheel is rotating at 120 rpm (revolutions per minute). What is its angular velocity in radians per second?

(i) 120 (ii) 240π (iii) 2 (iv) π (v) 2π (vi) 3π (vii) 4π

(b) (*5 points*) At $t = 0$ a flywheel is rotating with angular velocity ω_0. It then undergoes uniform angular acceleration for a time t_1, at the end of which the angular velocity is ω_1. How many revolutions did the flywheel make during this time interval?

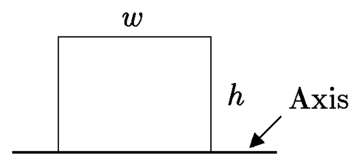
(i) $\omega_0 t$ (ii) $\omega_1 t$ (iii) $\frac{1}{2}(\omega_0 + \omega_1)t$ (iv) $\dfrac{\omega_0 t}{2\pi}$ (v) $\dfrac{\omega_1 t}{2\pi}$ (vi) $\dfrac{(\omega_0 + \omega_1)t}{4\pi}$

(c) (*5 points*) Five small balls, each of mass M, are attached to a massless rigid rod of length $4b$. One ball is at the center, one ball is at each end, and one ball is a distance b from the center in each direction, as shown. What is the moment of inertia of this object for rotation about an axis through the center of the rod and perpendicular to it?



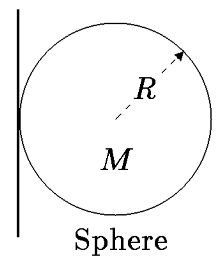
(i) $2Mb^2$ (ii) $5Mb^2$ (iii) $8Mb^2$ (iv) $10Mb^2$ (v) $12Mb^2$

(d) (*5 points*) What is the moment of inertia I of a thin rectangular plate of mass M and dimensions $w \times h$, pivoted about one of the edges of length w? (Remember that there is a table of moments of inertia on the formula sheet.)



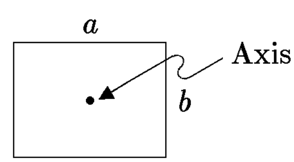
(i) $\frac{1}{3}Mw^2$ (ii) $\frac{1}{3}Mh^2$ (iii) $\frac{1}{2}Mw^2$ (iv) $\frac{1}{2}Mh^2$ (v) $\frac{1}{3}M(w^2+h^2)$

(e) (*5 points*) What is the moment of inertia I of a sphere of mass M and radius R, pivoted about a rod that is tangent to the surface of the sphere?



(i) $\frac{2}{5}MR^2$ (ii) $\frac{3}{5}MR^2$ (iii) $\frac{4}{5}MR^2$ (iv) MR^2 (v) $\frac{6}{5}MR^2$ (vi) $\frac{7}{5}MR^2$

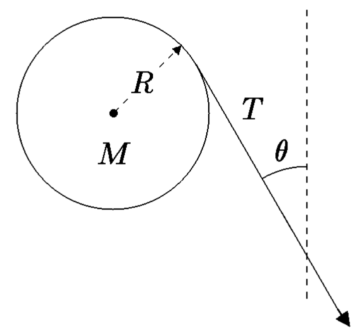
(f) (*5 points*) What is the moment of inertia I of a thin rectangular plate of mass M and dimensions $a \times b$, pivoted about an axis perpendicular to the plate and through its center?



(i) $\frac{1}{2}Mab$ (ii) $\frac{1}{12}M(a^2+b^2)$ (iii) $\frac{1}{3}M(a^2+b^2)$ (iv) $\frac{1}{2}M(a^2+b^2)$ (v) $\frac{7}{12}M(a^2+b^2)$

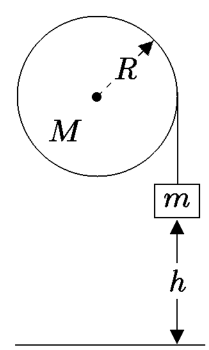
— *Problem 1 Continues* —

(g) (*5 points*) A wheel, which consists of a solid uniform cylindrical disk of radius R and mass M, is pivoted about a fixed axle through the axis of the cylinder. A massless inextensible rope is wound around the rim of the wheel so that it cannot slip. The rope is pulled with a tension T at angle θ with respect to the vertical, as shown. What is the magnitude of the angular acceleration α of the wheel about the pivot? Neglect all friction.



