MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 Spring 2005

WEEKLY QUIZ 2 SOLUTIONS

Friday, February 11, 2005

Corrected Version, February 19, 2005: Typo fixed in 4(d) solution

FAMILY (LAST) NAME

GIVEN (FIRST) NAME

STUDENT ID NUMBER

Please check (✔) your class ☞

	L01	MTW 10:00	Walter Lewin
	L02	MTW 11:00	Walter Lewin
	L03	MTW 2:00	Min Chen
	L04	MTW 3:00	Min Chen

INSTRUCTIONS:

1. The FORMULA SHEET is in the back of this exam. You may tear it off. There is also an extra BLANK PAGE in case you need it.
2. This is a closed book exam. CALCULATORS, BOOKS, and NOTES are NOT ALLOWED.
3. Unless otherwise stated, to earn full credit you must show a valid DERIVATION and/or EXPLANATION of your answer, and you must express it in terms of the GIVEN VARIABLES.

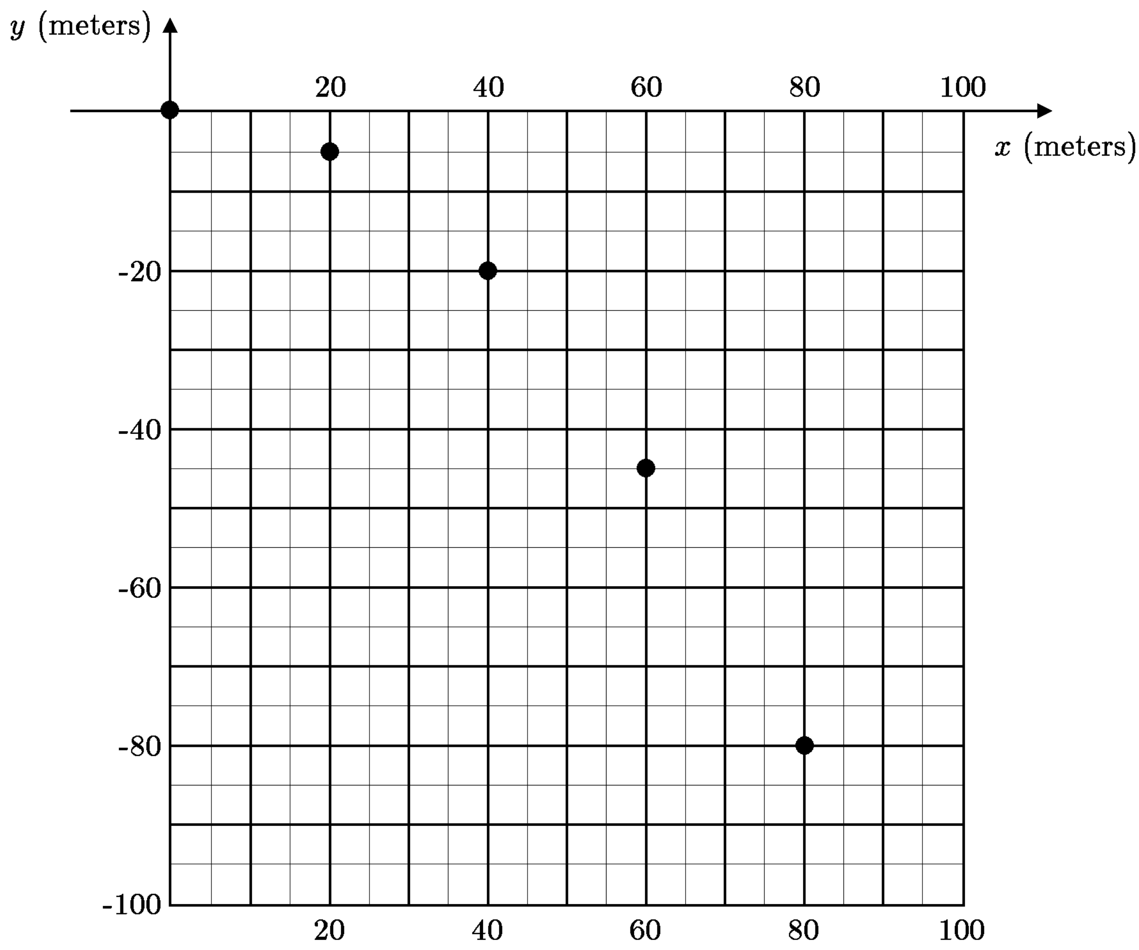
Problem	**Maximum**	**Score**	**Grader**
1	25		
2	25		
3	25		
4	25		
TOTAL	100		

