MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 Spring 2005

WEEKLY QUIZ 11

Friday, April 29, 2005

FAMILY (LAST) NAME

GIVEN (FIRST) NAME

STUDENT ID NUMBER

Please check (✔) your class

	L01	MTW 10:00	Walter Lewin
	L02	MTW 11:00	Walter Lewin
	L03	MTW 2:00	Min Chen
	L04	MTW 3:00	Min Chen

INSTRUCTIONS:

1. The FORMULA SHEET is in the back of this exam. You may tear it off. There is also an extra BLANK PAGE in case you need it.
2. This is a closed book exam. CALCULATORS, BOOKS, and NOTES are NOT ALLOWED.
3. Unless otherwise stated, to earn full credit you must show a valid DERIVATION and/or EXPLANATION of your answer, and you must express it in terms of the GIVEN VARIABLES.

ANNOUNCEMENT MADE AT THE QUIZ

Problem 1(e): Answers (i) and (iii) are identical. Circling one is equivalent to circling the other.

Problem	**Maximum**	**Score**	**Grader**
1	20		
2	20		
3	30		
4	30		
TOTAL	100		

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Problem 1: Basic concepts about gravity and orbits (*20 points*)

(a) Two planets, called Uno and Duo, are in circular orbits about the same star. The orbit of Duo, however, has twice the radius of the orbit of Uno. The ratio of the period T_D of Duo's orbit to the period T_U of Uno's orbit is given by $T_D/T_U =$

(i) $1/2$ (ii) $1/\sqrt{2}$ (iii) 1 (iv) $\sqrt{2}$ (v) 2 (vi) $2\sqrt{2}$ (vii) 4

(b) (*4 points*) Two completely different planets, which happen to also be called Uno and Duo, are each composed of the same material, which we will assume has a fixed density. The planet Duo has a radius twice as large as that of Uno. The ratio of the gravitational acceleration g_D at the surface of Duo to the gravitational acceleration g_U at the surface of Uno is given by $g_D/g_U =$

(i) $1/2$ (ii) $1/\sqrt{2}$ (iii) 1 (iv) $\sqrt{2}$ (v) 2 (vi) $2\sqrt{2}$ (vii) 4

(c) (*4 points*) An astronaut of mass m is a member of the crew of a space shuttle orbiting the earth at an altitude h. If the radius of the earth is R and g is the acceleration due to gravity at the surface of the earth, the magnitude of the gravitational force on the astronaut is:

(i) mg (ii) $mg\,\frac{R}{R+h}$ (iii) $mg\,\frac{(R+h)}{R}$ (iv) $mg\,\frac{R^2}{(R+h)^2}$ (v) $mg\,\frac{(R+h)^2}{R^2}$

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(d) (*4 points*) Two spherical planets, each of uniform density and total mass M, are separated from each other by a distance R. If they were point masses, Newton's law of gravity tells us that the force that each would exert on the other would have magnitude GM^2/R^2. They are not point masses, however, but instead each has a radius a, which is not negligible compared to R. When this nonzero size is taken into account the magnitude of the force between the planets is (choose one)

(i) larger than GM^2/R^2 when the spheres almost touch, but approaches GM^2/R^2 when $R \gg a$;

(ii) smaller than GM^2/R^2 when the spheres almost touch, but approaches GM^2/R^2 when $R \gg a$;

(iii) equal to GM^2/R^2 as long as the spheres are not touching.

(e) (*4 points*) A space probe measures the gravitational acceleration caused by a spherical astronomical object to be g_0, at a distance R_0 from its center. If the object is a black hole, what would be its Schwarzschild radius R_S?

