MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 Spring 2005

# WEEKLY QUIZ 9

**Friday, April 8, 2005**

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

FAMILY (LAST) NAME

| | | | | | | | | | | | | | | | |
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GIVEN (FIRST) NAME

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|

STUDENT ID NUMBER

Please check (✔) your class ☞

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|---|---|---|---|
| | L01 | MTW 10:00 | Walter Lewin |
| | L02 | MTW 11:00 | Walter Lewin |
| | L03 | MTW 2:00 | Min Chen |
| | L04 | MTW 3:00 | Min Chen |

**INSTRUCTIONS:**

1. The FORMULA SHEET is in the back of this exam. You may tear it off. There is also an extra BLANK PAGE in case you need it.
2. This is a closed book exam. CALCULATORS, BOOKS, and NOTES are NOT ALLOWED.
3. Unless otherwise stated, to earn full credit you must show a valid DERIVATION and/or EXPLANATION of your answer, and you must express it in terms of the GIVEN VARIABLES.

| **Problem** | **Maximum** | **Score** | **Grader** |
|---|---|---|---|
| 1 | 30 | | |
| 2 | 40 | | |
| 3 | 30 | | |
| **TOTAL** | 100 | | |

**ANNOUNCEMENTS MADE AT THE QUIZ**

Problem 1(c): You should choose only one of the 5 possible answers. Choice (iv) should read: "When no external torque acts on the system."

Problem 2(b): Use the center of the wheel as the origin for defining the torque.

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Note that Problem 1 is worth 30 points, not 40 points as it says at the start of the problem.

Name ______________________

**Problem 1: Basic concepts about the rotation of rigid bodies (*40 points*)**

Mark your answer by circling it. For these short answer problems you need not show your work, and there will be no partial credit. *Warning: some problems may contain irrelevant information.*

(a) (*5 points*) A uniform solid cylinder and a uniform solid sphere have the same mass and radius, respectively $M$, and $R$. They roll without slipping down identical inclines. Which one reaches the bottom first? Briefly explain your answer.

(b) (*5 points*) If the cylinder and the sphere start with the same angular velocity and they roll without slipping up identical slopes, which one reaches the higher height? Briefly explain your answer.

(c) (*5 points*) When is the angular momentum of a system constant?

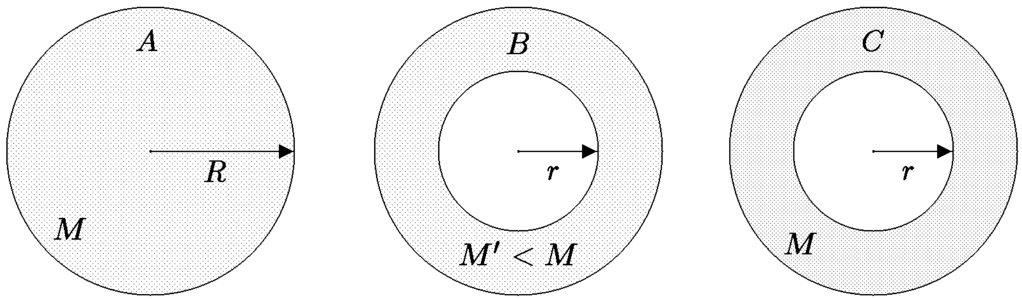
- (i) When the total kinetic energy is constant.
- (ii) When no net external force acts on the system.
- (iii) When the linear momentum and the energy are constant.
- (iv) When no torque acts on the system.
- (v) When the moment of inertia is constant.

(d) (*5 points*) A uniform cylinder of radius $R$ and mass $M$ rolls without slipping on a horizontal surface with a linear speed $v$. What is its kinetic energy?