Name ______________________________

Problem 1: Short answer questions (*25 points*)

(a) A simple trajectory (*10 points*)

A ball is thrown from a cliff, with an initial velocity that is exactly horizontal, with a magnitude of 20 meter/second. It travels under the influence of gravity, and for numerical simplicity we use the approximate value $g = 10$ meter/second^2 for the acceleration of gravity. Use a coordinate system in which the ball is thrown from [0, 0, 0], and travels initially in the positive x-direction. Ignoring all frictional effects, indicate the trajectory followed by the ball by putting a dot on the following graph at the location of the ball at $t = 1$, 2, 3, and 4 seconds.



$$x = v_0 t = (20 \text{ m/s})\, t$$

$$y = -\frac{1}{2} g t^2 = -\frac{1}{2}(10 \text{ m/s}^2)\, t^2 = -(5 \text{ m/s}^2)\, t^2 \ .$$

t	x	y
1 s	20 m	-5 m
2 s	40 m	-20 m
3 s	60 m	-45 m
4 s	80 m	-80 m

Problem 1, continued

(b) (*10 points*) If $\vec{\mathbf{a}} = (2\text{ m})\,\hat{\boldsymbol{\imath}} - (4\text{ m})\,\hat{\boldsymbol{\jmath}} + (4\text{ m})\,\hat{\boldsymbol{k}}$ and $\vec{\mathbf{b}} = (3\text{ m})\,\hat{\boldsymbol{\imath}} - (2\text{ m})\,\hat{\boldsymbol{k}}$, find:

(i) (*2 points*) $\vec{\mathbf{a}} + \vec{\mathbf{b}} = \boxed{(5\text{ m})\,\hat{\boldsymbol{\imath}} - (4\text{ m})\,\hat{\boldsymbol{\jmath}} + (2\text{ m})\,\hat{\boldsymbol{k}}\ .}$

(ii) (*2 points*) $\vec{\mathbf{a}} - \vec{\mathbf{b}} = \boxed{-(1\text{ m})\,\hat{\boldsymbol{\imath}} - (4\text{ m})\,\hat{\boldsymbol{\jmath}} + (6\text{ m})\,\hat{\boldsymbol{k}}\ .}$

(iii) (*2 points*) $\vec{\mathbf{a}} + 2\vec{\mathbf{b}} = \boxed{(8\text{ m})\,\hat{\boldsymbol{\imath}} - (4\text{ m})\,\hat{\boldsymbol{\jmath}}\ .}$

(iv) (*2 points*) $|\vec{\mathbf{a}}| = \sqrt{(2\text{ m})^2 + (4\text{ m})^2 + (4\text{ m})^2} = \sqrt{36\text{ m}^2} = \boxed{6\text{ m}\ .}$

(v) (*2 points*) Construct a unit vector in the direction of $\vec{\mathbf{a}}$.

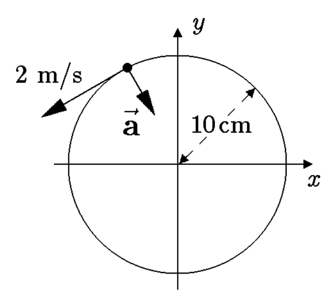
$$\hat{a} = \frac{\vec{\mathbf{a}}}{|\vec{\mathbf{a}}|} = \frac{1}{6\text{ m}}\vec{\mathbf{a}} = \boxed{\frac{1}{3}\,\hat{\boldsymbol{\imath}} - \frac{2}{3}\,\hat{\boldsymbol{\jmath}} + \frac{2}{3}\,\hat{\boldsymbol{k}}\ .}$$

(c) (*5 points*) Suppose that a particle is moving in a circle of radius 10 cm, at a speed of 2 m/s.

(i) (*3 points*) What is the magnitude of the acceleration?

$$a = \frac{v^2}{r} = \frac{(2\text{ m/s})^2}{0.1\text{ m}} = \boxed{40\text{ m/s}^2\ .}$$

(ii) (*2 points*) On the diagram below, draw an arrow showing the direction of the acceleration when the particle is at the position shown by the dot.