(i) $\dfrac{2R_0^2 g_0}{c^2}$ (ii) $\dfrac{2R_0^3 g_0^2}{c^4}$ (iii) $\dfrac{2R_0^2 g_0}{c^2}$ (iv) $\dfrac{2g_0}{R_0^2 c^2}$ (v) $2R_0\sqrt{\dfrac{R_0 g_0}{c^2}}$

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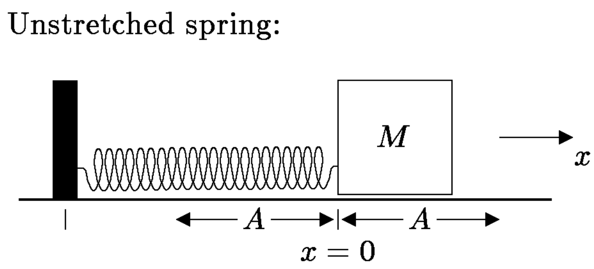
Problem 2: Basic concepts about periodic motion (*20 points*)



(a) (*4 points*) A block attached to a spring moves along the x axis on a frictionless horizontal table, executing simple harmonic motion, as shown above. The oscillations are centered at $x = 0$, and have an amplitude A. As the block crosses $x = 0$, it is observed to have speed v_0. What is the period T of the oscillation?

(i) $\dfrac{v_0}{A}$ (ii) $2\pi\dfrac{v_0}{A}$ (iii) $\dfrac{A^2}{v_0}$ (iv) $2\pi\dfrac{A^2}{v_0}$ (v) $\dfrac{A}{v_0}$ (vi) $\dfrac{2A}{v_0}$ (vii) $\dfrac{\pi A}{v_0}$ (viii) $\dfrac{2\pi A}{v_0}$

(b) (*4 points*) Now consider two blocks which, like the block in part (a), are each attached to a spring and moving along the x axis on a frictionless horizontal table, executing simple harmonic motion. They both oscillate with the same amplitude A, but the motion of the 2nd block has an angular frequency ω_2 which is twice as large as the angular frequency ω_1 for the first block. Let v_1 denote the speed of the first block when it crosses $x = A/2$, and let v_2 denote the speed of the second block when it crosses $x = A/2$. What is the ratio v_2/v_1?

(i) $1/2$ (ii) $1/\sqrt{2}$ (iii) 1 (iv) $\sqrt{2}$ (v) 2 (vi) $2\sqrt{2}$ (vii) 4

(c) (*4 points*) For the same two blocks described in part (b), let a_1 denote the acceleration of the first block when it crosses $x = A/2$, and let a_2 denote the acceleration of the second block when it crosses $x = A/2$. What is the ratio a_2/a_1?

(i) $1/2$ (ii) $1/\sqrt{2}$ (iii) 1 (iv) $\sqrt{2}$ (v) 2 (vi) $2\sqrt{2}$ (vii) 4

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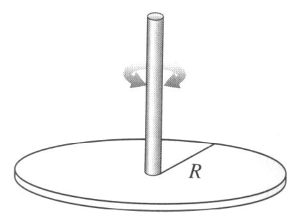
(d) (*4 points*) A pendulum is described by an angle θ, which measures its deviation from the vertical. It undergoes simple harmonic motion with frequency f and amplitude θ_m. At $t = 0$ the pendulum is at its maximum angle, $\theta = \theta_m$. Which of the following functions $\theta(t)$ describes the motion of the pendulum?

(i) $\theta_m \sin(ft)$ (ii) $\theta_m \sin(2\pi ft)$ (iii) $\theta_m \cos(ft)$ (iv) $\theta_m \cos(2\pi ft)$

(v) $\frac{1}{\sqrt{2}}\theta_m \sin(ft)$ (vi) $\frac{1}{\sqrt{2}}\theta_m \sin(2\pi ft)$ (vii) $\frac{1}{\sqrt{2}}\theta_m \cos(ft)$ (viii) $\frac{1}{\sqrt{2}}\theta_m \cos(2\pi ft)$

(e) (*4 points*) A horizontal circular disk of radius R and mass M has uniform density, and is suspended from a thread to create a torsional pendulum. When the thread twists through an angle θ, it creates a restoring torque $\boldsymbol{\tau} = -\kappa\theta$. What is the frequency f of this pendulum?



