MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 Spring 2005

WEEKLY QUIZ 10

Friday, April 15, 2005

Corrected Version, April 21, 2005: Diagram for Problem 1(b) was replaced

FAMILY (LAST) NAME

GIVEN (FIRST) NAME

STUDENT ID NUMBER

Please check (✔) your class

	L01	MTW 10:00	Walter Lewin
	L02	MTW 11:00	Walter Lewin
	L03	MTW 2:00	Min Chen
	L04	MTW 3:00	Min Chen

INSTRUCTIONS:

1. The FORMULA SHEET is in the back of this exam. You may tear it off. There is also an extra BLANK PAGE in case you need it.
2. This is a closed book exam. CALCULATORS, BOOKS, and NOTES are NOT ALLOWED.
3. Unless otherwise stated, to earn full credit you must show a valid DERIVATION and/or EXPLANATION of your answer, and you must express it in terms of the GIVEN VARIABLES.

Problem	**Maximum**	**Score**	**Grader**
1	25		
2	30		
3	20		
4	25		
TOTAL	100		

ANNOUNCEMENTS MADE (AND NOT MADE) AT THE QUIZ

Problem 2: The distances d and h are both measured along the direction of the ladder (i.e., they are not measured vertically).

Problem 3: The moment of inertia I is the moment of inertia for rotation about the axis of symmetry of the cylinder (where the center of mass is located).

Problem 1(b): It was not announced at the quiz, but it should have been specified that the moment of inertia under discussion is that for rotation about the center of the sphere.

Problem 4: The diagram was not drawn correctly. The bottom end of the line indicated by h should have been at ground level, the same level as the bottom of the wheels. On part (c) it should have said to neglect the mass of the wheels, relative to the mass of the car.

Name ______________________

Problem 1: Basic concepts about the rotation of rigid bodies (*25 points*)

Mark your answer by circling it. For these short answer problems you need not show your work, and there will be no partial credit. *Warning: some problems may contain irrelevant information.*

(a) (*5 points*) When is the angular momentum of a system constant? Choose one.

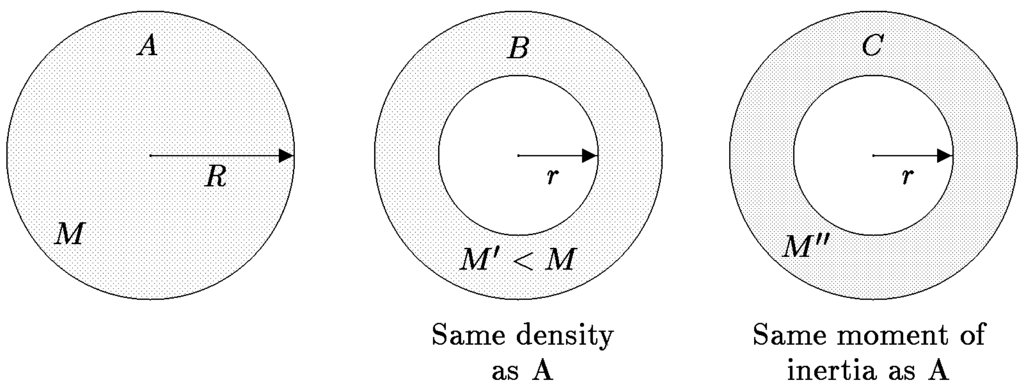
(i) When the total kinetic energy is constant.

(ii) When no net external force acts on the system.

(iii) When the linear momentum and the energy are constant.

(iv) When no external torque acts on the system.

(v) When the moment of inertia is constant.



(b) (*5 points*) Consider three spheres, labeled A, B, and C, all with the same radius R. Sphere A is a uniform, solid sphere of mass M. Sphere B is made of the same material, and hence has the same density, but it has a hollow cavity of radius r, where $r < R$. Its mass is M'. Sphere C also has a hollow cavity of radius r, but it is made of a different material so that its mass M'' is just the right value to give sphere C the same moment of inertia as sphere A. List the three masses, M, M', and M'', from largest to smallest, noting if any or all of them are equal.