(i) $\frac{1}{4}Mv^2$ (ii) $\frac{1}{2}Mv^2$ (iii) $\frac{1}{2}Mv^2 + MR^2v$ (iv) $\frac{3}{4}Mv^2$ (v) $\frac{2}{5}Mv^2$ (vi) $Mv^2$

— *Problem 1 Continues* —

Name ____________________



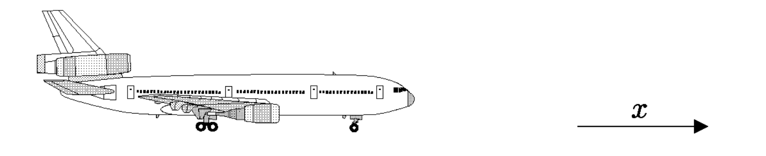

(e) (*5 points*) Consider three spheres, labeled A, B, and C, all with the same radius $R$. Sphere A is a uniform, solid sphere of mass $M$. Sphere B is made of the same material, and hence has the same density, but it has a hollow cavity of radius $r$, where $r < R$. Its mass $M'$ is therefore less than $M$. Sphere C also has a hollow cavity of radius $r$, but it is made of a denser material so that its mass is $M$, the same as sphere A. Denote the moments of inertia of the three spheres as $I_{\text{A}}$, $I_{\text{B}}$, and $I_{\text{C}}$. List these moments of inertia from largest to smallest, noting if any or all of them are equal.

(f) (*5 points*) Suppose that the three spheres in part (e) are all allowed to roll down the same incline, each starting from rest. Assume that they roll without slipping, but that all dissipative forces, such as air friction or rolling friction, are negligible. List the three spheres (A, B, and C) in the order in which they reach the bottom, again noting if any or all of them are equal.

Name

**Problem 2: Spinning wheels on an aircraft as it lands (*40 points*)**

An aircraft lands at a speed $v_0$. Before it touches down, its wheels are not rotating. After touching down, the wheels skid for a period of time, and afterwards roll without slipping. Assume that each wheel has radius $R$ and moment of inertia $I$ and supports a weight $Mg$ (which includes the weight of the wheel), and that the pilot does not apply reverse thrust until the aircraft is no longer skidding. Assume further that the coefficient of friction between the wheels and the runway is $\mu_s$ for static friction and $\mu_k$ for kinetic friction. For purposes of describing directions, assume that we are standing alongside the runway, and that the plane is moving to the right, in the positive $x$-direction. For each part, your answer may include symbols that represent the answers to previous parts, whether or not you have correctly answered those previous parts.




(a) (*6 points*) Immediately after the plane touches the ground, what is the magnitude $F$ and direction of the force of friction acting on each wheel?

(b) (*6 points*) At the same time as in (a), what is the torque $\tau$ due to friction acting on each wheel? Is this torque clockwise or counterclockwise as seen from our vantage point alongside the runway?

(c) (*6 points*) While the wheels are skidding, what is the velocity $v_x(t)$ of the plane? Take $t = 0$ as the instant when the wheels make contact with the ground.

(d) (*6 points*) During the same period as in (c), what is the angular velocity $\omega(t)$ of the wheels?

(e) (*6 points*) What is the speed $v_f$ of the plane when it stops skidding?

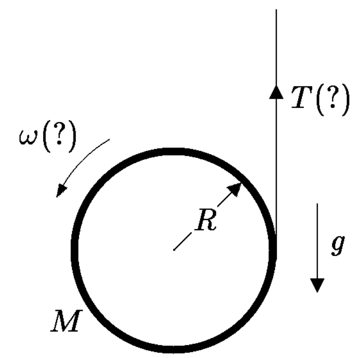
(f) (*5 points*) Now suppose that as soon as the wheels touch the ground, the pilot applies a reverse thrust from the engines, resulting in a net force of magnitude $F_d$ ($F_d > 0$) to the left. We will assume that $F_d$ can be treated as a constant over the relevant time period, and we will assume that the engines are positioned so that the thrust exerts no torque about the center of mass of the plane. In this case, what would be the velocity $v'_x(t)$ of the plane during the period when the wheels are skidding. As in part (c), take $t = 0$ as the instant when the wheels make contact with the ground.

(g) (*5 points*) During the same period as in (f), what is the angular velocity $\omega'(t)$ of the wheels?