(i) $\frac{T}{\pi MR}$ (ii) $\frac{T \sin\theta}{\pi MR}$ (iii) $\frac{T \cos\theta}{\pi MR}$ (iv) $\frac{2T}{MR}$ (v) $\frac{2T \sin\theta}{MR}$ (vi) $\frac{2T \cos\theta}{MR}$

(h) (*5 points*) A solid uniform cylinder of mass M and radius R is pivoted about a fixed horizontal rod. A massless inextensible string is wrapped around it, and attached to a block of mass m which is initially at a height h above the floor. The acceleration of gravity is g, directed downward. The block is released from rest. By what total angle $\Delta\theta$ (in radians) has the cylinder turned when the block hits the floor?



(i) $\frac{M}{m}$ (ii) $\sqrt{1 + \frac{2M}{m}}$ (iii) $\sqrt{1 + \frac{h}{2R}}$ (iv) $\frac{h}{R}$

(v) $\frac{MR}{mh}$ (vi) $\left(\frac{MR}{mh}\right)^2$

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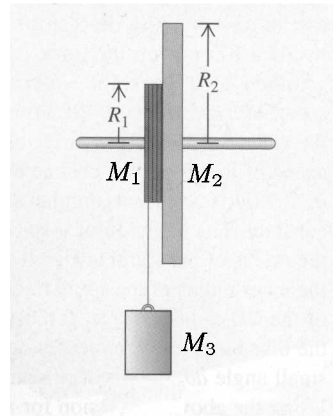
Problem 2: Two disks welded to a common shaft (*30 points*)

Two uniform disks, one with radius R_1 and mass M_1 and the other with radius $R_2 > R_1$ and mass M_2, are welded together and mounted on a frictionless axle through their common center.



(a) (*6 points*) What is the total moment of inertia I_{tot} of the two-disk system about the axis of rotation?

(b) (*10 points*) A light string is wrapped around the edge of the smaller disk (disk 1), and a block of mass M_3 is suspended from the free end of the string. If the block is released from rest at a distance ℓ above the floor, what is its speed v just before it strikes the floor? (You may express your answer in terms of I_{tot} along with the given variables, whether or not you answered part (a).)

(c) (*4 points*) Now suppose that the string is wrapped around the larger disk, the one of radius R_2, and is again released from rest at a distance ℓ above the floor. In this case, what is the speed v' of the block just before it strikes the floor.

(d) (*5 points*) In which case, (b) or (c), is the final speed of the block the greatest? Explain, in a sentence or two, why this is so.

(e) (*5 points*) In which case, (b) or (c), is the final angular velocity of the axle and the two disks the greatest? Explain, in a sentence or two, why this is so.

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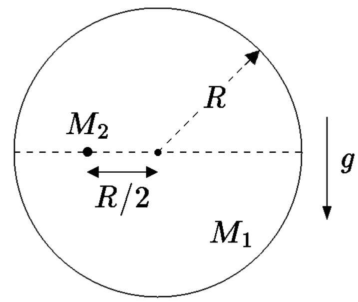
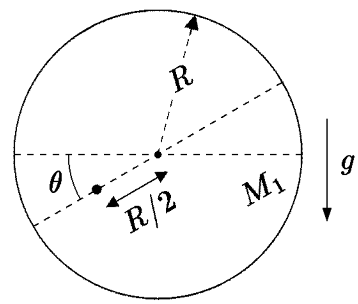
Problem 3: A pivoted disk with attached weights (*30 points*)

A uniform disk of mass M_1 and radius R is pivoted on a frictionless horizontal axle through its center. A small marble of mass M_2 is attached to the disk at radius $R/2$, at the same height as the axle. Assume that the marble is small enough to be treated as a point mass. The acceleration of gravity is downward, with magnitude g.



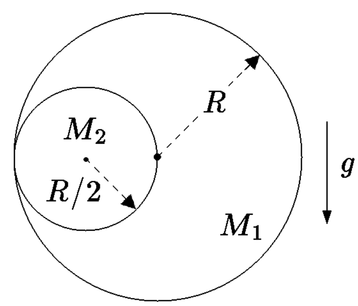
(a) (*8 points*) If this system is released from rest to rotate about the pivot, what will be the angular acceleration α_0 of the disk immediately after it is released?