(c) (*5 points*) Suppose that the three spheres in part (b) are all allowed to roll down the same incline, each starting from rest. Assume that they roll without slipping, but that all dissipative forces, such as air friction or rolling friction, are negligible. List the three spheres (A, B, and C) in the order in which they reach the bottom, again noting if any or all of them are equal.

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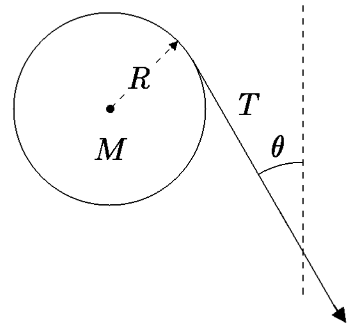
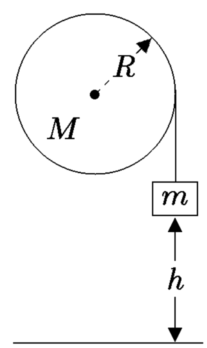
(d) (*5 points*) A wheel, which consists of a solid uniform cylindrical disk of radius R and mass M, is pivoted about a fixed axle through the axis of the cylinder. A massless inextensible rope is wound around the rim of the wheel so that it cannot slip. The rope is pulled with a tension T at angle θ with respect to the vertical, as shown. What is the magnitude of the angular acceleration α of the wheel about the pivot? Neglect all friction.



(i) $\frac{2T}{MR}$ (ii) $\frac{T\sin\theta}{\pi MR}$ (iii) $\frac{T\cos\theta}{\pi MR}$ (iv) $\frac{T}{\pi MR}$ (v) $\frac{2T\sin\theta}{MR}$ (vi) $\frac{2T\cos\theta}{MR}$

(e) (*5 points*) A solid uniform cylinder of mass M and radius R is pivoted about a fixed horizontal rod. A massless inextensible string is wrapped around it, and attached to a block of mass m which is initially at a height h above the floor. The acceleration of gravity is g, directed downward. The block is released from rest. By what total angle $\Delta\theta$ (in radians) has the cylinder turned when the block hits the floor?



(i) $\frac{M}{m}$ (ii) $\sqrt{1+\frac{2M}{m}}$ (iii) $\frac{h}{R}$ (iv) $\sqrt{1+\frac{h}{2R}}$

(v) $\frac{MR}{mh}$ (vi) $\left(\frac{MR}{mh}\right)^2$

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Problem 2: Will the ladder slip? (*30 points*)

A uniform ladder of mass m and length ℓ rests against a smooth wall, making an angle θ with respect to the ground. Assume that there is negligible friction between the ladder and the wall. A do-it-yourself enthusiast of mass M stands on the ladder a distance d from the bottom. The force of gravity acts downward with acceleration g, where $g > 0$. Assume that the ladder is stationary, i.e., that it does not slip.

(a) (*5 points*) What is the magnitude N_g of the upward normal force that the ground exerts on the ladder?

(b) (*5 points*) Does the frictional force that the ground exerts on the ladder point toward the wall, or away from the wall? Explain briefly.

(c) (*10 points*) Let F denote the magnitude of the force of friction that the ground exerts on the ladder, and let N_w denote the magnitude of the normal force that the wall exerts on the ladder. Write down two equations which can be solved to determine these two variables. The equations should be written in terms of any or all of the variables F, N_w, m, ℓ, θ, M, d, g, and N_g.

(d) (*5 points*) What is the minimum coefficient of friction between the ladder and the ground that is required in order that the ladder will not slip? Whether or not you have answered parts (a) and (c), you can use any of the variables N_w, N_g, and F in expressing your answer, as well as any of the given variables.

Now suppose that the ladder is not uniform, but instead is tapered so that there is more material near the bottom than near the top. We are not told the detailed geometry, but we are told that the center of mass of the ladder lies a distance h from the bottom. (The mass of the ladder is still m, and its overall length is still ℓ.)