(i) $\frac{R}{2\pi}\sqrt{\frac{M}{2\kappa}}$ (ii) $2\pi R\sqrt{\frac{M}{2\kappa}}$ (iii) $\frac{R}{2\pi}\sqrt{\frac{M}{\kappa}}$ (iv) $2\pi R\sqrt{\frac{M}{\kappa}}$

(v) $\frac{1}{2\pi R}\sqrt{\frac{2\kappa}{M}}$ (vi) $\frac{2\pi}{R}\sqrt{\frac{2\kappa}{M}}$ (vii) $\frac{1}{2\pi R}\sqrt{\frac{\kappa}{M}}$ (viii) $\frac{2\pi}{R}\sqrt{\frac{\kappa}{M}}$

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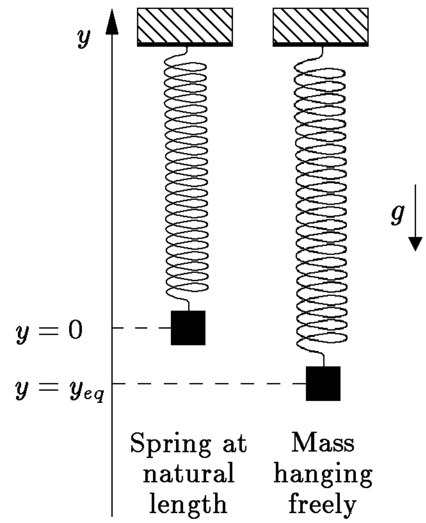
Problem 3: A mass suspended on a spring (*30 points*)

A block of mass M is suspended from a massless ideal spring of spring constant k. The coordinate system is defined so that y is directed vertically upwards and $y = 0$ when the spring is at its natural (i.e., unstretched) length. The mass is first positioned at $y = 0$, and then lowered gently until it is hanging freely from the spring.



(a) (*3 points*) What is the value of y_{eq}, the coordinate of the equilibrium point at which the block comes to rest?

For the remaining parts you may treat y_{eq} as if it were a given variable, whether or not you answered (a) correctly.

(b) (*3 points*) When the block comes to rest, what is its gravitational potential energy U_g, compared to its value when the block was at $y = 0$?

(c) (*3 points*) When the block comes to rest, what is the potential energy U_s stored in the spring, compared to its value when the block was at $y = 0$?

(d) As the block was lowered from $y = 0$ to its equilibrium position,

(i) (*2 points*) What was the total work done on the block?

(ii) (*2 points*) What was the work done by gravity?

(iii) (*2 points*) What was the work done by the spring force?

(iv) (*2 points*) Was there any other agent that did work on the block? If so, what was that agent, and how much work did it do?

Now suppose that instead of being lowered gently, the mass is simply released from rest at $y = 0$ and allowed to fall under the combined effects of gravity and the spring force.

(e) (*3 points*) At what speed is it moving when it crosses $y = y_{eq}$?

(f) (*5 points*) What is the period T of the oscillations of the mass?

(g) (*5 points*) What is the amplitude of these oscillations? That is, how far up or down will the mass move from its average position?

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Problem 4: Space Shuttle Maneuvers (*30 points*)

A space shuttle of mass M_S travels in a circular orbit, of radius R_0, about the Earth. Let M_E denote the mass of the Earth, and let G denote Newton's constant. Treat the Earth as if it were at rest in an inertial reference frame, and assume that the shuttle is high enough so that atmospheric drag can be ignored.

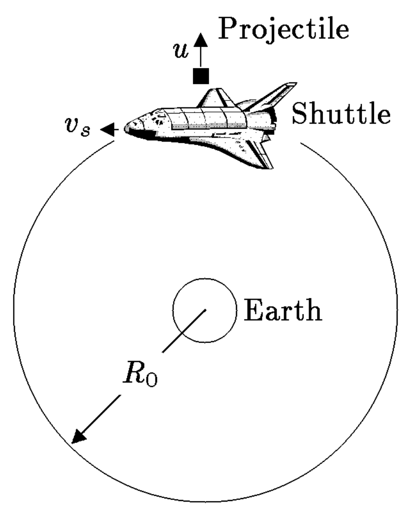
(a) (*5 points*) What is the magnitude a of the acceleration of the shuttle? Express your answer in terms of some or all of the variables G, M_E, M_S, and R_0. What is the direction of the acceleration?

(b) (*4 points*) What is the speed v_S of the shuttle in its orbit? Express your answer **ONLY** in terms of the magnitude of the acceleration a and the radius R_0 of the orbit.

