MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 — Spring 2005

WEEKLY QUIZ 3 SOLUTIONS

Friday, February 18, 2005

Corrected Version, February 19, 2005: Added announcements made at the quiz

FAMILY (LAST) NAME

GIVEN (FIRST) NAME

STUDENT ID NUMBER

Please check (✔) your class

	L01	MTW 10:00	Walter Lewin
	L02	MTW 11:00	Walter Lewin
	L03	MTW 2:00	Min Chen
	L04	MTW 3:00	Min Chen

INSTRUCTIONS:

1. The FORMULA SHEET is in the back of this exam. You may tear it off. There is also an extra BLANK PAGE in case you need it.
2. This is a closed book exam. CALCULATORS, BOOKS, and NOTES are NOT ALLOWED.
3. Unless otherwise stated, to earn full credit you must show a valid DERIVATION and/or EXPLANATION of your answer, and you must express it in terms of the GIVEN VARIABLES.

Problem	**Maximum**	**Score**	**Grader**
1	25		
2	25		
3	25		
4	25		
TOTAL	100		

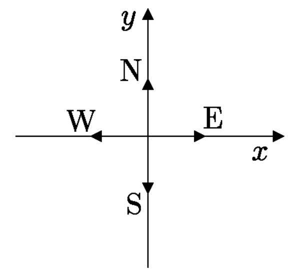
ANNOUNCEMENTS MADE AT THE QUIZ

Problem 3(c): Remember that the rope has mass M and length ℓ.

Problem 4: Neglect the mass of the cables.

Name ______________________

Problem 1: Flying in the wind (*25 points*)



For this problem we will use a coordinate system in which the compass directions — North, South, East, and West — are oriented with respect to the x- and y-axes as shown on the right. Suppose that an airplane is traveling due east, at a speed v_{air} relative to the air. There is a wind blowing uniformly toward the south, at a speed v_{wind}.

(a) (*6 points*) What is the velocity $\vec{\mathbf{v}}_{\text{ground}}$ of the plane relative to the ground? Express your answer as a vector of the form $\vec{\mathbf{v}}_{\text{ground}} = ?\hat{\boldsymbol{\imath}} + ?\hat{\boldsymbol{\jmath}}$.

The key item here is the definition of relative velocity, which is defined on the formula sheet as a difference in velocities; i.e., the velocity of some object A relative to some object B is defined by $\vec{\mathbf{v}}_{A,B} \equiv \vec{\mathbf{v}}_A - \vec{\mathbf{v}}_B$*, where* $\vec{\mathbf{v}}_A$ *and* $\vec{\mathbf{v}}_B$ *are measured in the same coordinate system. Applying that definition to this case,*

$$\vec{\mathbf{v}}_{\text{plane,air}} = \vec{\mathbf{v}}_{\text{plane,ground}} - \vec{\mathbf{v}}_{\text{air,ground}} \ ,$$

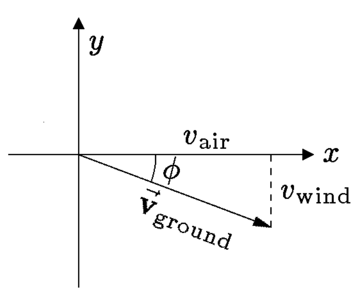
so

$$\begin{aligned} \vec{\mathbf{v}}_{\text{ground}} &\equiv \vec{\mathbf{v}}_{\text{plane,ground}} \\ &= \vec{\mathbf{v}}_{\text{air,ground}} + \vec{\mathbf{v}}_{\text{plane,air}} \\ &= \boxed{v_{\text{air}}\hat{\boldsymbol{\imath}} - v_{\text{wind}}\hat{\boldsymbol{\jmath}} \ .} \end{aligned}$$

(b) (*6 points*) What is the speed of the plane relative to the ground?

$$\text{Speed} = \left|\vec{\mathbf{v}}_{\text{ground}}\right| = \boxed{\sqrt{v_{\text{air}}^2 + v_{\text{wind}}^2} \ .}$$

(c) (*6 points*) What is the angle of the plane's ground velocity relative to east? Be sure to specify if the angle is north of east or south of east.