When a particle moves in a circle at a constant speed, the acceleration is centripetal, i.e., towards the center of the circle.

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Problem 2: Tracing the flight of bees (*25 points*)

At time $t = 0$, three bees are located at the origin of a coordinate system. From that time onward, the first bee travels at a constant velocity $[0, 0, v_a]$. The second bee has an initial velocity $[v_b, 0, 0]$, and accelerates with a uniform acceleration $[0, a, 0]$. The third bee flies at a constant velocity, and collides with the second bee at time $t = t_f$. Note that $[x, y, z]$ has the same meaning as $x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}} + z\hat{\boldsymbol{k}}$; you may express your answers in either notation.

(a) (*6 points*) Find the displacement vector $\vec{\mathbf{r}}_1(t)$ which describes the position of the first bee as a function of time, valid for times $t > 0$.

$$\vec{\mathbf{r}}_1(t) = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_1\, t + \frac{1}{2}\vec{\mathbf{a}}_1\, t^2 = [0,0,0] + [0,0,v_a\, t] + [0,0,0] = \boxed{[0,0,v_a t]\ .}$$

(b) (*6 points*) Find the displacement vector $\vec{\mathbf{r}}_2(t)$ which describes the position of the second bee as a function of time, valid for $t > 0$.

$$\vec{\mathbf{r}}_2(t) = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_2\, t + \frac{1}{2}\vec{\mathbf{a}}_2\, t^2 = [0,0,0] + [v_b t, 0, 0] + \frac{1}{2}[0,a,0]t^2 = \boxed{\left[v_b t, \frac{1}{2}at^2, 0\right]\ .}$$

(c) (*6 points*) Find the velocity vector $\vec{\mathbf{v}}_2(t)$ of the second bee as a function of time (for $t > 0$).

$$\vec{\mathbf{v}}_2(t) = \frac{\mathrm{d}\vec{\mathbf{r}}_2(t)}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left[v_b t, \frac{1}{2}at^2, 0\right] = \boxed{[v_b, at, 0]\ .}$$

where I used the fact that a vector in Cartesian coordinates is differentiated by differentiating each component separately, and of course the fact that $\frac{\mathrm{d}}{\mathrm{d}t}t^2 = 2t$.

(d) (*7 points*) With what ***speed*** v_3 does the third bee travel?

Be sure to express all your answers in terms of the given variables, v_a, v_b, a, and t_f.

At time t_f, the 2nd bee is at

$$\vec{\mathbf{r}}_2(t_f) = \left[v_b t_f, \frac{1}{2}at_f^2, 0\right] \ .$$

Since the 3rd bee collides with the 2nd at this time, the 3rd bee must be at the same position. Since this bee also started at $[0, 0, 0]$ and traveled at a constant velocity, that velocity must be

$$\vec{\mathbf{v}}_3 = \frac{\Delta\vec{\mathbf{r}}}{\Delta t} = \frac{\vec{\mathbf{r}}_2(t_f)}{t_f} = \left[v_b, \frac{1}{2}at_f, 0\right] \ .$$

The speed is then

$$v_3 = \left|\vec{\mathbf{v}}_3\right| = \boxed{\sqrt{v_b^2 + \frac{1}{4}a^2t_f^2} \ .}$$

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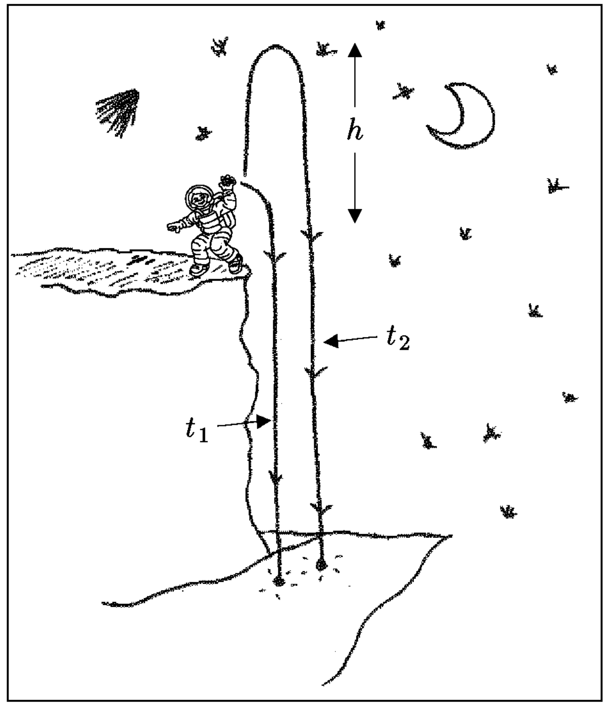
Problem 3: Vertical trajectories on an exotic planet (*30 points*):