In the following questions, you may express your answer to each part in terms of the given variables and the symbols representing the answers to any previous parts, regardless of whether the previous parts were answered correctly.

(c) (*5 points*) What is the magnitude L_S of the angular momentum of the shuttle about the center of the Earth?

Suppose that the shuttle fires a projectile of mass m in a direction radially outward from the Earth, with a speed u. That is, the projectile is given a velocity, **relative to the satellite,** which has magnitude u and is directed opposite to the direction of the Earth. Assume that the projectile is light enough (i.e., $m \ll M_S$) so that the recoil of the shuttle can be ignored.



(d) (*5 points*) Immediately after the projectile is ejected, what is its total mechanical energy? Define the gravitational potential energy so that it would vanish for an object infinitely far from the Earth.

(e) (*3 points*) Immediately after the projectile is ejected, what is the magnitude L_m of its angular momentum about the center of the Earth?

(f) (*8 points*) Let R_{max} denote the maximum distance from the center of the Earth that the projectile will reach, and let v_{max} denote the speed of the projectile (relative to the inertial frame of the Earth) when it reaches the maximum distance. (Note that v_{max} is not the maximum velocity.) Write down two equations which can be solved to find R_{max} and v_{max}. Do not solve the equations.

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QUIZ 11
FORMULA SHEET
Quiz Date: Friday, April 29, 2005

For motion in one dimension:

$$v_{\rm av} \equiv \frac{\Delta x}{\Delta t} \qquad \text{Average velocity;}$$

$$v \equiv \frac{{\rm d}x}{{\rm d}t} \qquad \text{Instantaneous velocity;}$$

For motion in three dimensions:

$$\vec{\mathbf{v}} \equiv \frac{{\rm d}\vec{\mathbf{r}}}{{\rm d}t}\,; \qquad \vec{\mathbf{a}} \equiv \frac{{\rm d}\vec{\mathbf{v}}}{{\rm d}t} = \frac{{\rm d}^2\vec{\mathbf{r}}}{{\rm d}t^2}\,; \qquad \vec{\mathbf{r}}(t_1) = \vec{\mathbf{r}}_0 + \int_0^{t_1} \vec{\mathbf{v}}\,{\rm d}t\,; \qquad \vec{\mathbf{v}}(t_1) = \vec{\mathbf{v}}_0 + \int_0^{t_1} \vec{\mathbf{a}}\,{\rm d}t\ .$$

For *constant* acceleration $\vec{\mathbf{a}}$, if $\vec{\mathbf{r}} = \vec{\mathbf{r}}_0$ and $\vec{\mathbf{v}} = \vec{\mathbf{v}}_0$ at time $t = 0$, then

$$\vec{\mathbf{v}}(t) = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t$$

$$\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \frac{1}{2}\vec{\mathbf{a}}t^2\ .$$

For one-dimensional motion with constant acceleration a:

$$v^2 = v_0^2 + 2a(x - x_0)\ .$$

For circular motion at constant speed v:

$$a = \frac{v^2}{r}\ ,$$

where r is the radius of the circle, and the acceleration is directed towards the center of the circle.

If an object has position $\vec{\mathbf{r}}$ and velocity $\vec{\mathbf{v}}$, its position and velocity relative to an observer with position $\vec{\mathbf{r}}_0$ and velocity $\vec{\mathbf{v}}_0$ are given respectively by

$$\vec{\mathbf{r}}' = \vec{\mathbf{r}} - \vec{\mathbf{r}}_0\ , \qquad \vec{\mathbf{v}}' = \vec{\mathbf{v}} - \vec{\mathbf{v}}_0\ .$$

Average velocity and acceleration are given by

$$\vec{\mathbf{v}}_{\rm average} \equiv \frac{\Delta\vec{\mathbf{r}}}{\Delta t}\ , \qquad \vec{\mathbf{a}}_{\rm average} \equiv \frac{\Delta\vec{\mathbf{v}}}{\Delta t}\ .$$

Mass, Acceleration, and Force:

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} \qquad \text{(Newton's second law);}$$

$$\vec{\mathbf{F}} = -\frac{GMm}{r^2}\hat{\boldsymbol{r}} \qquad \text{(the gravitational force between two particles);}$$

$$\vec{\mathbf{F}} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}\hat{\boldsymbol{r}} \qquad \text{(the electrostatic force between two particles);}$$

$$F_x = -kx \qquad \text{(Hooke's law);}$$

where $\hat{\boldsymbol{r}}$ is a unit vector pointing from the particle which is the source of the force, toward the particle on which the force is acting.

