MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 Spring 2005

WEEKLY QUIZ 12

Friday, May 6, 2005

FAMILY (LAST) NAME

GIVEN (FIRST) NAME

STUDENT ID NUMBER

Please check (✔) your class ☞

	L01	MTW 10:00	Walter Lewin
	L02	MTW 11:00	Walter Lewin
	L03	MTW 2:00	Min Chen
	L04	MTW 3:00	Min Chen

INSTRUCTIONS:

1. The FORMULA SHEET is in the back of this exam. You may tear it off. There is also an extra BLANK PAGE in case you need it.
2. This is a closed book exam. CALCULATORS, BOOKS, and NOTES are NOT ALLOWED.
3. Unless otherwise stated, to earn full credit you must show a valid DERIVATION and/or EXPLANATION of your answer, and you must express it in terms of the GIVEN VARIABLES.

Problem	**Maximum**	**Score**	**Grader**
1	25		
2	35		
3	40		
TOTAL	100		

Name ______________________________

Problem 1: Basic concepts about special relativity (*25 points*)

(a) (*5 points*) A rocket flies at a relativistic speed v in a straight line from space station A to space station B, which are at rest relative to each other. The space stations keep time on a system of clocks which are synchronized in their common rest frame. Measured on the clock inside the rocket, the elapsed time for the trip is Δt. If t_A and t_B denote the time of the take-off and landing, respectively, on the space station clocks, what is $t_B - t_A$? Let γ denote $1/\sqrt{1 - v^2/c^2}$.

(i) Δt (ii) $\gamma \Delta t$ (iii) $\Delta t/\gamma$ (iv) $\gamma^2 \Delta t$ (v) $\Delta t/\gamma^2$

(b) (*5 points*) A measuring rod with length ℓ_0 in its own rest frame is pulled across the surface of a horizontal table at relativistic speed v, along the direction of its length. At a fixed time in the rest frame of the table, two elves make scratch marks in the table at the locations of the two endpoints of the rod. When measured in the rest frame of the table, how far apart are these marks? Again let γ denote $1/\sqrt{1 - v^2/c^2}$.

(i) ℓ_0 (ii) $\gamma \ell_0$ (iii) ℓ_0/γ (iv) $\gamma^2 \ell_0$ (v) ℓ_0/γ^2

(c) (*5 points*) Consider the same measuring rod as described in (b), being pulled across a horizontal table at relativistic speed v. This time imagine elves that are riding on the rod, and have their elvish wristwatches synchronized in the frame of the rod. At the same time according to these wristwatches, the elves carve marks in the table at both ends of the rod. In this case, when measured in the rest frame of the table, how far away are the marks? As usual, let γ denote $1/\sqrt{1 - v^2/c^2}$.

(i) ℓ_0 (ii) $\gamma \ell_0$ (iii) ℓ_0/γ (iv) $\gamma^2 \ell_0$ (v) ℓ_0/γ^2

— *Problem 1 Continues* —

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(d) (*5 points*) A very high speed train moves along a straight track from your right to your left. The train has length ℓ_0 in its own rest frame. There are clocks at the front and rear of the train, which are synchronized with each other in the rest frame of the train. At a fixed time in your reference frame, observers who happen to be situated at the two ends of the train note the time readings on the two clocks, calling them $t_{\rm left}$ and $t_{\rm right}$, referring to the left and right ends of the train, respectively, as seen by you. What is $\Delta t \equiv t_{\rm left} - t_{\rm right}$?

(i) $\dfrac{\gamma v \ell_0}{c^2}$ (ii) $\dfrac{v \ell_0}{c^2}$ (iii) $\dfrac{v \ell_0}{\gamma c^2}$ (iv) $-\dfrac{\gamma v \ell_0}{c^2}$ (v) $-\dfrac{v \ell_0}{c^2}$ (vi) $-\dfrac{v \ell_0}{\gamma c^2}$

(vii) $\dfrac{\gamma v^2 \ell_0}{c^2}$ (viii) $\dfrac{v^2 \ell_0}{c^2}$ (ix) $\dfrac{v^2 \ell_0}{\gamma c^2}$ (x) $-\dfrac{\gamma v^2 \ell_0}{c^2}$ (xi) $-\dfrac{v^2 \ell_0}{c^2}$ (xii) $-\dfrac{v^2 \ell_0}{\gamma c^2}$

(e) (*5 points*) Two twins, Karla and Marla, are born in the year 2300. In 2318, Karla takes off on a spaceship which travels at a relativistic speed in a straight line for many years, and then stops, reverses its direction, and returns to Earth in the year 2360. When the 60-year-old Marla looks at her twin sister Karla, she is immediately jealous that her sister looks (and really is) 20 years younger than she is, due to relativistic time dilation. However, just as Karla has been moving at high speed relative to Marla, Marla has been moving at high speed relative to Karla. So what does Karla see when she looks at her sister Marla? Choose the best answer.