A crewman on the starship Enterprise is on shore leave on a distant planet. He drops a rock from the top of a cliff and observes that it takes time t_1 to reach the bottom. He now throws another rock vertically upwards so that it reaches a height h above the cliff before dropping down the cliff face. The second rock takes a total time t_2 to reach the bottom of the cliff, starting from the time it leaves the crewman's hand. The planet has a very thin atmosphere which offers negligible air resistance. How high is the cliff, and what is the value of g on this planet?



DO NOT CARRY OUT THE ALGEBRA for this problem, but instead write down a set of equations that could be solved to obtain the answer.

CLEARLY ENCLOSE IN BOXES the set of equations that needs to be solved. (5 points will be taken off for any irrelevant boxed equation, so don't box everything!)

BE SURE TO DEFINE any variables you introduce into the discussion, beyond the variables t_1, t_2, h, and g, which have already been defined.

SOLUTION METHOD 1 (following study guide):

Let H = height of cliff above ground.

Let v_0 = initial upward speed of second rock.

Let t_0 = time at which 2nd rock reaches maximum height.

There are now four unknowns, g, H, v_0, and t_0, so we need four equations. If z is the vertical coordinate, measured from ground level, then the statement that the first rock hits the ground at time t_1 is written

$$z(t_1) = \boxed{H - \frac{1}{2} g t_1^2 = 0 \ ,}$$

and the statement that the second rock hits the ground at time t_2 is written

$$z(t_2) = \boxed{H + v_0 t_2 - \frac{1}{2} g t_2^2 = 0 \ .}$$

If the 2nd rock reaches its maximum height at time t_0, then it must have zero velocity at this time, so

$$v(t_0) = \boxed{v_0 - g t_0 = 0 \ .}$$

Finally, the condition that the height of the 2nd rock above the ground is $H + h$ at time t_0 is

$$z(t_0) = \boxed{H + h = H + v_0 t_0 - \frac{1}{2} g t_0^2 \ .}$$

SOLUTION METHOD 2:

Let H = height of cliff above ground.

Let v_0 = initial upward speed of second rock.

This time we have only three unknowns, g, H, and v_0, so we need only three equations. If z is the vertical coordinate, measured from ground level, then the statement that the first rock hits the ground at time t_1 is written

$$z(t_1) = \boxed{H - \frac{1}{2}gt_1^2 = 0 \ ,}$$

and the statement that the second rock hits the ground at time t_2 is written

$$z(t_2) = \boxed{H + v_0 t_2 - \frac{1}{2}gt_2^2 = 0 \ .}$$

The condition that the 2nd rock reaches a maximum height h above the cliff is

$$\boxed{v_0^2 = 2gh \ .}$$

EXTENSION:

You were asked only to write the equations above, but now that the quiz is over you might want to solve them. If you do, you should find

$$\boxed{g = \frac{8t_2^2 h}{\left(t_2^2 - t_1^2\right)^2}}$$

and

$$\boxed{H = \frac{4t_2^2 t_1^2 h}{\left(t_2^2 - t_1^2\right)^2} \ .}$$

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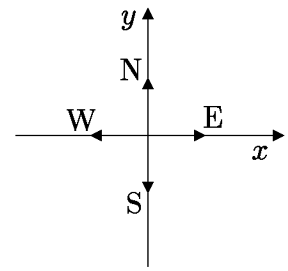
Problem 4: Flying in the wind (*25 points*)



For this problem we will use a coordinate system in which the compass directions — North, South, East, and West — are oriented with respect to the x- and y-axes as shown on the right. Suppose that an airplane is traveling due north, at a speed v_{air} relative to the air. There is a wind blowing uniformly toward the west, at a speed v_{wind}.

(a) (*6 points*) What is the velocity $\vec{\mathbf{v}}_{\text{ground}}$ of the plane relative to the ground? Express your answer as a vector of the form $\vec{\mathbf{v}}_{\text{ground}} = ?\hat{\boldsymbol{\imath}} + ?\hat{\boldsymbol{\jmath}}$.