Friction:

$$\left|\vec{\mathbf{F}}_k\right| = \mu_k \left|\vec{\mathbf{N}}\right| \qquad \text{(kinetic friction);}$$

$$\left|\vec{\mathbf{F}}_s\right| \leq \mu_s \left|\vec{\mathbf{N}}\right| \qquad \text{(static friction).}$$

Kinetic Energy, Work, and Potential Energy:

Description	**1 Dimension**	**3 Dimensions**
Work done by a constant force $\vec{\mathbf{F}}$	$W \equiv F\Delta x$	$W \equiv \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{\Delta r}}$
Work done by a varying force $\vec{\mathbf{F}}$	$W \equiv \int F(x)\,\mathrm{d}x$	$W \equiv \int_{\vec{\mathbf{r}}_1}^{\vec{\mathbf{r}}_2} \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{dr}}$
Potential energy derived from force $\vec{\mathbf{F}}$	$U(x_p) \equiv U_0 - \int_{x_0}^{x_p} F\,\mathrm{d}x$	$U(\vec{\mathbf{r}}_p) \equiv U_0 - \int_{\vec{\mathbf{r}}_0}^{\vec{\mathbf{r}}_p} \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{dr}}$
Force derived from potential energy	$F = -\frac{\mathrm{d}U}{\mathrm{d}x}$	$\vec{\mathbf{F}} = \left[-\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y}, -\frac{\partial U}{\partial z}\right]$

$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \equiv \lvert\vec{\mathbf{a}}\rvert\lvert\vec{\mathbf{b}}\rvert \cos\theta = a_x b_x + a_y b_y + a_z b_z$	(scalar (or dot) product of two vectors);
$E_k \equiv \frac{1}{2}mv^2$	(kinetic energy of a particle);
$W_{\text{tot}} = E_{k,f} - E_{k,i}$	(work-energy theorem: always true if W_{tot} includes work due to all forces; a non-rigid object can do work on itself!);
$E_{k,i} + U_i + W_{\text{other}} = E_{k,f} + U_f$	(generalized work-energy theorem: always true if W_{other} includes work due to all forces not included in U);
$\frac{1}{2}mv^2 + U(x) = \text{constant}$	(conservation of mechanical energy: true in the absence of dissipative forces);
$\frac{1}{2}mv^2 + mgh = \frac{1}{2}mv_0^2$	(conservation of mechanical energy for a projectile: true in the absence of dissipative forces);
$W = \frac{1}{2}kx^2$	(work to compress a spring);
$U = \frac{1}{2}kx^2$	(potential energy for spring force);
$W = mgh$	(work to lift a body near the surface of the Earth);
$U = mgh$	(gravitational potential energy, near the surface of the Earth);
$U = -\frac{GMm}{r}$	(gravitational potential energy, spherical bodies);
$U = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r}$	(electrostatic potential energy, spherical charges).

Momentum, Center of Mass, and Systems of Particles:

$\vec{\mathbf{F}}_{\text{AB}} = -\vec{\mathbf{F}}_{\text{BA}}$	(Newton's third law);
$\vec{\mathbf{p}} \equiv m\vec{\mathbf{v}}$	(momentum);
$\frac{\mathrm{d}\vec{\mathbf{P}}_{\text{tot}}}{\mathrm{d}t} = 0$	(conservation of momentum in absence of external force)
$\vec{\mathbf{F}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}$	(Newton's second law in terms of momentum);
$\vec{\mathbf{r}}_{\text{cm}} \equiv \frac{1}{M_{\text{tot}}}\sum_i m_i \vec{\mathbf{r}}_i$	(position of center of mass);