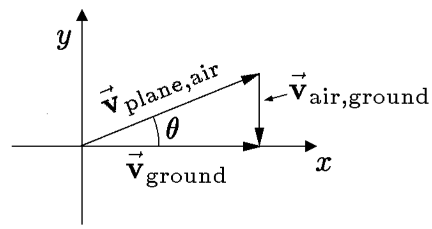
$$\vec{\mathbf{v}}_{\text{ground}} = v_{\text{air}}\hat{\boldsymbol{\imath}} - v_{\text{wind}}\hat{\boldsymbol{\jmath}}$$

$$\tan\phi = \frac{v_{\text{wind}}}{v_{\text{air}}}$$

$$\boxed{\phi = \arctan\left(\frac{v_{\text{wind}}}{v_{\text{air}}}\right) \ , \text{ south of east.}}$$

(d) (*7 points*) Keeping the same speed relative to the air, v_{air}, the pilot turns the plane at just the right angle so that its velocity relative to the ground is due east. At what angle relative to east does he turn the plane? Be sure to specify if the angle is north of east or south of east.

The vector relationship



$$\vec{\mathbf{v}}_{\text{ground}} = \vec{\mathbf{v}}_{\text{air,ground}} + \vec{\mathbf{v}}_{\text{plane,air}}$$

continues to hold, since it follows from the definition of relative velocity. The wind velocity has not changed, so $\vec{\mathbf{v}}_{\text{air,ground}} = -v_{\text{wind}}\,\hat{\boldsymbol{\jmath}}$*, and we are told that* $\left|\vec{\mathbf{v}}_{\text{plane,air}}\right| = v_{\text{air}}$*. This time, however, the direction of* $\vec{\mathbf{v}}_{\text{plane,air}}$ *is modified, so that* $\vec{\mathbf{v}}_{\text{ground}}$ *points due east. The picture is then as shown on the right. The angle* θ *is then given by*

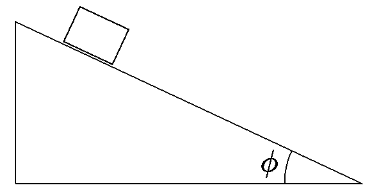
$$\sin\theta = \frac{\left|\vec{\mathbf{v}}_{\text{air,ground}}\right|}{\left|\vec{\mathbf{v}}_{\text{plane,air}}\right|} = \frac{v_{\text{wind}}}{v_{\text{air}}} \ ,$$

so

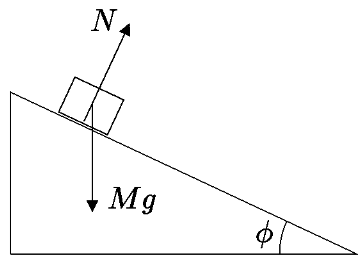
$$\boxed{\theta = \arcsin\left(\frac{v_{\text{wind}}}{v_{\text{air}}}\right) \ , \ \textit{north of east.}}$$

Problem 2: The inclined plane (*25 points*)

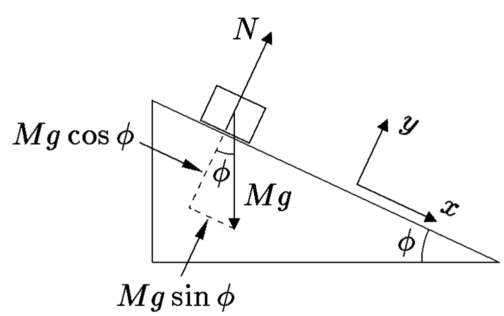
A block slides on a frictionless inclined plane, at an angle ϕ relative to the horizontal. Neglect air resistance.



(a) (*8 points*) Draw a free body diagram showing all the forces acting on the block. Be sure to label each force.