(b) (*4 points*) After the disk has rotated through an angle θ, what will be the angular acceleration α_1?

(c) (*6 points*) What will be the maximum angular velocity $\omega_{\rm max}$ that the disk will reach in its subsequent motion?

(d) (*7 points*) Now consider the situation if the marble is replaced by a disk of radius $R/2$, with the same mass M_2, located with its center at the same place where the marble was located in part (a). (When calculating the torque on this disk, you can use the fact that the torque caused by gravity can be calculated as if the force of gravity were a single force acting at the center of mass of the object.) For this case, find the angular acceleration α'_0 immediately after the system is released from rest.



(e) (*5 points*) For the case described in part (d), what will be the maximum angular velocity $\omega'_{\rm max}$ that the disk will reach in its subsequent motion?

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QUIZ 8
FORMULA SHEET
Quiz Date: Friday, April 1, 2005

For motion in one dimension:

$$v_{\rm av} \equiv \frac{\Delta x}{\Delta t} \qquad \text{Average velocity;}$$

$$v \equiv \frac{{\rm d}x}{{\rm d}t} \qquad \text{Instantaneous velocity;}$$

For motion in three dimensions:

$$\vec{\mathbf{v}} \equiv \frac{{\rm d}\vec{\mathbf{r}}}{{\rm d}t}\,; \qquad \vec{\mathbf{a}} \equiv \frac{{\rm d}\vec{\mathbf{v}}}{{\rm d}t} = \frac{{\rm d}^2\vec{\mathbf{r}}}{{\rm d}t^2}\,; \qquad \vec{\mathbf{r}}(t_1) = \vec{\mathbf{r}}_0 + \int_0^{t_1} \vec{\mathbf{v}}\,{\rm d}t\,; \qquad \vec{\mathbf{v}}(t_1) = \vec{\mathbf{v}}_0 + \int_0^{t_1} \vec{\mathbf{a}}\,{\rm d}t\ .$$

For *constant* acceleration $\vec{\mathbf{a}}$, if $\vec{\mathbf{r}} = \vec{\mathbf{r}}_0$ and $\vec{\mathbf{v}} = \vec{\mathbf{v}}_0$ at time $t = 0$, then

$$\vec{\mathbf{v}}(t) = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t$$

$$\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \frac{1}{2}\vec{\mathbf{a}}t^2\ .$$

For one-dimensional motion with constant acceleration a:

$$v^2 = v_0^2 + 2a(x - x_0)\ .$$

For circular motion at constant speed v:

$$a = \frac{v^2}{r}\ ,$$

where r is the radius of the circle, and the acceleration is directed towards the center of the circle.

If an object has position $\vec{\mathbf{r}}$ and velocity $\vec{\mathbf{v}}$, its position and velocity relative to an observer with position $\vec{\mathbf{r}}_0$ and velocity $\vec{\mathbf{v}}_0$ are given respectively by

$$\vec{\mathbf{r}}' = \vec{\mathbf{r}} - \vec{\mathbf{r}}_0\ , \qquad \vec{\mathbf{v}}' = \vec{\mathbf{v}} - \vec{\mathbf{v}}_0\ .$$

Average velocity and acceleration are given by

$$\vec{\mathbf{v}}_{\rm average} \equiv \frac{\Delta\vec{\mathbf{r}}}{\Delta t}\ , \qquad \vec{\mathbf{a}}_{\rm average} \equiv \frac{\Delta\vec{\mathbf{v}}}{\Delta t}\ .$$

Mass, Acceleration, and Force:

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} \qquad \text{(Newton's second law);}$$

$$\vec{\mathbf{F}} = -\frac{GMm}{r^2}\hat{\boldsymbol{r}} \qquad \text{(the gravitational force between two particles);}$$

$$\vec{\mathbf{F}} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}\hat{\boldsymbol{r}} \qquad \text{(the electrostatic force between two particles);}$$

$$F_x = -kx \qquad \text{(Hooke's law);}$$

where $\hat{\boldsymbol{r}}$ is a unit vector pointing from the particle which is the source of the force, toward the particle on which the force is acting.

