MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

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QUIZ 11
FORMULA SHEET
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For motion in one dimension:

$$v_{\rm av} \equiv \frac{\Delta x}{\Delta t} \qquad \text{Average velocity;}$$

$$v \equiv \frac{{\rm d}x}{{\rm d}t} \qquad \text{Instantaneous velocity;}$$

For motion in three dimensions:

$$\vec{\mathbf{v}} \equiv \frac{{\rm d}\vec{\mathbf{r}}}{{\rm d}t}\,; \qquad \vec{\mathbf{a}} \equiv \frac{{\rm d}\vec{\mathbf{v}}}{{\rm d}t} = \frac{{\rm d}^2\vec{\mathbf{r}}}{{\rm d}t^2}\,; \qquad \vec{\mathbf{r}}(t_1) = \vec{\mathbf{r}}_0 + \int_0^{t_1} \vec{\mathbf{v}}\,{\rm d}t\,; \qquad \vec{\mathbf{v}}(t_1) = \vec{\mathbf{v}}_0 + \int_0^{t_1} \vec{\mathbf{a}}\,{\rm d}t\ .$$

For *constant* acceleration $\vec{\mathbf{a}}$, if $\vec{\mathbf{r}} = \vec{\mathbf{r}}_0$ and $\vec{\mathbf{v}} = \vec{\mathbf{v}}_0$ at time $t = 0$, then

$$\vec{\mathbf{v}}(t) = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t$$

$$\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \frac{1}{2}\vec{\mathbf{a}}t^2\ .$$

For one-dimensional motion with constant acceleration a:

$$v^2 = v_0^2 + 2a(x - x_0)\ .$$

For circular motion at constant speed v:

$$a = \frac{v^2}{r}\ ,$$

where r is the radius of the circle, and the acceleration is directed towards the center of the circle.

If an object has position $\vec{\mathbf{r}}$ and velocity $\vec{\mathbf{v}}$, its position and velocity relative to an observer with position $\vec{\mathbf{r}}_0$ and velocity $\vec{\mathbf{v}}_0$ are given respectively by

$$\vec{\mathbf{r}}' = \vec{\mathbf{r}} - \vec{\mathbf{r}}_0\ , \qquad \vec{\mathbf{v}}' = \vec{\mathbf{v}} - \vec{\mathbf{v}}_0\ .$$

Average velocity and acceleration are given by

$$\vec{\mathbf{v}}_{\rm average} \equiv \frac{\Delta\vec{\mathbf{r}}}{\Delta t}\ , \qquad \vec{\mathbf{a}}_{\rm average} \equiv \frac{\Delta\vec{\mathbf{v}}}{\Delta t}\ .$$

Mass, Acceleration, and Force:

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} \qquad \text{(Newton's second law);}$$

$$\vec{\mathbf{F}} = -\frac{GMm}{r^2}\hat{\boldsymbol{r}} \qquad \text{(the gravitational force between two particles);}$$

$$\vec{\mathbf{F}} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}\hat{\boldsymbol{r}} \qquad \text{(the electrostatic force between two particles);}$$

$$F_x = -kx \qquad \text{(Hooke's law);}$$

where $\hat{\boldsymbol{r}}$ is a unit vector pointing from the particle which is the source of the force, toward the particle on which the force is acting.

Friction:

$$\left|\vec{\mathbf{F}}_k\right| = \mu_k \left|\vec{\mathbf{N}}\right| \qquad \text{(kinetic friction);}$$

$$\left|\vec{\mathbf{F}}_s\right| \leq \mu_s \left|\vec{\mathbf{N}}\right| \qquad \text{(static friction).}$$

Kinetic Energy, Work, and Potential Energy:

Description	**1 Dimension**	**3 Dimensions**
Work done by a constant force $\vec{\mathbf{F}}$	$W \equiv F\Delta x$	$W \equiv \vec{\mathbf{F}} \cdot \overrightarrow{\boldsymbol{\Delta}\mathbf{r}}$
Work done by a varying force $\vec{\mathbf{F}}$	$W \equiv \int F(x)\,\mathrm{d}x$	$W \equiv \int_{\vec{\mathbf{r}}_1}^{\vec{\mathbf{r}}_2} \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{dr}}$
Potential energy derived from force $\vec{\mathbf{F}}$	$U(x_p) \equiv U_0 - \int_{x_0}^{x_p} F\,\mathrm{d}x$	$U(\vec{\mathbf{r}}_p) \equiv U_0 - \int_{\vec{\mathbf{r}}_0}^{\vec{\mathbf{r}}_p} \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{dr}}$
Force derived from potential energy	$F = -\frac{\mathrm{d}U}{\mathrm{d}x}$	$\vec{\mathbf{F}} = \left[-\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y}, -\frac{\partial U}{\partial z}\right]$

$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \equiv \vert\vec{\mathbf{a}}\vert\vert\vec{\mathbf{b}}\vert \cos\theta = a_x b_x + a_y b_y + a_z b_z$	(scalar (or dot) product of two vectors);
$E_k \equiv \frac{1}{2} m v^2$	(kinetic energy of a particle);
$W_{\text{tot}} = E_{k,f} - E_{k,i}$	(work-energy theorem: always true if W_{tot} includes work due to all forces; a non-rigid object can do work on itself!);
$E_{k,i} + U_i + W_{\text{other}} = E_{k,f} + U_f$	(generalized work-energy theorem: always true if W_{other} includes work due to all forces not included in U);
$\frac{1}{2} m v^2 + U(x) = \text{constant}$	(conservation of mechanical energy: true in the absence of dissipative forces);
$\frac{1}{2} m v^2 + mgh = \frac{1}{2} m v_0^2$	(conservation of mechanical energy for a projectile: true in the absence of dissipative forces);
$W = \frac{1}{2} k x^2$	(work to compress a spring);
$U = \frac{1}{2} k x^2$	(potential energy for spring force);
$W = mgh$	(work to lift a body near the surface of the Earth);
$U = mgh$	(gravitational potential energy, near the surface of the Earth);
$U = -\frac{GMm}{r}$	(gravitational potential energy, spherical bodies);
$U = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r}$	(electrostatic potential energy, spherical charges).

Momentum, Center of Mass, and Systems of Particles:

$\vec{\mathbf{F}}_{\text{AB}} = -\vec{\mathbf{F}}_{\text{BA}}$	(Newton's third law);
$\vec{\mathbf{p}} \equiv m\vec{\mathbf{v}}$	(momentum);
$\frac{\mathrm{d}\vec{\mathbf{P}}_{\text{tot}}}{\mathrm{d}t} = 0$	(conservation of momentum in absence of external force)
$\vec{\mathbf{F}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}$	(Newton's second law in terms of momentum);
$\vec{\mathbf{r}}_{\text{cm}} \equiv \frac{1}{M_{\text{tot}}} \sum_i m_i \vec{\mathbf{r}}_i$	(position of center of mass);

$$\vec{\mathbf{v}}_{\rm cm} \equiv \frac{{\rm d}\vec{\mathbf{r}}_{\rm cm}}{{\rm d}t} = \frac{1}{M_{\rm tot}} \sum_i m_i \vec{\mathbf{v}}_i \qquad \text{(velocity of center of mass);}$$

$$\vec{\mathbf{F}}^{\rm ext}_{\rm tot} = M_{\rm tot}\vec{\mathbf{a}}_{\rm cm} = \frac{{\rm d}\vec{\mathbf{P}}_{\rm tot}}{{\rm d}t} \qquad \text{(acceleration of a system of particles);}$$

$$\vec{\mathbf{P}}_{\rm tot} = \sum_i m_i \vec{\mathbf{v}}_i = M_{\rm tot}\vec{\mathbf{v}}_{\rm cm} \qquad \text{(momentum of a system of particles);}$$