Name

Name ______________________

**Problem 3: A cylinder on a string (*30 points*)**

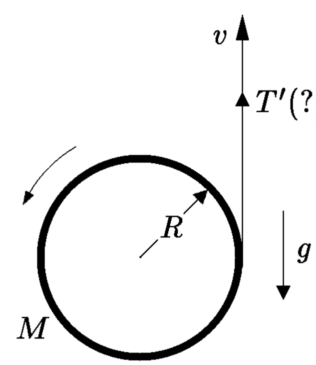



A hollow cylinder (i.e., a cylinder for which all the mass is concentrated in a wall of negligible thickness) of radius $R$ and mass $M$ is wrapped with an inextensible string of negligible mass. One end of the string is tied to the ceiling, and the cylinder is allowed to fall with its axis horizontal, as the string unrolls. Take the acceleration of gravity as $g$, downward, with $g > 0$.

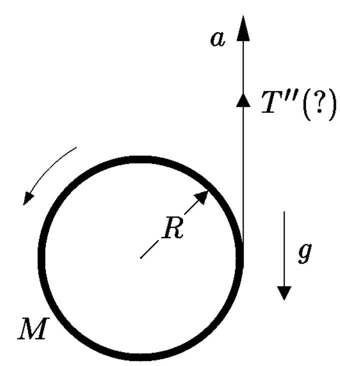
(a) (*10 points*) Find the angular velocity $\omega$ of the cylinder after it falls a distance $\ell$, starting from rest with the string taut.

(b) (*10 points*) Find the tension $T$ in the string, as the cylinder is falling.

(c) (*5 points*) Now suppose that instead of the string being tied to the ceiling, it is being held by a person who pulls the end upward with a constant speed $v$. What is the tension $T'$ in the string in this case?




(d) (*5 points*) Now suppose that instead of pulling the string upward with a constant speed, the person pulls the end of the string upward with a constant acceleration $a$. What is the tension $T''$ of the string in this case?



Name

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# QUIZ 9
# FORMULA SHEET
### Quiz Date: Friday, April 8, 2005

For motion in one dimension:

$$v_{\rm av} \equiv \frac{\Delta x}{\Delta t} \qquad \text{Average velocity;}$$

$$v \equiv \frac{{\rm d}x}{{\rm d}t} \qquad \text{Instantaneous velocity;}$$

For motion in three dimensions:

$$\vec{\mathbf{v}} \equiv \frac{{\rm d}\vec{\mathbf{r}}}{{\rm d}t}\,; \qquad \vec{\mathbf{a}} \equiv \frac{{\rm d}\vec{\mathbf{v}}}{{\rm d}t} = \frac{{\rm d}^2\vec{\mathbf{r}}}{{\rm d}t^2}\,; \qquad \vec{\mathbf{r}}(t_1) = \vec{\mathbf{r}}_0 + \int_0^{t_1} \vec{\mathbf{v}}\,{\rm d}t\,; \qquad \vec{\mathbf{v}}(t_1) = \vec{\mathbf{v}}_0 + \int_0^{t_1} \vec{\mathbf{a}}\,{\rm d}t\ .$$

For *constant* acceleration $\vec{\mathbf{a}}$, if $\vec{\mathbf{r}} = \vec{\mathbf{r}}_0$ and $\vec{\mathbf{v}} = \vec{\mathbf{v}}_0$ at time $t = 0$, then

$$\vec{\mathbf{v}}(t) = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t$$

$$\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \frac{1}{2}\vec{\mathbf{a}}t^2\ .$$

For one-dimensional motion with constant acceleration $a$:

$$v^2 = v_0^2 + 2a(x - x_0)\ .$$

For circular motion at constant speed $v$:

$$a = \frac{v^2}{r}\ ,$$

where $r$ is the radius of the circle, and the acceleration is directed towards the center of the circle.