Friction:

$$\left|\vec{\mathbf{F}}_k\right| = \mu_k \left|\vec{\mathbf{N}}\right| \qquad \text{(kinetic friction);}$$

$$\left|\vec{\mathbf{F}}_s\right| \le \mu_s \left|\vec{\mathbf{N}}\right| \qquad \text{(static friction).}$$

Kinetic Energy, Work, and Potential Energy:

Description	**1 Dimension**	**3 Dimensions**
Work done by a constant force $\vec{\mathbf{F}}$	$W \equiv F\Delta x$	$W \equiv \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{\Delta r}}$
Work done by a varying force $\vec{\mathbf{F}}$	$W \equiv \int F(x)\,\mathrm{d}x$	$W \equiv \int_{\vec{\mathbf{r}}_1}^{\vec{\mathbf{r}}_2} \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{dr}}$
Potential energy derived from force $\vec{\mathbf{F}}$	$U(x_p) \equiv U_0 - \int_{x_0}^{x_p} F\,\mathrm{d}x$	$U(\vec{\mathbf{r}}_p) \equiv U_0 - \int_{\vec{\mathbf{r}}_0}^{\vec{\mathbf{r}}_p} \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{dr}}$
Force derived from potential energy	$F = -\frac{\mathrm{d}U}{\mathrm{d}x}$	$\vec{\mathbf{F}} = \left[-\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y}, -\frac{\partial U}{\partial z}\right]$

$$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \equiv |\vec{\mathbf{a}}||\vec{\mathbf{b}}| \cos\theta = a_x b_x + a_y b_y + a_z b_z$$ (scalar (or dot) product of two vectors);

$$E_k \equiv \frac{1}{2} m v^2$$ (kinetic energy of a particle);

$$W_{\text{tot}} = E_{k,f} - E_{k,i}$$ (work-energy theorem: always true if W_{tot} includes work due to all forces; a non-rigid object can do work on itself!);

$$E_{k,i} + U_i + W_{\text{other}} = E_{k,f} + U_f$$ (generalized work-energy theorem: always true if W_{other} includes work due to all forces not included in U);

$$\frac{1}{2} m v^2 + U(x) = \text{constant}$$ (conservation of mechanical energy: true in the absence of dissipative forces);

$$\frac{1}{2} m v^2 + mgh = \frac{1}{2} m v_0^2$$ (conservation of mechanical energy for a projectile: true in the absence of dissipative forces);

$$W = \frac{1}{2} k x^2$$ (work to compress a spring);

$$U = \frac{1}{2} k x^2$$ (potential energy for spring force);

$$W = mgh$$ (work to lift a body near the surface of the Earth);

$$U = mgh$$ (gravitational potential energy, near the surface of the Earth);

$$U = -\frac{GMm}{r}$$ (gravitational potential energy, spherical bodies);

$$U = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r}$$ (electrostatic potential energy, spherical charges).

Momentum, Center of Mass, and Systems of Particles:

$$\vec{\mathbf{F}}_{\text{AB}} = -\vec{\mathbf{F}}_{\text{BA}}$$ (Newton's third law);

$$\vec{\mathbf{p}} \equiv m\vec{\mathbf{v}}$$ (momentum);

$$\frac{\mathrm{d}\vec{\mathbf{P}}_{\text{tot}}}{\mathrm{d}t} = 0$$ (conservation of momentum in absence of external force)

$$\vec{\mathbf{F}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}$$ (Newton's second law in terms of momentum);

$$\vec{\mathbf{r}}_{\text{cm}} \equiv \frac{1}{M_{\text{tot}}} \sum_i m_i \vec{\mathbf{r}}_i$$ (position of center of mass);

$$\vec{\mathbf{v}}_{\rm cm} \equiv \frac{\mathrm{d}\vec{\mathbf{r}}_{\rm cm}}{\mathrm{d}t} = \frac{1}{M_{\rm tot}} \sum_i m_i \vec{\mathbf{v}}_i \qquad \text{(velocity of center of mass);}$$

$$\vec{\mathbf{F}}^{\rm ext}_{\rm tot} = M_{\rm tot}\vec{\mathbf{a}}_{\rm cm} = \frac{\mathrm{d}\vec{\mathbf{P}}_{\rm tot}}{\mathrm{d}t} \qquad \text{(acceleration of a system of particles);}$$