(i) To Karla it looks like Marla is younger, because in her own reference frame Karla has been at rest, and Marla has been moving at high speed.

(ii) To Karla it looks like they are the same age, because the time dilation effect is only apparent, not real.

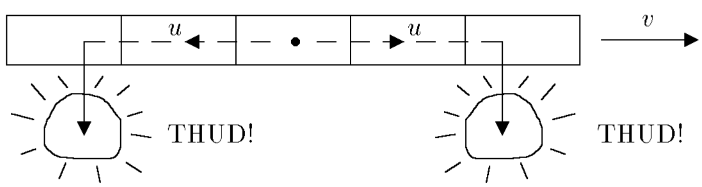
(iii) Karla agrees with Marla that Marla has aged 20 years more than Karla. This asymmetry can be explained because it was really Karla who was in motion, while Marla was always at rest.

(iv) Karla agrees with Marla that Marla has aged 20 years more than Karla. This asymmetry can be explained because Karla has undergone dramatic acceleration when she began her space journey, when the spaceship reversed its direction, and when it came to rest on Earth. Marla, on the other hand, has undergone no such acceleration.

(v) Karla agrees with Marla that Marla has aged 20 years more than Karla. This asymmetry can be explained because Marla spent her whole life on Earth, while Karla was in deep space where relativistic effects can occur.

Problem 2: Dynamic Duo Jumps Off a Fast Train (*35 points*)

Batman and Robin are standing next to each other on the roof of a train, which is moving at a steady velocity of magnitude v. (Since this is a relativity problem, v is of course not negligible compared to c. Amtrak would have a hard time keeping up.) The superheroes simultaneously set their stop-watches to zero, and then by prearrangement they start running along the train in opposite directions at speed u, measured in the frame of reference of the train. They each run for a time Δt, as measured on their own stop-watches, and then they each jump from the train. They hit the ground with a sharp thud, and come instantly to a stop. Miraculously neither member of the Dynamic Duo is hurt. Assume that the time required for the jump is negligible, so that they land instantaneously directly below the point from which they jumped.



(a) (*5 points*) As measured in the frame of reference of the train, how far apart are Batman and Robin when they jump?

(b) (*10 points*) As measured on the ground, how far apart are the two points at which the pair hit the ground?

(c) (*10 points*) As seen from the ground, do the two superheroes hit the ground simultaneously? If not, calculate the time difference.

(d) (*10 points*) From the point of view of an observer on the ground, what is the distance between the point where Batman began to run, and the point where he jumps. Assume that Batman is the member of the pair who runs forward.

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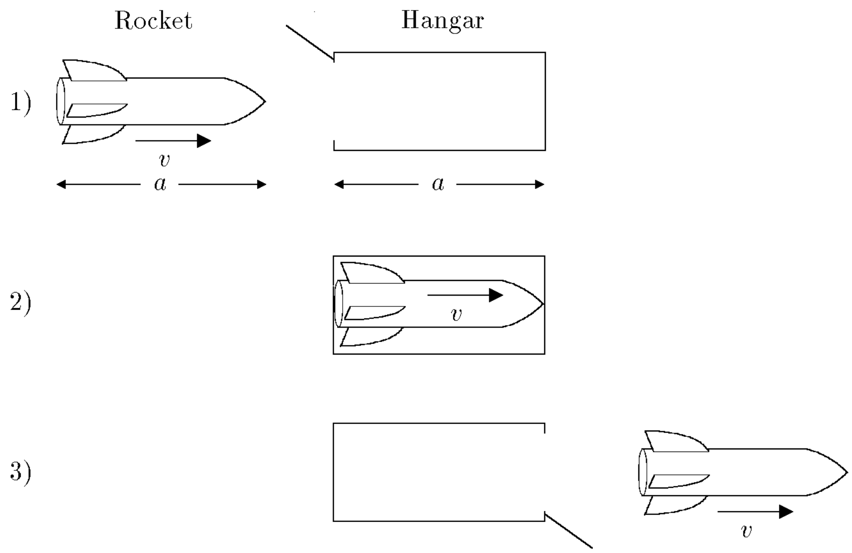
Problem 3: A Rocket Ship Squeezed in a Hangar (*40 points*)