If an object has position $\vec{\mathbf{r}}$ and velocity $\vec{\mathbf{v}}$, its position and velocity relative to an observer with position $\vec{\mathbf{r}}_0$ and velocity $\vec{\mathbf{v}}_0$ are given respectively by

$$\vec{\mathbf{r}}' = \vec{\mathbf{r}} - \vec{\mathbf{r}}_0\ , \qquad \vec{\mathbf{v}}' = \vec{\mathbf{v}} - \vec{\mathbf{v}}_0\ .$$

Average velocity and acceleration are given by

$$\vec{\mathbf{v}}_{\rm average} \equiv \frac{\Delta\vec{\mathbf{r}}}{\Delta t}\ , \qquad \vec{\mathbf{a}}_{\rm average} \equiv \frac{\Delta\vec{\mathbf{v}}}{\Delta t}\ .$$

**Mass, Acceleration, and Force:**

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} \qquad \text{(Newton's second law);}$$

$$\vec{\mathbf{F}} = -\frac{GMm}{r^2}\hat{\boldsymbol{r}} \qquad \text{(the gravitational force between two particles);}$$

$$\vec{\mathbf{F}} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}\hat{\boldsymbol{r}} \qquad \text{(the electrostatic force between two particles);}$$

$$F_x = -kx \qquad \text{(Hooke's law);}$$

where $\hat{\boldsymbol{r}}$ is a unit vector pointing from the particle which is the source of the force, toward the particle on which the force is acting.

**Friction:**

$$\left|\vec{\mathbf{F}}_k\right| = \mu_k \left|\vec{\mathbf{N}}\right| \qquad \text{(kinetic friction);}$$

$$\left|\vec{\mathbf{F}}_s\right| \le \mu_s \left|\vec{\mathbf{N}}\right| \qquad \text{(static friction).}$$

**Kinetic Energy, Work, and Potential Energy:**

| **Description** | **1 Dimension** | **3 Dimensions** |
|---|---|---|
| Work done by a constant force $\vec{\mathbf{F}}$ | $W \equiv F\Delta x$ | $W \equiv \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{\Delta r}}$ |
| Work done by a varying force $\vec{\mathbf{F}}$ | $W \equiv \int F(x)\,\mathrm{d}x$ | $W \equiv \int_{\vec{\mathbf{r}}_1}^{\vec{\mathbf{r}}_2} \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{dr}}$ |
| Potential energy derived from force $\vec{\mathbf{F}}$ | $U(x_p) \equiv U_0 - \int_{x_0}^{x_p} F\,\mathrm{d}x$ | $U(\vec{\mathbf{r}}_p) \equiv U_0 - \int_{\vec{\mathbf{r}}_0}^{\vec{\mathbf{r}}_p} \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{dr}}$ |
| Force derived from potential energy | $F = -\frac{\mathrm{d}U}{\mathrm{d}x}$ | $\vec{\mathbf{F}} = \left[-\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y}, -\frac{\partial U}{\partial z}\right]$ |

| | |
|---|---|
| $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \equiv \lvert\vec{\mathbf{a}}\rvert\lvert\vec{\mathbf{b}}\rvert \cos\theta = a_x b_x + a_y b_y + a_z b_z$ | (scalar (or dot) product of two vectors); |
| $E_k \equiv \frac{1}{2}mv^2$ | (kinetic energy of a particle); |
| $W_{\text{tot}} = E_{k,f} - E_{k,i}$ | (work-energy theorem: always true if $W_{\text{tot}}$ includes work due to all forces; a non-rigid object can do work on itself!); |
| $E_{k,i} + U_i + W_{\text{other}} = E_{k,f} + U_f$ | (generalized work-energy theorem: always true if $W_{\text{other}}$ includes work due to all forces not included in $U$); |
| $\frac{1}{2}mv^2 + U(x) = \text{constant}$ | (conservation of mechanical energy: true in the absence of dissipative forces); |
| $\frac{1}{2}mv^2 + mgh = \frac{1}{2}mv_0^2$ | (conservation of mechanical energy for a projectile: true in the absence of dissipative forces); |
| $W = \frac{1}{2}kx^2$ | (work to compress a spring); |
| $U = \frac{1}{2}kx^2$ | (potential energy for spring force); |
| $W = mgh$ | (work to lift a body near the surface of the Earth); |
| $U = mgh$ | (gravitational potential energy, near the surface of the Earth); |
| $U = -\frac{GMm}{r}$ | (gravitational potential energy, spherical bodies); |
| $U = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r}$ | (electrostatic potential energy, spherical charges). |