$$K_{\rm tot} = \frac{1}{2} M_{\rm tot} v^2_{\rm cm} + \sum_i \frac{1}{2} m_i \left(\vec{\mathbf{v}}_i - \vec{\mathbf{v}}_{\rm cm}\right)^2 \qquad \text{(K.E. of a system of particles);}$$

$$\vec{\mathbf{J}} = \int_{t_1}^{t_2} \vec{\mathbf{F}}\,{\rm d}t = \int_{t_1}^{t_2} \frac{{\rm d}\vec{\mathbf{p}}}{{\rm d}t}\,{\rm d}t = \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1 \qquad \text{(impulse-momentum theorem).}$$

Rotation in Two Dimensions:

Most of the equations for this topic are most easily remembered in the context of the analogous equations for linear motion in one dimension:

TRANSLATION (one dimension)		ROTATION (about fixed axis)	
Name	Symbol	Name	Symbol
Position	x	Orientation	θ
Velocity	$v = \frac{{\rm d}x}{{\rm d}t}$	Angular velocity	$\omega = \frac{{\rm d}\theta}{{\rm d}t}$
Acceleration	$a = \frac{{\rm d}v}{{\rm d}t}$	Angular acceleration	$\alpha = \frac{{\rm d}\omega}{{\rm d}t}$
Mass	$M = \sum_i m_i$	Moment of inertia	$I = \sum_i m_i R_i^2$
Force	F	Torque	$\boldsymbol{\tau} = F_\perp R$ $= \pm\lvert\vec{\mathbf{F}}\rvert R_\perp$
Force equation	$\sum_i \vec{\mathbf{F}}^{\rm ext} = M\vec{\mathbf{a}}_{\rm cm}$	Torque equation	$\sum_i \boldsymbol{\tau}^{\rm ext} = I\alpha$
Momentum	$p = Mv$	Angular momentum	$L = I\omega$
Kinetic energy	$\frac{1}{2}Mv^2$	Kinetic energy	$\frac{1}{2}I\omega^2$
Work done	$\vec{\mathbf{F}} \cdot \overrightarrow{\boldsymbol{\Delta r}}$	Work done	$\boldsymbol{\tau}\,\Delta\theta$

Other equations about rotation in two dimensions:

$v_r = 0\ ; \quad v_\perp = R\omega$	(velocity of point on rotating body);
$a_r = -\dfrac{v^2}{R} = -R\omega^2\ ; \quad a_\perp = R\alpha$	(acceleration of point on rotating body);
$v = \pm R\lvert\omega\rvert$	(rolling without slipping);
$\left.\begin{aligned} \sum \vec{\mathbf{F}}^{\text{ext}} &= M\vec{\mathbf{a}}_{\text{cm}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} \\ \sum \boldsymbol{\tau}^{\text{ext}} &= I_{\text{cm}}\alpha = \frac{\mathrm{d}L}{\mathrm{d}t} \end{aligned}\right\}$	(combined translational and rotational motion);
$K_{\text{tot}} = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2$	(kinetic energy for combined translational and rotational motion);
$I_\parallel = I_{\text{cm}} + Md^2$	(parallel-axis theorem);
$I_z = I_x + I_y$	(perpendicular-axis theorem).

TABLE OF STANDARD MOMENTS OF INERTIA:

Slender uniform rod of length ℓ, axis through center and perpendicular to axis of rod	$\frac{1}{12}m\ell^2$
Rectangular plate with dimensions $a \times b$, axis along one of the b edges	$\frac{1}{3}ma^2$
Thin-walled hollow cylinder of radius R, axis along axis of cylinder	mR^2
Uniform solid cylinder of radius R, axis along axis of cylinder	$\frac{1}{2}mR^2$
Thin-walled hollow sphere of radius R, axis through center	$\frac{2}{3}mR^2$
Solid uniform sphere of radius R, axis through center	$\frac{2}{5}mR^2$