(e) (*5 points*) For this case, write down two equations which can be solved to determine F (the magnitude of the force of friction that the ground exerts on the ladder) and N_w (the magnitude of the normal force that the wall exerts on the ladder). The equations should be written in terms of any or all of the variables F, N_w, m, ℓ, h, θ, M, d, g, and N_g.

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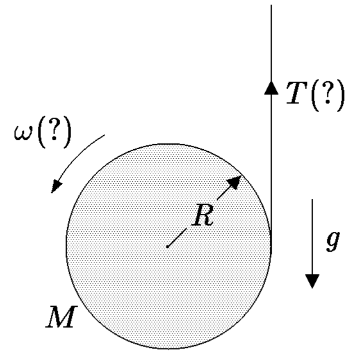
Problem 3: A cylinder on a string (*20 points*)

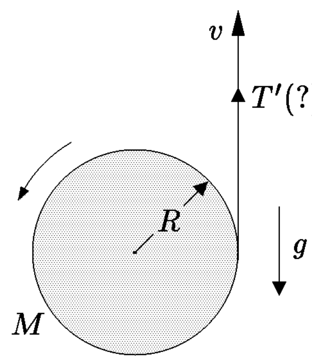
A cylinder of radius R and mass M is wrapped with an inextensible string of negligible mass. The density of the cylinder is not uniform, but depends on the distance from the axis. The moment of inertia I therefore cannot be calculated in terms of M and R, but is to be taken as a given variable. One end of the string is tied to the ceiling, and the cylinder is allowed to fall with its axis horizontal, as the string unrolls. Take the acceleration of gravity as g, downward, with $g > 0$.



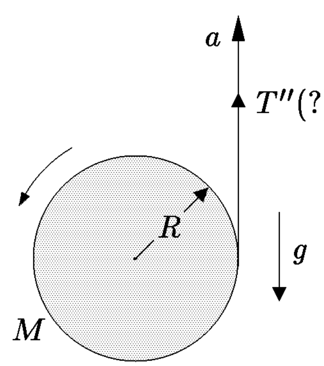
(a) (*5 points*) Find the angular velocity ω of the cylinder after it falls a distance ℓ, starting from rest with the string taut.

(b) (*5 points*) Find the tension T in the string, as the cylinder is falling.

(c) (*5 points*) Now suppose that instead of the string being tied to the ceiling, it is being held by a person who pulls the end upward with a constant speed v. What is the tension T' in the string in this case?



(d) (*5 points*) Now suppose that instead of pulling the string upward with a constant speed, the person pulls the end of the string upward with a constant acceleration a. What is the tension T'' of the string in this case?



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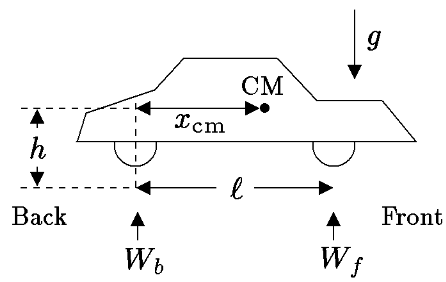
Problem 4: Distribution of weight on the wheels of a car (*25 points*)

A car of total mass M stands at rest on level ground. The car is symmetric left and right, so the wheels on the left side can be considered equivalent to the wheels on the right side. The distance between the front and back wheels is ℓ, and the center of mass of the car is located a distance h from the ground, and a horizontal distance $x_{\rm cm}$ from a point directly above the contact point of the back wheel, as shown in the diagram. Gravity acts downward, with acceleration g, with $g > 0$.