**Momentum, Center of Mass, and Systems of Particles:**

| | |
|---|---|
| $\vec{\mathbf{F}}_{\text{AB}} = -\vec{\mathbf{F}}_{\text{BA}}$ | (Newton's third law); |
| $\vec{\mathbf{p}} \equiv m\vec{\mathbf{v}}$ | (momentum); |
| $\frac{\mathrm{d}\vec{\mathbf{P}}_{\text{tot}}}{\mathrm{d}t} = 0$ | (conservation of momentum in absence of external force) |
| $\vec{\mathbf{F}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}$ | (Newton's second law in terms of momentum); |
| $\vec{\mathbf{r}}_{\text{cm}} \equiv \frac{1}{M_{\text{tot}}}\sum_i m_i\vec{\mathbf{r}}_i$ | (position of center of mass); |

$$\vec{\mathbf{v}}_{\rm cm} \equiv \frac{d\vec{\mathbf{r}}_{\rm cm}}{dt} = \frac{1}{M_{\rm tot}} \sum_i m_i \vec{\mathbf{v}}_i \qquad \text{(velocity of center of mass);}$$

$$\vec{\mathbf{F}}^{\rm ext}_{\rm tot} = M_{\rm tot} \vec{\mathbf{a}}_{\rm cm} = \frac{d\vec{\mathbf{P}}_{\rm tot}}{dt} \qquad \text{(acceleration of a system of particles);}$$

$$\vec{\mathbf{P}}_{\rm tot} = \sum_i m_i \vec{\mathbf{v}}_i = M_{\rm tot} \vec{\mathbf{v}}_{\rm cm} \qquad \text{(momentum of a system of particles);}$$

$$K_{\rm tot} = \frac{1}{2} M_{\rm tot} v^2_{\rm cm} + \sum_i \frac{1}{2} m_i \left( \vec{\mathbf{v}}_i - \vec{\mathbf{v}}_{\rm cm} \right)^2 \qquad \text{(K.E. of a system of particles);}$$

$$\vec{\mathbf{J}} = \int_{t_1}^{t_2} \vec{\mathbf{F}}\, dt = \int_{t_1}^{t_2} \frac{d\vec{\mathbf{p}}}{dt}\, dt = \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1 \qquad \text{(impulse-momentum theorem).}$$

**Rotation in Two Dimensions:**

Most of the equations for this topic are most easily remembered in the context of the analogous equations for linear motion in one dimension:

| TRANSLATION (one dimension) | | ROTATION (about fixed axis) | |
|---|---|---|---|
| Name | Symbol | Name | Symbol |
| Position | $x$ | Orientation | $\theta$ |
| Velocity | $v = \frac{dx}{dt}$ | Angular velocity | $\omega = \frac{d\theta}{dt}$ |
| Acceleration | $a = \frac{dv}{dt}$ | Angular acceleration | $\alpha = \frac{d\omega}{dt}$ |
| Mass | $M = \sum_i m_i$ | Moment of inertia | $I = \sum_i m_i R_i^2$ |
| Force | $F$ | Torque | $\boldsymbol{\tau} = F_\perp R$<br>$= \pm\lvert\vec{\mathbf{F}}\rvert R_\perp$ |
| Force equation | $\sum_i \vec{\mathbf{F}}^{\rm ext} = M\vec{\mathbf{a}}_{\rm cm}$ | Torque equation | $\sum_i \boldsymbol{\tau}^{\rm ext} = I\alpha$ |
| Momentum | $p = Mv$ | Angular momentum | $L = I\omega$ |
| Kinetic energy | $\frac{1}{2}Mv^2$ | Kinetic energy | $\frac{1}{2}I\omega^2$ |
| Work done | $\vec{\mathbf{F}} \cdot \overrightarrow{\boldsymbol{\Delta r}}$ | Work done | $\boldsymbol{\tau}\, \Delta\theta$ |