The key item here is the definition of relative velocity, which is defined on the formula sheet as a difference in velocities; i.e., the velocity of some object A relative to some object B is defined by $\vec{\mathbf{v}}_{A,B} \equiv \vec{\mathbf{v}}_A - \vec{\mathbf{v}}_B$, *where* $\vec{\mathbf{v}}_A$ *and* $\vec{\mathbf{v}}_B$ *are measured in the same coordinate system. Applying that definition to this case,*

$$\vec{\mathbf{v}}_{\text{plane,air}} = \vec{\mathbf{v}}_{\text{plane,ground}} - \vec{\mathbf{v}}_{\text{air,ground}} \ ,$$

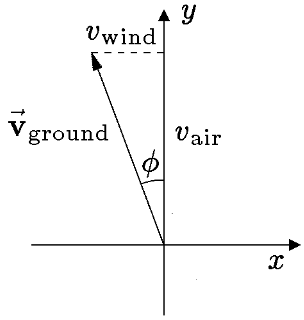
so

$$\begin{aligned} \vec{\mathbf{v}}_{\text{ground}} &\equiv \vec{\mathbf{v}}_{\text{plane,ground}} \\ &= \vec{\mathbf{v}}_{\text{air,ground}} + \vec{\mathbf{v}}_{\text{plane,air}} \\ &= \boxed{-v_{\text{wind}}\hat{\boldsymbol{\imath}} + v_{\text{air}}\hat{\boldsymbol{\jmath}} \ .} \end{aligned}$$

(b) (*6 points*) What is the speed of the plane relative to the ground?

$$\text{Speed} = \left|\vec{\mathbf{v}}_{\text{ground}}\right| = \boxed{\sqrt{v_{\text{wind}}^2 + v_{\text{air}}^2} \ .}$$

(c) (*6 points*) What is the angle of the plane's ground velocity relative to north? Be sure to specify if the angle is east of north or west of north.



$$\vec{\mathbf{v}}_{\text{ground}} = -v_{\text{wind}}\hat{\boldsymbol{\imath}} + v_{\text{air}}\hat{\boldsymbol{\jmath}}$$

$$\tan\phi = \frac{v_{\text{wind}}}{v_{\text{air}}}$$

$$\boxed{\phi = \arctan\left(\frac{v_{\text{wind}}}{v_{\text{air}}}\right) \text{, west of north.}}$$

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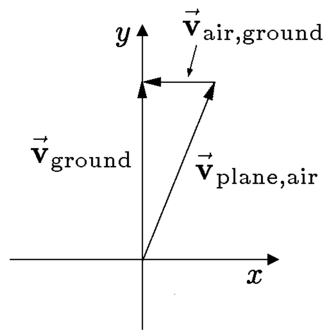
(d) (*7 points*) Keeping the same speed relative to the air, $v_{\rm air}$, the pilot turns the plane at just the right angle so that its velocity relative to the ground is due north. What is the ground speed (i.e., magnitude of the velocity relative to the ground) of the plane?

The vector relationship

$$\vec{\mathbf{v}}_{\rm ground} = \vec{\mathbf{v}}_{\rm air,ground} + \vec{\mathbf{v}}_{\rm plane,air}$$

continues to hold, since it follows from the definition of relative velocity. The wind velocity has not changed, so $\vec{\mathbf{v}}_{\rm air,ground} = -v_{\rm wind}\,\hat{\imath}$, *and we are told that* $\left|\vec{\mathbf{v}}_{\rm plane,air}\right| = v_{\rm air}$. *This time, however, the direction of* $\vec{\mathbf{v}}_{\rm plane,air}$ *is modified, so that* $\vec{\mathbf{v}}_{\rm ground}$ *points due north. The picture is then as shown on the right. The Pythagorean theorem then implies that*



$$\left|\vec{\mathbf{v}}_{\rm ground}\right| = \sqrt{\left|\vec{\mathbf{v}}_{\rm plane,air}\right|^2 - \left|\vec{\mathbf{v}}_{\rm air,ground}\right|^2}$$

$$= \boxed{\sqrt{v_{\rm air}^2 - v_{\rm wind}^2}\ ,}$$

which indicates that a wind perpendicular to the flight path slows the airplane down.