A rocket ship moves at speed v through a hangar which is at rest, and which has a door at either end. As observed in the rest frame (*i.e.*,the reference frame of the hangar), both the rocket ship and the hangar have exactly the same length, a. Picture (1) above shows a snapshot diagram of the rocket approaching the hangar, with the left door open. All three pictures are snapshot diagrams in the rest frame of the hangar, indicating the location of all objects at a constant time in that rest frame. In picture (2) the rocket has moved completely inside the hangar; the left door has been closed, and for an instant the rocket is completely enclosed. The door on the right is immediately opened, however, just in time to prevent it from being smashed by the rocket's nose cone. Picture (3) shows the rocket well outside the hangar.

(a) (*5 points*) What is the length ℓ_R of the rocket in its own rest frame?

(b) (*5 points*) What is the length ℓ_H of the hangar in the rest frame of the rocket?

In the subsequent parts, you may treat ℓ_R and ℓ_H as given variables, whether or not you have answered (a) and (b) correctly.

(c) (*3 points*) Assume that the rocket has clocks at both ends, and that these clocks are synchronized in the reference frame of the rocket. In picture (2), when both doors are instantaneously closed and the rocket is entirely contained inside the hangar (as seen in the reference frame of the hangar), the time on the clock at the tail reads t_1, and the clock at the nose cone reads t_2. Is t_2 less than t_1, equal to t_1, or greater than t_1?

(d) (*7 points*) Give an expression for $\Delta t \equiv t_2 - t_1$ in terms of the given variables.

— *Problem 3 Continues* —

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(e) (*10 points*) Draw a snapshot diagram in the frame of the ***rocket***, at the instant that the door at the tail of the rocket is closed.

(f) (*5 points*) At the instant the hangar door at the rear of the rocket is closed, a light signal is sent from the rear of the rocket toward the front. In the frame of reference of the rocket, how long does it take for the signal to reach the front of the rocket?

(g) (*5 points*) In the rest frame (*i.e.*, the reference frame of the hangar), how long does it take for the signal to reach the front of the rocket?

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For motion in one dimension:

$$v_{\rm av} \equiv \frac{\Delta x}{\Delta t} \qquad \text{Average velocity;}$$

$$v \equiv \frac{{\rm d}x}{{\rm d}t} \qquad \text{Instantaneous velocity;}$$

For motion in three dimensions:

$$\vec{\mathbf{v}} \equiv \frac{{\rm d}\vec{\mathbf{r}}}{{\rm d}t}\,; \qquad \vec{\mathbf{a}} \equiv \frac{{\rm d}\vec{\mathbf{v}}}{{\rm d}t} = \frac{{\rm d}^2\vec{\mathbf{r}}}{{\rm d}t^2}\,; \qquad \vec{\mathbf{r}}(t_1) = \vec{\mathbf{r}}_0 + \int_0^{t_1} \vec{\mathbf{v}}\,{\rm d}t\,; \qquad \vec{\mathbf{v}}(t_1) = \vec{\mathbf{v}}_0 + \int_0^{t_1} \vec{\mathbf{a}}\,{\rm d}t\ .$$

For *constant* acceleration $\vec{\mathbf{a}}$, if $\vec{\mathbf{r}} = \vec{\mathbf{r}}_0$ and $\vec{\mathbf{v}} = \vec{\mathbf{v}}_0$ at time $t = 0$, then

$$\vec{\mathbf{v}}(t) = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t$$

$$\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \frac{1}{2}\vec{\mathbf{a}}t^2\ .$$

For one-dimensional motion with constant acceleration a:

$$v^2 = v_0^2 + 2a(x - x_0)\ .$$

For circular motion at constant speed v:

$$a = \frac{v^2}{r}\ ,$$

where r is the radius of the circle, and the acceleration is directed towards the center of the circle.