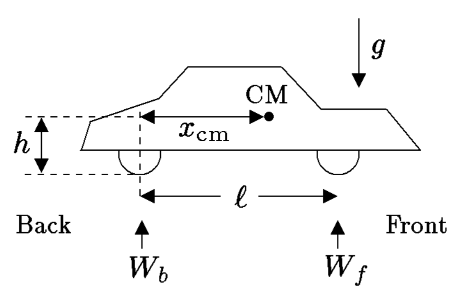


(a) (*10 points*) The ground exerts a total upward normal force of magnitude W_f on the two front wheels, evenly distributed between the left and the right. The ground exerts a total upward normal force of magnitude W_b on the back wheels, also evenly distributed between the left and the right. Find expressions for W_f and W_b in terms of some or all of the given variables M, g, ℓ, $x_{\rm cm}$, and h.

(b) (*5 points*) Now suppose that the car is moving forward, on a level road, at a constant speed v. In this case we will let W_f' denote the magnitude of the total upward normal force on the front wheels, and W_b' denote the magnitude of the total upward normal force on the back wheels. Find expressions for W_f' and W_b' in terms of some or all of the variables v, W_f, W_b, M, g, ℓ, $x_{\rm cm}$, and h. Note that the variables W_f and W_b may appear in your answer, whether or not you answered part (a).

(c) (*10 points*) Now suppose that the car starts from rest at time $t = 0$, and accelerates forward with a uniform acceleration of magnitude a, directed to the right. Assume that the road is level, and that the car remains level as it accelerates. Assume that the wheels roll without slipping, and neglect any dissipative forces, such as air friction or rolling friction. For this part use W_f'' to denote the magnitude of the total upward normal force on the front wheels, and W_b'' denote the magnitude of the total upward normal force on the back wheels. Write down two equations that can be solved to determine W_f'' and W_b'' in terms of any or all of the variables a, W_f, W_b, W_f', W_b', M, g, ℓ, $x_{\rm cm}$, h, and t.

Correction: As stated on the cover sheet, the diagram for this problem was not drawn correctly. The correct version is:



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Physics 8.01 Spring 2005

QUIZ 10
FORMULA SHEET
Quiz Date: Friday, April 15, 2005

For motion in one dimension:

$$v_{\rm av} \equiv \frac{\Delta x}{\Delta t} \qquad \text{Average velocity;}$$

$$v \equiv \frac{{\rm d}x}{{\rm d}t} \qquad \text{Instantaneous velocity;}$$

For motion in three dimensions:

$$\vec{\mathbf{v}} \equiv \frac{{\rm d}\vec{\mathbf{r}}}{{\rm d}t}\,; \qquad \vec{\mathbf{a}} \equiv \frac{{\rm d}\vec{\mathbf{v}}}{{\rm d}t} = \frac{{\rm d}^2\vec{\mathbf{r}}}{{\rm d}t^2}\,; \qquad \vec{\mathbf{r}}(t_1) = \vec{\mathbf{r}}_0 + \int_0^{t_1} \vec{\mathbf{v}}\,{\rm d}t\,; \qquad \vec{\mathbf{v}}(t_1) = \vec{\mathbf{v}}_0 + \int_0^{t_1} \vec{\mathbf{a}}\,{\rm d}t\ .$$

For *constant* acceleration $\vec{\mathbf{a}}$, if $\vec{\mathbf{r}} = \vec{\mathbf{r}}_0$ and $\vec{\mathbf{v}} = \vec{\mathbf{v}}_0$ at time $t = 0$, then

$$\vec{\mathbf{v}}(t) = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t$$

$$\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \frac{1}{2}\vec{\mathbf{a}}t^2\ .$$

For one-dimensional motion with constant acceleration a:

$$v^2 = v_0^2 + 2a(x - x_0)\ .$$

For circular motion at constant speed v:

$$a = \frac{v^2}{r}\ ,$$

where r is the radius of the circle, and the acceleration is directed towards the center of the circle.