$$\vec{\mathbf{v}}_{\rm cm} \equiv \frac{d\vec{\mathbf{r}}_{\rm cm}}{dt} = \frac{1}{M_{\rm tot}} \sum_i m_i \vec{\mathbf{v}}_i \qquad \text{(velocity of center of mass);}$$

$$\vec{\mathbf{F}}^{\rm ext}_{\rm tot} = M_{\rm tot}\vec{\mathbf{a}}_{\rm cm} = \frac{d\vec{\mathbf{P}}_{\rm tot}}{dt} \qquad \text{(acceleration of a system of particles);}$$

$$\vec{\mathbf{P}}_{\rm tot} = \sum_i m_i \vec{\mathbf{v}}_i = M_{\rm tot}\vec{\mathbf{v}}_{\rm cm} \qquad \text{(momentum of a system of particles);}$$

$$K_{\rm tot} = \frac{1}{2} M_{\rm tot} v^2_{\rm cm} + \sum_i \frac{1}{2} m_i \left(\vec{\mathbf{v}}_i - \vec{\mathbf{v}}_{\rm cm}\right)^2 \qquad \text{(K.E. of a system of particles);}$$

$$\vec{\mathbf{J}} = \int_{t_1}^{t_2} \vec{\mathbf{F}}\, dt = \int_{t_1}^{t_2} \frac{d\vec{\mathbf{p}}}{dt}\, dt = \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1 \qquad \text{(impulse-momentum theorem).}$$

Rotation in Two Dimensions:

Most of the equations for this topic are most easily remembered in the context of the analogous equations for linear motion in one dimension:

TRANSLATION (one dimension)		ROTATION (about fixed axis)	
Name	Symbol	Name	Symbol
Position	x	Orientation	θ
Velocity	$v = \frac{dx}{dt}$	Angular velocity	$\omega = \frac{d\theta}{dt}$
Acceleration	$a = \frac{dv}{dt}$	Angular acceleration	$\alpha = \frac{d\omega}{dt}$
Mass	$M = \sum_i m_i$	Moment of inertia	$I = \sum_i m_i R_i^2$
Force	F	Torque	$\boldsymbol{\tau} = F_\perp R$ $= \pm\lvert\vec{\mathbf{F}}\rvert R_\perp$
Force equation	$\sum_i \vec{\mathbf{F}}^{\rm ext} = M\vec{\mathbf{a}}_{\rm cm}$	Torque equation	$\sum_i \boldsymbol{\tau}^{\rm ext} = I\alpha$
Momentum	$p = Mv$	Angular momentum	$L = I\omega$
Kinetic energy	$\frac{1}{2}Mv^2$	Kinetic energy	$\frac{1}{2}I\omega^2$
Work done	$\vec{\mathbf{F}} \cdot \overrightarrow{\boldsymbol{\Delta r}}$	Work done	$\boldsymbol{\tau}\,\Delta\theta$

Other equations about rotation in two dimensions:

$v_r = 0 \;; \quad v_\perp = R\omega$	(velocity of point on rotating body);
$a_r = -\dfrac{v^2}{R} = -R\omega^2 \;; \quad a_\perp = R\alpha$	(acceleration of point on rotating body);
$v = \pm R\lvert\omega\rvert$	(rolling without slipping);
$\left.\begin{aligned} \sum \vec{\mathbf{F}}^{\text{ext}} &= M\vec{\mathbf{a}}_{\text{cm}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} \\ \sum \boldsymbol{\tau}^{\text{ext}} &= I_{\text{cm}}\alpha = \frac{\mathrm{d}L}{\mathrm{d}t} \end{aligned}\right\}$	(combined translational and rotational motion);
$K_{\text{tot}} = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2$	(kinetic energy for combined translational and rotational motion);
$I_{\parallel} = I_{\text{cm}} + Md^2$	(parallel-axis theorem);
$I_z = I_x + I_y$	(perpendicular-axis theorem).