If an object has position $\vec{\mathbf{r}}$ and velocity $\vec{\mathbf{v}}$, its position and velocity relative to an observer with position $\vec{\mathbf{r}}_0$ and velocity $\vec{\mathbf{v}}_0$ are given respectively by

$$\vec{\mathbf{r}}' = \vec{\mathbf{r}} - \vec{\mathbf{r}}_0\ , \qquad \vec{\mathbf{v}}' = \vec{\mathbf{v}} - \vec{\mathbf{v}}_0\ .$$

Average velocity and acceleration are given by

$$\vec{\mathbf{v}}_{\rm average} \equiv \frac{\Delta\vec{\mathbf{r}}}{\Delta t}\ , \qquad \vec{\mathbf{a}}_{\rm average} \equiv \frac{\Delta\vec{\mathbf{v}}}{\Delta t}\ .$$

Mass, Acceleration, and Force:

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} \qquad \text{(Newton's second law);}$$

$$\vec{\mathbf{F}} = -\frac{GMm}{r^2}\hat{\boldsymbol{r}} \qquad \text{(the gravitational force between two particles);}$$

$$\vec{\mathbf{F}} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}\hat{\boldsymbol{r}} \qquad \text{(the electrostatic force between two particles);}$$

$$F_x = -kx \qquad \text{(Hooke's law);}$$

where $\hat{\boldsymbol{r}}$ is a unit vector pointing from the particle which is the source of the force, toward the particle on which the force is acting.

Friction:

$$\left|\vec{\mathbf{F}}_k\right| = \mu_k \left|\vec{\mathbf{N}}\right| \qquad \text{(kinetic friction);}$$

$$\left|\vec{\mathbf{F}}_s\right| \leq \mu_s \left|\vec{\mathbf{N}}\right| \qquad \text{(static friction).}$$

Kinetic Energy, Work, and Potential Energy:

Description	**1 Dimension**	**3 Dimensions**
Work done by a constant force $\vec{\mathbf{F}}$	$W \equiv F\Delta x$	$W \equiv \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{\Delta r}}$
Work done by a varying force $\vec{\mathbf{F}}$	$W \equiv \int F(x)\,\mathrm{d}x$	$W \equiv \int_{\vec{\mathbf{r}}_1}^{\vec{\mathbf{r}}_2} \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{dr}}$
Potential energy derived from force $\vec{\mathbf{F}}$	$U(x_p) \equiv U_0 - \int_{x_0}^{x_p} F\,\mathrm{d}x$	$U(\vec{\mathbf{r}}_p) \equiv U_0 - \int_{\vec{\mathbf{r}}_0}^{\vec{\mathbf{r}}_p} \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{dr}}$
Force derived from potential energy	$F = -\frac{\mathrm{d}U}{\mathrm{d}x}$	$\vec{\mathbf{F}} = \left[-\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y}, -\frac{\partial U}{\partial z}\right]$

$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \equiv \lvert\vec{\mathbf{a}}\rvert\lvert\vec{\mathbf{b}}\rvert \cos\theta$ $= a_x b_x + a_y b_y + a_z b_z$	(scalar (or dot) product of two vectors);
$E_k \equiv \frac{1}{2}mv^2$	(kinetic energy of a particle);
$W_{\text{tot}} = E_{k,f} - E_{k,i}$	(work-energy theorem: always true if W_{tot} includes work due to all forces; a non-rigid object can do work on itself!);
$E_{k,i} + U_i + W_{\text{other}}$ $= E_{k,f} + U_f$	(generalized work-energy theorem: always true if W_{other} includes work due to all forces not included in U);
$\frac{1}{2}mv^2 + U(x) = \text{constant}$	(conservation of mechanical energy: true in the absence of dissipative forces);
$\frac{1}{2}mv^2 + mgh = \frac{1}{2}mv_0^2$	(conservation of mechanical energy for a projectile: true in the absence of dissipative forces);
$W = \frac{1}{2}kx^2$	(work to compress a spring);
$U = \frac{1}{2}kx^2$	(potential energy for spring force);
$W = mgh$	(work to lift a body near the surface of the Earth);
$U = mgh$	(gravitational potential energy, near the surface of the Earth);
$U = -\frac{GMm}{r}$	(gravitational potential energy, spherical bodies);
$U = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r}$	(electrostatic potential energy, spherical charges).