Rotations in Vector Notation:

$$c_x = a_y b_z - a_z b_y \ ; \quad c_y = a_z b_x - a_x b_z \ ; \quad c_z = a_x b_y - a_y b_x \ .$$
(vector cross product, component form);

$$|\vec{\mathbf{c}}| = |\vec{\mathbf{a}}||\vec{\mathbf{b}}| \sin\theta$$
(magnitude of vector cross product);

$$\vec{\mathbf{v}} = \vec{\boldsymbol{\omega}} \times \vec{\mathbf{r}}$$
(velocity of atom in rotating body with a fixed point);

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_P + \vec{\boldsymbol{\omega}} \times \left(\vec{\mathbf{r}} - \vec{\mathbf{r}}_P\right)$$
(velocity of atom in rotating body, general case);

$$\vec{\mathbf{L}} = \sum_i \vec{\mathbf{r}}_i \times \vec{\mathbf{p}}_i$$
(angular momentum, as vector product);

$$\vec{\boldsymbol{\tau}} = \sum_i \vec{\mathbf{r}}_i \times \vec{\mathbf{F}}_i$$
(vector torque, as vector product);

$$\vec{\boldsymbol{\tau}} = \frac{\mathrm{d}\vec{\mathbf{L}}}{\mathrm{d}t}$$
(torque equation);

$$\left.\begin{aligned} \sum \vec{\mathbf{F}}^{\mathrm{ext}} &= M\vec{\mathbf{a}}_{\mathrm{cm}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} \\ \sum \vec{\boldsymbol{\tau}}^{\mathrm{ext}} &= \frac{\mathrm{d}\vec{\mathbf{L}}_{\mathrm{cm}}}{\mathrm{d}t} \end{aligned}\right\}$$
(combined translational and rotational motion);

$$\vec{\mathbf{L}} = \vec{\mathbf{r}}_{\mathrm{cm}} \times \vec{\mathbf{p}}_{\mathrm{tot}} + \sum_i \vec{\mathbf{r}}_{\mathrm{rel},i} \times m_i \vec{\mathbf{v}}_{\mathrm{rel},i}$$
(angular momentum decomposition);

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}}_{\mathrm{cm}} \times \vec{\mathbf{F}}_{\mathrm{tot}} + \sum_i \vec{\mathbf{r}}_{\mathrm{rel},i} \times \vec{\mathbf{F}}_i$$
(torque decomposition).

For Static Bodies:

$$\sum \vec{\mathbf{F}}^{\mathrm{ext}} = 0$$
(total external force vanishes);

$$\sum \vec{\boldsymbol{\tau}}^{\mathrm{ext}} = 0$$
(total external torque about ANY point vanishes).

Gravitation:

$$F_g = \frac{Gm_1m_2}{r^2}$$ (Newton's law of gravity);

$$g = \frac{Gm_E}{R_E^2}$$ (Acceleration due to gravity at Earth's surface);

$$U = -\frac{Gm_Em}{r}$$ (Gravitational potential energy);

$$v = \sqrt{\frac{Gm_E}{r}}$$ (Speed in cicular orbit);

$$T = \frac{2\pi r}{v} = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}}$$ (Period of a circular orbit);

$$R_S = \frac{2GM}{c^2}$$ (Schwarzschild radius).

Periodic Motion:

$$f = \frac{1}{T}$$ (Frequency and period);

$$\omega = 2\pi f = \frac{2\pi}{T}$$ (angular frequency);

$$F_x = -kx$$ (simple harmonic oscillator);

$$\omega = \sqrt{\frac{k}{m}}$$ (angular frequency of simple harmonic oscillator);

$$x = A\cos(\omega t + \phi)$$ (motion of simple harmonic oscillator);

$$\omega = \sqrt{\frac{\kappa}{I}}$$ (angular frequency ω in terms of torsion constant κ and moment of inertia I)