(b) (*9 points*) What is the magnitude and direction of the acceleration that the block experiences?



Choose x- and y-axes that are parallel and perpendicular to the inclined plane, as shown in the diagram. Then the force of gravity can be resolved explicitly as

$$\vec{\mathbf{W}} = M\vec{\mathbf{g}} = Mg\sin\phi\,\hat{\boldsymbol{\imath}} - Mg\cos\phi\,\hat{\boldsymbol{\jmath}}\ ,$$

and the normal force becomes

$$\vec{\mathbf{N}} = N\,\hat{\boldsymbol{\jmath}}\ .$$

Thus the x-component of $\vec{\mathbf{F}} = M\vec{\mathbf{a}}$ *becomes*

$$Mg\sin\phi = Ma_x\ ,\ \text{which implies } a_x = g\sin\phi\ .$$

There is no motion in the y-direction, so

$$\boxed{\text{the acceleration has magnitude } g\sin\phi \text{ and points down the slope.}}$$

(c) (*8 points*) What is the magnitude and direction of the force that the inclined plane exerts on the block?

The rigidity of the inclined plane prevents the block from moving in the y-direction, so there is no acceleration in the y-direction. Thus the y-component of $\vec{\mathbf{F}} = M\vec{\mathbf{a}}$ *can be written*

$$N - Mg\cos\phi = 0 \ ,$$

so

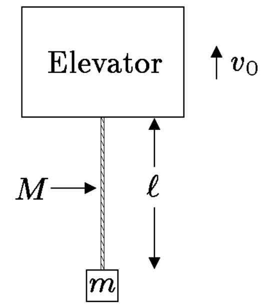
$$N = Mg\cos\phi \ .$$

Thus,

the normal force has magnitude $Mg\cos\phi$ *and points along the upward normal to the plane.*

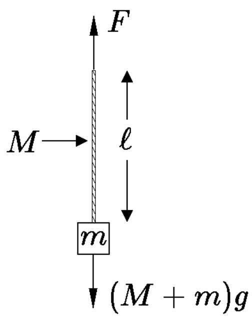
Name ____________________

Problem 3: A rope dangling from an elevator (*25 points*)



A rope of length ℓ and mass M is suspended from the bottom of an elevator. A block of mass m is attached to the bottom of the rope. For parts (a) and (b) of this problem, the elevator is moving upward at a uniform speed v_0.

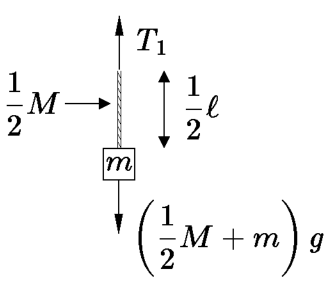
(a) (*8 points*) What is the magnitude F of the force that the elevator applies to the top of the rope?



Isolating the system consisting of the rope plus the block, the free body diagram is shown on the left. The only forces are the force applied by the elevator, F, and the downward force of gravity, $(M+m)g$. Since the velocity is uniform, there is no acceleration, so the net force must be zero. Therefore

$$F = (M+m)g \ .$$

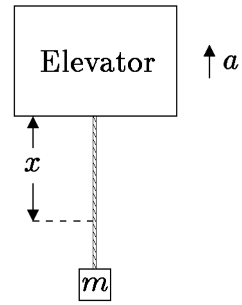
(b) (*8 points*) What is the tension T_1 of the rope at its midpoint?