$$\vec{\mathbf{P}}_{\rm tot} = \sum_i m_i \vec{\mathbf{v}}_i = M_{\rm tot}\vec{\mathbf{v}}_{\rm cm} \qquad \text{(momentum of a system of particles);}$$

$$K_{\rm tot} = \frac{1}{2} M_{\rm tot} v_{\rm cm}^2 + \sum_i \frac{1}{2} m_i \left(\vec{\mathbf{v}}_i - \vec{\mathbf{v}}_{\rm cm}\right)^2 \qquad \text{(K.E. of a system of particles);}$$

$$\vec{\mathbf{J}} = \int_{t_1}^{t_2} \vec{\mathbf{F}}\,\mathrm{d}t = \int_{t_1}^{t_2} \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}\,\mathrm{d}t = \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1 \qquad \text{(impulse-momentum theorem).}$$

Rotation in Two Dimensions:

Most of the equations for this topic are most easily remembered in the context of the analogous equations for linear motion in one dimension:

TRANSLATION (one dimension)		ROTATION (about fixed axis)	
Name	Symbol	Name	Symbol
Position	x	Orientation	θ
Velocity	$v = \frac{\mathrm{d}x}{\mathrm{d}t}$	Angular velocity	$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}$
Acceleration	$a = \frac{\mathrm{d}v}{\mathrm{d}t}$	Angular acceleration	$\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t}$
Mass	$M = \sum_i m_i$	Moment of inertia	$I = \sum_i m_i R_i^2$
Force	F	Torque	$\boldsymbol{\tau} = F_\perp R = \pm\lvert\vec{\mathbf{F}}\rvert R_\perp$
Force equation	$\sum_i \vec{\mathbf{F}}^{\rm ext} = M\vec{\mathbf{a}}_{\rm cm}$	Torque equation	$\sum_i \boldsymbol{\tau}^{\rm ext} = I\alpha$
Kinetic energy	$\frac{1}{2}Mv^2$	Kinetic energy	$\frac{1}{2}I\omega^2$
Work done	$\vec{\mathbf{F}} \cdot \overrightarrow{\boldsymbol{\Delta r}}$	Work done	$\boldsymbol{\tau}\,\Delta\theta$

Other equations about rotation in two dimensions:

$$v_r = 0\ ;\quad v_\perp = R\omega \qquad \text{(velocity of point on rotating body);}$$

$$a_r = -\frac{v^2}{R} = -R\omega^2\ ;\quad a_\perp = R\alpha \qquad \text{(acceleration of point on rotating body);}$$

$$v = \pm R|\omega| \qquad \text{(rolling without slipping);}$$

$$\left.\begin{aligned} \sum \vec{\mathbf{F}}^{\text{ext}} &= M\vec{\mathbf{a}}_{\text{cm}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} \\ \sum \boldsymbol{\tau}^{\text{ext}} &= I_{\text{cm}}\alpha = \frac{\mathrm{d}L}{\mathrm{d}t} \end{aligned}\right\} \qquad \text{(combined translational and rotational motion);}$$

$$K_{\text{tot}} = \tfrac{1}{2}Mv_{\text{cm}}^2 + \tfrac{1}{2}I_{\text{cm}}\omega^2 \qquad \text{(kinetic energy for combined translational and rotational motion);}$$

$$I_\parallel = I_{\text{cm}} + Md^2 \qquad \text{(parallel-axis theorem);}$$

$$I_z = I_x + I_y \qquad \text{(perpendicular-axis theorem).}$$

TABLE OF STANDARD MOMENTS OF INERTIA:

Slender uniform rod of length ℓ, axis through center and perpendicular to axis of rod	$\frac{1}{12}m\ell^2$
Rectangular plate with dimensions $a \times b$, axis along one of the b edges	$\frac{1}{3}ma^2$
Thin-walled hollow cylinder of radius R, axis along axis of cylinder	mR^2
Uniform solid cylinder of radius R, axis along axis of cylinder	$\frac{1}{2}mR^2$
Thin-walled hollow sphere of radius R, axis through center	$\frac{2}{3}mR^2$
Solid uniform sphere of radius R, axis through center	$\frac{2}{5}mR^2$