Momentum, Center of Mass, and Systems of Particles:

$\vec{\mathbf{F}}_{\text{AB}} = -\vec{\mathbf{F}}_{\text{BA}}$	(Newton's third law);
$\vec{\mathbf{p}} \equiv m\vec{\mathbf{v}}$	(momentum);
$\frac{\mathrm{d}\vec{\mathbf{P}}_{\text{tot}}}{\mathrm{d}t} = 0$	(conservation of momentum in absence of external force)
$\vec{\mathbf{F}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}$	(Newton's second law in terms of momentum);
$\vec{\mathbf{r}}_{\text{cm}} \equiv \frac{1}{M_{\text{tot}}}\sum_i m_i\vec{\mathbf{r}}_i$	(position of center of mass);

$$\vec{\mathbf{v}}_{\rm cm} \equiv \frac{d\vec{\mathbf{r}}_{\rm cm}}{dt} = \frac{1}{M_{\rm tot}} \sum_i m_i \vec{\mathbf{v}}_i \qquad \text{(velocity of center of mass);}$$

$$\vec{\mathbf{F}}^{\rm ext}_{\rm tot} = M_{\rm tot} \vec{\mathbf{a}}_{\rm cm} = \frac{d\vec{\mathbf{P}}_{\rm tot}}{dt} \qquad \text{(acceleration of a system of particles);}$$

$$\vec{\mathbf{P}}_{\rm tot} = \sum_i m_i \vec{\mathbf{v}}_i = M_{\rm tot} \vec{\mathbf{v}}_{\rm cm} \qquad \text{(momentum of a system of particles);}$$

$$K_{\rm tot} = \frac{1}{2} M_{\rm tot} v^2_{\rm cm} + \sum_i \frac{1}{2} m_i \left(\vec{\mathbf{v}}_i - \vec{\mathbf{v}}_{\rm cm} \right)^2 \qquad \text{(K.E. of a system of particles);}$$

$$\vec{\mathbf{J}} = \int_{t_1}^{t_2} \vec{\mathbf{F}}\, dt = \int_{t_1}^{t_2} \frac{d\vec{\mathbf{p}}}{dt}\, dt = \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1 \qquad \text{(impulse-momentum theorem).}$$

Rotation in Two Dimensions:

Most of the equations for this topic are most easily remembered in the context of the analogous equations for linear motion in one dimension:

TRANSLATION (one dimension)		ROTATION (about fixed axis)	
Name	Symbol	Name	Symbol
Position	x	Orientation	θ
Velocity	$v = \frac{dx}{dt}$	Angular velocity	$\omega = \frac{d\theta}{dt}$
Acceleration	$a = \frac{dv}{dt}$	Angular acceleration	$\alpha = \frac{d\omega}{dt}$
Mass	$M = \sum_i m_i$	Moment of inertia	$I = \sum_i m_i R_i^2$
Force	F	Torque	$\boldsymbol{\tau} = F_\perp R$ $= \pm\lvert\vec{\mathbf{F}}\rvert R_\perp$
Force equation	$\sum_i \vec{\mathbf{F}}^{\rm ext} = M\vec{\mathbf{a}}_{\rm cm}$	Torque equation	$\sum_i \boldsymbol{\tau}^{\rm ext} = I\alpha$
Momentum	$p = Mv$	Angular momentum	$L = I\omega$
Kinetic energy	$\frac{1}{2}Mv^2$	Kinetic energy	$\frac{1}{2}I\omega^2$
Work done	$\vec{\mathbf{F}} \cdot \overrightarrow{\boldsymbol{\Delta r}}$	Work done	$\boldsymbol{\tau}\,\Delta\theta$

Other equations about rotation in two dimensions:

$v_r = 0 \; ; \quad v_\perp = R\omega$	(velocity of point on rotating body);
$a_r = -\dfrac{v^2}{R} = -R\omega^2 \; ; \quad a_\perp = R\alpha$	(acceleration of point on rotating body);
$v = \pm R\lvert\omega\rvert$	(rolling without slipping);
$\left.\begin{aligned} \sum \vec{\mathbf{F}}^{\rm ext} &= M\vec{\mathbf{a}}_{\rm cm} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} \\ \sum \boldsymbol{\tau}^{\rm ext} &= I_{\rm cm}\alpha = \frac{\mathrm{d}L}{\mathrm{d}t} \end{aligned}\right\}$	(combined translational and rotational motion);
$K_{\rm tot} = \frac{1}{2}Mv_{\rm cm}^2 + \frac{1}{2}I_{\rm cm}\omega^2$	(kinetic energy for combined translational and rotational motion);
$I_\parallel = I_{\rm cm} + Md^2$	(parallel-axis theorem);
$I_z = I_x + I_y$	(perpendicular-axis theorem).