If an object has position $\vec{\mathbf{r}}$ and velocity $\vec{\mathbf{v}}$, its position and velocity relative to an observer with position $\vec{\mathbf{r}}_0$ and velocity $\vec{\mathbf{v}}_0$ are given respectively by

$$\vec{\mathbf{r}}' = \vec{\mathbf{r}} - \vec{\mathbf{r}}_0\ , \qquad \vec{\mathbf{v}}' = \vec{\mathbf{v}} - \vec{\mathbf{v}}_0\ .$$

Average velocity and acceleration are given by

$$\vec{\mathbf{v}}_{\rm average} \equiv \frac{\Delta\vec{\mathbf{r}}}{\Delta t}\ , \qquad \vec{\mathbf{a}}_{\rm average} \equiv \frac{\Delta\vec{\mathbf{v}}}{\Delta t}\ .$$

Mass, Acceleration, and Force:

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} \qquad \text{(Newton's second law);}$$

$$\vec{\mathbf{F}} = -\frac{GMm}{r^2}\hat{\boldsymbol{r}} \qquad \text{(the gravitational force between two particles);}$$

$$\vec{\mathbf{F}} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}\hat{\boldsymbol{r}} \qquad \text{(the electrostatic force between two particles);}$$

$$F_x = -kx \qquad \text{(Hooke's law);}$$

where $\hat{\boldsymbol{r}}$ is a unit vector pointing from the particle which is the source of the force, toward the particle on which the force is acting.

Friction:

$$\left|\vec{\mathbf{F}}_k\right| = \mu_k \left|\vec{\mathbf{N}}\right| \qquad \text{(kinetic friction);}$$

$$\left|\vec{\mathbf{F}}_s\right| \le \mu_s \left|\vec{\mathbf{N}}\right| \qquad \text{(static friction).}$$

Kinetic Energy, Work, and Potential Energy:

Description	**1 Dimension**	**3 Dimensions**
Work done by a constant force $\vec{\mathbf{F}}$	$W \equiv F\Delta x$	$W \equiv \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{\Delta r}}$
Work done by a varying force $\vec{\mathbf{F}}$	$W \equiv \int F(x)\,\mathrm{d}x$	$W \equiv \int_{\vec{\mathbf{r}}_1}^{\vec{\mathbf{r}}_2} \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{dr}}$
Potential energy derived from force $\vec{\mathbf{F}}$	$U(x_p) \equiv U_0 - \int_{x_0}^{x_p} F\,\mathrm{d}x$	$U(\vec{\mathbf{r}}_p) \equiv U_0 - \int_{\vec{\mathbf{r}}_0}^{\vec{\mathbf{r}}_p} \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{dr}}$
Force derived from potential energy	$F = -\frac{\mathrm{d}U}{\mathrm{d}x}$	$\vec{\mathbf{F}} = \left[-\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y}, -\frac{\partial U}{\partial z}\right]$

$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \equiv \lvert\vec{\mathbf{a}}\rvert\lvert\vec{\mathbf{b}}\rvert \cos\theta$ $= a_x b_x + a_y b_y + a_z b_z$	(scalar (or dot) product of two vectors);
$E_k \equiv \frac{1}{2}mv^2$	(kinetic energy of a particle);
$W_{\rm tot} = E_{k,f} - E_{k,i}$	(work-energy theorem: always true if $W_{\rm tot}$ includes work due to all forces; a non-rigid object can do work on itself!);
$E_{k,i} + U_i + W_{\rm other}$ $= E_{k,f} + U_f$	(generalized work-energy theorem: always true if $W_{\rm other}$ includes work due to all forces not included in U);
$\frac{1}{2}mv^2 + U(x) = \text{constant}$	(conservation of mechanical energy: true in the absence of dissipative forces);
$\frac{1}{2}mv^2 + mgh = \frac{1}{2}mv_0^2$	(conservation of mechanical energy for a projectile: true in the absence of dissipative forces);
$W = \frac{1}{2}kx^2$	(work to compress a spring);
$U = \frac{1}{2}kx^2$	(potential energy for spring force);
$W = mgh$	(work to lift a body near the surface of the Earth);
$U = mgh$	(gravitational potential energy, near the surface of the Earth);
$U = -\frac{GMm}{r}$	(gravitational potential energy, spherical bodies);
$U = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r}$	(electrostatic potential energy, spherical charges).