TABLE OF STANDARD MOMENTS OF INERTIA:

Slender uniform rod of length ℓ, axis through center and perpendicular to axis of rod	$\frac{1}{12}m\ell^2$
Rectangular plate with dimensions $a \times b$, axis along one of the b edges	$\frac{1}{3}ma^2$
Thin-walled hollow cylinder of radius R, axis along axis of cylinder	mR^2
Uniform solid cylinder of radius R, axis along axis of cylinder	$\frac{1}{2}mR^2$
Thin-walled hollow sphere of radius R, axis through center	$\frac{2}{3}mR^2$
Solid uniform sphere of radius R, axis through center	$\frac{2}{5}mR^2$

Rotations in Vector Notation:

$$c_x = a_y b_z - a_z b_y \ ; \quad c_y = a_z b_x - a_x b_z \ ; \quad c_z = a_x b_y - a_y b_x \ .$$
(vector cross product, component form);

$$|\vec{\mathbf{c}}| = |\vec{\mathbf{a}}||\vec{\mathbf{b}}| \sin\theta$$
(magnitude of vector cross product);

$$\vec{\mathbf{v}} = \vec{\boldsymbol{\omega}} \times \vec{\mathbf{r}}$$
(velocity of atom in rotating body with a fixed point);

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_P + \vec{\boldsymbol{\omega}} \times \left(\vec{\mathbf{r}} - \vec{\mathbf{r}}_P\right)$$
(velocity of atom in rotating body, general case);

$$\vec{\mathbf{L}} = \sum_i \vec{\mathbf{r}}_i \times \vec{\mathbf{p}}_i$$
(angular momentum, as vector product);

$$\vec{\boldsymbol{\tau}} = \sum_i \vec{\mathbf{r}}_i \times \vec{\mathbf{F}}_i$$
(vector torque, as vector product);

$$\vec{\boldsymbol{\tau}} = \frac{\mathrm{d}\vec{\mathbf{L}}}{\mathrm{d}t}$$
(torque equation);

$$\left.\begin{aligned} \sum \vec{\mathbf{F}}^{\text{ext}} &= M\vec{\mathbf{a}}_{\text{cm}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} \\ \sum \vec{\boldsymbol{\tau}}^{\text{ext}} &= \frac{\mathrm{d}\vec{\mathbf{L}}_{\text{cm}}}{\mathrm{d}t} \end{aligned}\right\}$$
(combined translational and rotational motion);

$$\vec{\mathbf{L}} = \vec{\mathbf{r}}_{\text{cm}} \times \vec{\mathbf{p}}_{\text{tot}} + \sum_i \vec{\mathbf{r}}_{\text{rel},i} \times m_i \vec{\mathbf{v}}_{\text{rel},i}$$
(angular momentum decomposition);

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}}_{\text{cm}} \times \vec{\mathbf{F}}_{\text{tot}} + \sum_i \vec{\mathbf{r}}_{\text{rel},i} \times \vec{\mathbf{F}}_i$$
(torque decomposition).

For Static Bodies:

$$\sum \vec{\mathbf{F}}^{\text{ext}} = 0$$
(total external force vanishes);

$$\sum \vec{\boldsymbol{\tau}}^{\text{ext}} = 0$$
(total external torque about ANY point vanishes).

Gravitation:

$$F_g = \frac{Gm_1m_2}{r^2}$$ (Newton's law of gravity);

$$g = \frac{Gm_E}{R_E^2}$$ (Acceleration due to gravity at Earth's surface);

$$U = -\frac{Gm_Em}{r}$$ (Gravitational potential energy);

$$v = \sqrt{\frac{Gm_E}{r}}$$ (Speed in cicular orbit);

$$T = \frac{2\pi r}{v} = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}}$$ (Period of a circular orbit);

$$R_S = \frac{2GM}{c^2}$$ (Schwarzschild radius).

Periodic Motion:

$$f = \frac{1}{T}$$ (Frequency and period);

$$\omega = 2\pi f = \frac{2\pi}{T}$$ (angular frequency);

$$F_x = -kx$$ (simple harmonic oscillator);

$$\omega = \sqrt{\frac{k}{m}}$$ (angular frequency of simple harmonic oscillator);

$$x = A\cos(\omega t + \phi)$$ (motion of simple harmonic oscillator);

$$\omega = \sqrt{\frac{\kappa}{I}}$$ (angular frequency ω in terms of torsion constant κ and moment of inertia I)