TABLE OF STANDARD MOMENTS OF INERTIA:

Slender uniform rod of length ℓ, axis through center and perpendicular to axis of rod	$\frac{1}{12}m\ell^2$
Rectangular plate with dimensions $a \times b$, axis along one of the b edges	$\frac{1}{3}ma^2$
Thin-walled hollow cylinder of radius R, axis along axis of cylinder	mR^2
Uniform solid cylinder of radius R, axis along axis of cylinder	$\frac{1}{2}mR^2$
Thin-walled hollow sphere of radius R, axis through center	$\frac{2}{3}mR^2$
Solid uniform sphere of radius R, axis through center	$\frac{2}{5}mR^2$

Rotations in Vector Notation:

$$c_x = a_y b_z - a_z b_y \ ; \quad c_y = a_z b_x - a_x b_z \ ; \quad c_z = a_x b_y - a_y b_x \ .$$ (vector cross product, component form);

$$|\vec{\mathbf{c}}| = |\vec{\mathbf{a}}||\vec{\mathbf{b}}| \sin\theta$$ (magnitude of vector cross product);

$$\vec{\mathbf{v}} = \vec{\boldsymbol{\omega}} \times \vec{\mathbf{r}}$$ (velocity of atom in rotating body with a fixed point);

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_P + \vec{\boldsymbol{\omega}} \times \left(\vec{\mathbf{r}} - \vec{\mathbf{r}}_P\right)$$ (velocity of atom in rotating body, general case);

$$\vec{\mathbf{L}} = \sum_i \vec{\mathbf{r}}_i \times \vec{\mathbf{p}}_i$$ (angular momentum, as vector product);

$$\vec{\boldsymbol{\tau}} = \sum_i \vec{\mathbf{r}}_i \times \vec{\mathbf{F}}_i$$ (vector torque, as vector product);

$$\vec{\boldsymbol{\tau}} = \frac{\mathrm{d}\vec{\mathbf{L}}}{\mathrm{d}t}$$ (torque equation);

$$\left.\begin{aligned} \sum \vec{\mathbf{F}}^{\mathrm{ext}} &= M\vec{\mathbf{a}}_{\mathrm{cm}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} \\ \sum \vec{\boldsymbol{\tau}}^{\mathrm{ext}} &= \frac{\mathrm{d}\vec{\mathbf{L}}_{\mathrm{cm}}}{\mathrm{d}t} \end{aligned}\right\}$$ (combined translational and rotational motion);

$$\vec{\mathbf{L}} = \vec{\mathbf{r}}_{\mathrm{cm}} \times \vec{\mathbf{p}}_{\mathrm{tot}} + \sum_i \vec{\mathbf{r}}_{\mathrm{rel},i} \times m_i \vec{\mathbf{v}}_{\mathrm{rel},i}$$ (angular momentum decomposition);

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}}_{\mathrm{cm}} \times \vec{\mathbf{F}}_{\mathrm{tot}} + \sum_i \vec{\mathbf{r}}_{\mathrm{rel},i} \times \vec{\mathbf{F}}_i$$ (torque decomposition).

For Static Bodies:

$$\sum \vec{\mathbf{F}}^{\mathrm{ext}} = 0$$ (total external force vanishes);

$$\sum \vec{\boldsymbol{\tau}}^{\mathrm{ext}} = 0$$ (total external torque about ANY point vanishes).

Gravitation:

$$F_g = \frac{Gm_1m_2}{r^2}$$ (Newton's law of gravity);

$$g = \frac{Gm_E}{R_E^2}$$ (Acceleration due to gravity at Earth's surface);

$$U = -\frac{Gm_Em}{r}$$ (Gravitational potential energy);

$$v = \sqrt{\frac{Gm_E}{r}}$$ (Speed in cicular orbit);

$$T = \frac{2\pi r}{v} = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}}$$ (Period of a circular orbit);

$$R_S = \frac{2GM}{c^2}$$ (Schwarzschild radius).