Other equations about rotation in two dimensions:

| | |
|---|---|
| $v_r = 0\ ;\quad v_\perp = R\omega$ | (velocity of point on rotating body); |
| $a_r = -\dfrac{v^2}{R} = -R\omega^2\ ;\quad a_\perp = R\alpha$ | (acceleration of point on rotating body); |
| $v = \pm R\lvert\omega\rvert$ | (rolling without slipping); |
| $\left.\begin{aligned}\sum \vec{\mathbf{F}}^{\text{ext}} &= M\vec{\mathbf{a}}_{\text{cm}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}\\ \sum \boldsymbol{\tau}^{\text{ext}} &= I_{\text{cm}}\alpha = \frac{\mathrm{d}L}{\mathrm{d}t}\end{aligned}\right\}$ | (combined translational and rotational motion); |
| $K_{\text{tot}} = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2$ | (kinetic energy for combined translational and rotational motion); |
| $I_\parallel = I_{\text{cm}} + Md^2$ | (parallel-axis theorem); |
| $I_z = I_x + I_y$ | (perpendicular-axis theorem). |

**TABLE OF STANDARD MOMENTS OF INERTIA:**

| | |
|---|---|
| Slender uniform rod of length $\ell$, axis through center and perpendicular to axis of rod | $\frac{1}{12}m\ell^2$ |
| Rectangular plate with dimensions $a \times b$, axis along one of the $b$ edges | $\frac{1}{3}ma^2$ |
| Thin-walled hollow cylinder of radius R, axis along axis of cylinder | $mR^2$ |
| Uniform solid cylinder of radius R, axis along axis of cylinder | $\frac{1}{2}mR^2$ |
| Thin-walled hollow sphere of radius $R$, axis through center | $\frac{2}{3}mR^2$ |
| Solid uniform sphere of radius $R$, axis through center | $\frac{2}{5}mR^2$ |

**Rotations in Vector Notation:**

$$c_x = a_y b_z - a_z b_y \ ;$$
$$c_y = a_z b_x - a_x b_z \ ;$$
$$c_z = a_x b_y - a_y b_x \ .$$
(vector cross product, component form);

$$|\vec{\mathbf{c}}| = |\vec{\mathbf{a}}||\vec{\mathbf{b}}| \sin\theta$$
(magnitude of vector cross product);

$$\vec{\mathbf{v}} = \vec{\boldsymbol{\omega}} \times \vec{\mathbf{r}}$$
(velocity of atom in rotating body with a fixed point);

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_P + \vec{\boldsymbol{\omega}} \times \left(\vec{\mathbf{r}} - \vec{\mathbf{r}}_P\right)$$
(velocity of atom in rotating body, general case);

$$\vec{\mathbf{L}} = \sum_i \vec{\mathbf{r}}_i \times \vec{\mathbf{p}}_i$$
(angular momentum, as vector product);

$$\vec{\boldsymbol{\tau}} = \sum_i \vec{\mathbf{r}}_i \times \vec{\mathbf{F}}_i$$
(vector torque, as vector product);

$$\vec{\boldsymbol{\tau}} = \frac{\mathrm{d}\vec{\mathbf{L}}}{\mathrm{d}t}$$
(torque equation);

$$\left.\begin{aligned} \sum \vec{\mathbf{F}}^{\mathrm{ext}} &= M\vec{\mathbf{a}}_{\mathrm{cm}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} \\ \sum \vec{\boldsymbol{\tau}}^{\mathrm{ext}} &= \frac{\mathrm{d}\vec{\mathbf{L}}_{\mathrm{cm}}}{\mathrm{d}t} \end{aligned}\right\}$$
(combined translational and rotational motion);

$$\vec{\mathbf{L}} = \vec{\mathbf{r}}_{\mathrm{cm}} \times \vec{\mathbf{p}}_{\mathrm{tot}} + \sum_i \vec{\mathbf{r}}_{\mathrm{rel},i} \times m_i \vec{\mathbf{v}}_{\mathrm{rel},i}$$
(angular momentum decomposition);

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}}_{\mathrm{cm}} \times \vec{\mathbf{F}}_{\mathrm{tot}} + \sum_i \vec{\mathbf{r}}_{\mathrm{rel},i} \times \vec{\mathbf{F}}_i$$
(torque decomposition).