Momentum, Center of Mass, and Systems of Particles:

$\vec{\mathbf{F}}_{\rm AB} = -\vec{\mathbf{F}}_{\rm BA}$	(Newton's third law);
$\vec{\mathbf{p}} \equiv m\vec{\mathbf{v}}$	(momentum);
$\frac{\mathrm{d}\vec{\mathbf{P}}_{\rm tot}}{\mathrm{d}t} = 0$	(conservation of momentum in absence of external force)
$\vec{\mathbf{F}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}$	(Newton's second law in terms of momentum);
$\vec{\mathbf{r}}_{\rm cm} \equiv \frac{1}{M_{\rm tot}}\sum_i m_i\vec{\mathbf{r}}_i$	(position of center of mass);

$$\vec{\mathbf{v}}_{\rm cm} \equiv \frac{\mathrm{d}\vec{\mathbf{r}}_{\rm cm}}{\mathrm{d}t} = \frac{1}{M_{\rm tot}} \sum_i m_i \vec{\mathbf{v}}_i \qquad \text{(velocity of center of mass);}$$

$$\vec{\mathbf{F}}^{\rm ext}_{\rm tot} = M_{\rm tot}\vec{\mathbf{a}}_{\rm cm} = \frac{\mathrm{d}\vec{\mathbf{P}}_{\rm tot}}{\mathrm{d}t} \qquad \text{(acceleration of a system of particles);}$$

$$\vec{\mathbf{P}}_{\rm tot} = \sum_i m_i \vec{\mathbf{v}}_i = M_{\rm tot}\vec{\mathbf{v}}_{\rm cm} \qquad \text{(momentum of a system of particles);}$$

$$K_{\rm tot} = \frac{1}{2} M_{\rm tot} v^2_{\rm cm} + \sum_i \frac{1}{2} m_i \left(\vec{\mathbf{v}}_i - \vec{\mathbf{v}}_{\rm cm}\right)^2 \qquad \text{(K.E. of a system of particles);}$$

$$\vec{\mathbf{J}} = \int_{t_1}^{t_2} \vec{\mathbf{F}}\,\mathrm{d}t = \int_{t_1}^{t_2} \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}\,\mathrm{d}t = \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1 \qquad \text{(impulse-momentum theorem).}$$

Rotation in Two Dimensions:

Most of the equations for this topic are most easily remembered in the context of the analogous equations for linear motion in one dimension:

TRANSLATION (one dimension)		ROTATION (about fixed axis)	
Name	Symbol	Name	Symbol
Position	x	Orientation	θ
Velocity	$v = \frac{\mathrm{d}x}{\mathrm{d}t}$	Angular velocity	$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}$
Acceleration	$a = \frac{\mathrm{d}v}{\mathrm{d}t}$	Angular acceleration	$\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t}$
Mass	$M = \sum_i m_i$	Moment of inertia	$I = \sum_i m_i R_i^2$
Force	F	Torque	$\boldsymbol{\tau} = F_\perp R$ $= \pm\lvert\vec{\mathbf{F}}\rvert R_\perp$
Force equation	$\sum_i \vec{\mathbf{F}}^{\rm ext} = M\vec{\mathbf{a}}_{\rm cm}$	Torque equation	$\sum_i \boldsymbol{\tau}^{\rm ext} = I\alpha$
Momentum	$p = Mv$	Angular momentum	$L = I\omega$
Kinetic energy	$\frac{1}{2}Mv^2$	Kinetic energy	$\frac{1}{2}I\omega^2$
Work done	$\vec{\mathbf{F}} \cdot \overrightarrow{\boldsymbol{\Delta r}}$	Work done	$\boldsymbol{\tau}\,\Delta\theta$

Other equations about rotation in two dimensions:

$v_r = 0$; $v_\perp = R\omega$	(velocity of point on rotating body);
$a_r = -\dfrac{v^2}{R} = -R\omega^2$; $a_\perp = R\alpha$	(acceleration of point on rotating body);
$v = \pm R\lvert\omega\rvert$	(rolling without slipping);
$\left.\begin{aligned} \sum \vec{\mathbf{F}}^{\text{ext}} &= M\vec{\mathbf{a}}_{\text{cm}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} \\ \sum \boldsymbol{\tau}^{\text{ext}} &= I_{\text{cm}}\alpha = \frac{\mathrm{d}L}{\mathrm{d}t} \end{aligned}\right\}$	(combined translational and rotational motion);
$K_{\text{tot}} = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2$	(kinetic energy for combined translational and rotational motion);
$I_{\parallel} = I_{\text{cm}} + Md^2$	(parallel-axis theorem);
$I_z = I_x + I_y$	(perpendicular-axis theorem).

TABLE OF STANDARD MOMENTS OF INERTIA:

Slender uniform rod of length ℓ, axis through center and perpendicular to axis of rod	$\frac{1}{12}m\ell^2$
Rectangular plate with dimensions $a \times b$, axis along one of the b edges	$\frac{1}{3}ma^2$
Thin-walled hollow cylinder of radius R, axis along axis of cylinder	mR^2
Uniform solid cylinder of radius R, axis along axis of cylinder	$\frac{1}{2}mR^2$
Thin-walled hollow sphere of radius R, axis through center	$\frac{2}{3}mR^2$
Solid uniform sphere of radius R, axis through center	$\frac{2}{5}mR^2$

Rotations in Vector Notation:

$$c_x = a_y b_z - a_z b_y \ ; \quad c_y = a_z b_x - a_x b_z \ ; \quad c_z = a_x b_y - a_y b_x \ .$$ (vector cross product, component form);

$$|\vec{\mathbf{c}}| = |\vec{\mathbf{a}}||\vec{\mathbf{b}}| \sin\theta$$ (magnitude of vector cross product);

$$\vec{\mathbf{v}} = \vec{\boldsymbol{\omega}} \times \vec{\mathbf{r}}$$ (velocity of atom in rotating body with a fixed point);

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_P + \vec{\boldsymbol{\omega}} \times \left(\vec{\mathbf{r}} - \vec{\mathbf{r}}_P\right)$$ (velocity of atom in rotating body, general case);

$$\vec{\mathbf{L}} = \sum_i \vec{\mathbf{r}}_i \times \vec{\mathbf{p}}_i$$ (angular momentum, as vector product);

$$\vec{\boldsymbol{\tau}} = \sum_i \vec{\mathbf{r}}_i \times \vec{\mathbf{F}}_i$$ (vector torque, as vector product);

$$\vec{\boldsymbol{\tau}} = \frac{\mathrm{d}\vec{\mathbf{L}}}{\mathrm{d}t}$$ (torque equation);

$$\left.\begin{aligned} \sum \vec{\mathbf{F}}^{\text{ext}} &= M\vec{\mathbf{a}}_{\text{cm}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} \\ \sum \vec{\boldsymbol{\tau}}^{\text{ext}} &= \frac{\mathrm{d}\vec{\mathbf{L}}_{\text{cm}}}{\mathrm{d}t} \end{aligned}\right\}$$ (combined translational and rotational motion);

$$\vec{\mathbf{L}} = \vec{\mathbf{r}}_{\text{cm}} \times \vec{\mathbf{p}}_{\text{tot}} + \sum_i \vec{\mathbf{r}}_{\text{rel},i} \times m_i \vec{\mathbf{v}}_{\text{rel},i}$$ (angular momentum decomposition);

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}}_{\text{cm}} \times \vec{\mathbf{F}}_{\text{tot}} + \sum_i \vec{\mathbf{r}}_{\text{rel},i} \times \vec{\mathbf{F}}_i$$ (torque decomposition).

For Static Bodies:

$$\sum \vec{\mathbf{F}}^{\text{ext}} = 0$$ (total external force vanishes);

$$\sum \vec{\boldsymbol{\tau}}^{\text{ext}} = 0$$ (total external torque about ANY point vanishes).