Periodic Motion:

$$f = \frac{1}{T}$$ (Frequency and period);

$$\omega = 2\pi f = \frac{2\pi}{T}$$ (angular frequency);

$$F_x = -kx$$ (simple harmonic oscillator);

$$\omega = \sqrt{\frac{k}{m}}$$ (angular frequency of simple harmonic oscillator);

$$x = A\cos(\omega t + \phi)$$ (motion of simple harmonic oscillator);

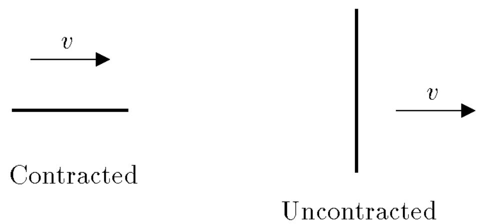
$$\omega = \sqrt{\frac{\kappa}{I}}$$ (angular frequency ω in terms of torsion constant κ and moment of inertia I)

Special Relativity:

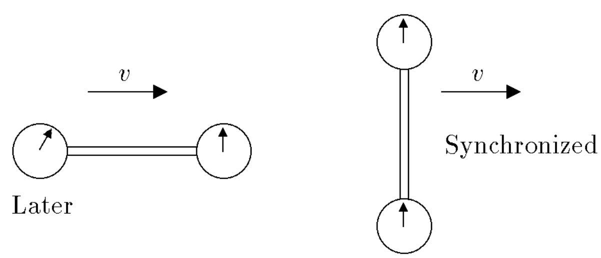
(1) TIME DILATION: Any clock which is moving at speed v relative to a given reference frame will appear (to an observer using that reference frame) to run slower than normal by a factor denoted by the Greek letter γ (gamma), and given by

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}} , \qquad \beta \equiv v/c .$$

(2) LORENTZ-FITZGERALD CONTRACTION: Any rod which is moving at a speed v along its length relative to a given reference frame will appear (to an observer using that reference frame) to be shorter than its normal length by the same factor γ. A rod which is moving perpendicular to its length does not undergo a change in apparent length.



(3) RELATIVITY OF SIMULTANEITY: Suppose a rod which has rest length ℓ_0 is equipped with a clock at each end. The clocks can be synchronized in the rest frame of the system by using light pulses. If the system moves at speed v along its length, then the trailing clock will appear to read a time which is later than the leading clock by an amount $\beta\ell_0/c$. If, on the other hand, the system moves perpendicular to its length, then the synchronization of the clocks is not disturbed.



There are a few minor qualifications that must be appended to the above statements. First, they hold only for inertial reference frames— they do not hold for rotating or accelerating reference frames. Any reference frame which moves at a uniform velocity relative to an inertial reference frame is also an inertial reference frame. Second, one must define the word "appear" which occurs in each of the three statements.

In plain English, the word "appear" normally refers to the perception of the human eyes. However, in these situations the perception of the human eyes would be very complicated. The complication is that one sees with light, and light travels with a less-than-infinite speed. Thus, when an object is moving toward you, the light which you see coming from the front of the object has left the object later than the light which you see coming from the back of the object. Effects of this kind lead to complicated distortions, which are not taken into account in the statements above. For purposes of interpreting these statements, one can imagine that each reference frame is covered by an infinite number of local observers, each of which observes only events so close that the time delay for light travel is negligible. Each observer carries a clock which has been synchronized with the others by light pulses. The "appearance" is then the description which is assembled after the fact by combining the reports of these local observers.

The Lorentz Transformation:

If an (x', y', z', t') inertial coordinate system is moving to the right (positive x direction) with speed v relative to an (x, y, z, t) inertial coordinate system, then the coordinates are related by

$$\begin{aligned} x' &= \gamma(x - vt) \\ t' &= \gamma\left(t - \frac{vx}{c^2}\right) \\ y' &= y \\ z' &= z \ . \end{aligned}$$

and

$$\begin{aligned} x &= \gamma(x' + vt') \\ t &= \gamma\left(t' + \frac{vx'}{c^2}\right) \\ y &= y' \\ z &= z' \ . \end{aligned}$$

Relativistic velocity addition and subtraction:

$$\begin{aligned} v'_x &= \frac{v_x - v}{1 - \frac{vv_x}{c^2}} \\ v_x &= \frac{v'_x + v}{1 + \frac{vv'_x}{c^2}} \ . \end{aligned}$$