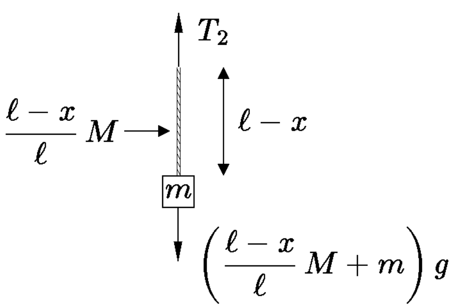
Isolating the system consisting of half the rope plus the block, the free body diagram is shown on the left. The only forces are the upward force applied by the top half of the rope, which is the tension T_1 at the midpoint, and the downward force of gravity, $\left(\frac{1}{2}M+m\right)g$. Since the velocity is uniform, there is no acceleration, so again the net force must be zero. Therefore

$$T_1 = \left(\frac{1}{2}M+m\right)g \ .$$

Now suppose that the elevator starts to accelerate upward at a fixed acceleration, of magnitude a.



(c) (*9 points*) What is the tension $T_2(x)$ in the rope at an arbitrary distance x below the elevator, where $0 \le x \le \ell$?



Isolating the system consisting the block plus a piece of rope of length $\ell - x$ (the piece whose end is a distance x below the elevator), the free body diagram is shown on the left. The only forces are the upward force applied by the top half of the rope, which is the tension $T_2(x)$ at a distance x below the elevator, and the downward force of gravity, $\left(\frac{\ell-x}{\ell} M + m\right) g$. Note that $(\ell - x)/\ell$ is the fraction of the rope that extends below the point at a distance x from the elevator. Since the acceleration is now upward with magnitude a, the vertical component of $\vec{\mathbf{F}} = M\vec{\mathbf{a}}$ becomes

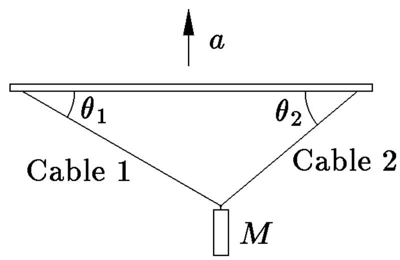
$$T_2 - \left(\frac{\ell - x}{\ell} M + m\right) g = \left(\frac{\ell - x}{\ell} M + m\right) a \ ,$$

so

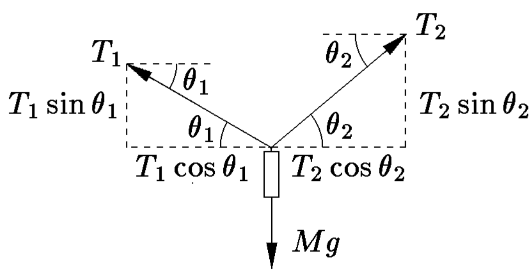
$$\boxed{T_2 = \left(\frac{\ell - x}{\ell} M + m\right) (g + a) \ .}$$

Problem 4: Lifting by helicopter (*25 points*)

A steel block of mass M is being lifted by a helicopter, which is accelerating upward with an acceleration of magnitude a. The cylinder is attached to a horizontal bar by two cables, which make angles of θ_1 and θ_2 with respect to the horizontal, as shown in the diagram.



Find an expression for T_1, the tension in cable 1, in terms of M, θ_1, θ_2, g, and a.



The force diagram, with the forces resolved into horizontal and vertical components, is shown above. There is no horizontal motion, so the horizontal forces must balance:

$$T_1 \cos\theta_1 = T_2 \cos\theta_2 \ . \tag{1}$$

The net vertical force must account for the vertical acceleration, and so must be equal to Ma:

$$T_1 \sin\theta_1 + T_2 \sin\theta_2 - Mg = Ma \ . \tag{2}$$

Substituting the value of T_2 obtained from Eq. (1),

$$T_1 \sin\theta_1 + T_1 \frac{\cos\theta_1}{\cos\theta_2} \sin\theta_2 = M(g+a) \ .$$

Now one just solves this equation for T_1:

$$T_1 \frac{\sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2}{\cos\theta_2} = M(g+a) \ .$$

By a trigonometric identity,

$$T_1 \frac{\sin(\theta_1 + \theta_2)}{\cos\theta_2} = M(g+a) \ ,$$

so finally

$$\boxed{T_1 = \frac{M(g+a)\cos\theta_2}{\sin(\theta_1 + \theta_2)} \ .}$$