MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 Spring 2005

WEEKLY QUIZ 4 SOLUTIONS

Quiz Date: Friday, February 25, 2005

FAMILY (LAST) NAME

GIVEN (FIRST) NAME

STUDENT ID NUMBER

Please check (✔) your class ☞

	L01	MTW 10:00	Walter Lewin
	L02	MTW 11:00	Walter Lewin
	L03	MTW 2:00	Min Chen
	L04	MTW 3:00	Min Chen

INSTRUCTIONS:

1. The FORMULA SHEET is in the back of this exam. You may tear it off. There is also an extra BLANK PAGE in case you need it.
2. This is a closed book exam. CALCULATORS, BOOKS, and NOTES are NOT ALLOWED.
3. Unless otherwise stated, to earn full credit you must show a valid DERIVATION and/or EXPLANATION of your answer, and you must express it in terms of the GIVEN VARIABLES.

ANNOUNCEMENT MADE AT THE QUIZ

Problem 1(d) and 1(e): The symbol F has no meaning for these parts, and therefore should not be included in the list of choices.

Problem	**Maximum**	**Score**	**Grader**
1	30		
2	40		
3	30		
TOTAL	100		

Name ______________________

Problem 1: Multiple choice questions about friction (*30 points*)

In this problem you will be asked to answer many short, multiple choice, questions pertaining to friction. In all parts you should assume that gravity is acting downward with acceleration g, with $g > 0$, and that air friction can be neglected. You can answer by circling your choice. Each question has one and only one correct answer.

(a) (*2 points*) A block of mass M rests on a level floor. The coefficient of static friction between the block and the floor is μ_s, and the coefficient of kinetic friction is μ_k. What is the magnitude of the force of friction acting on the block?

$\boxed{\text{(i) } 0;}$ (ii) Mg; (iii) $\mu_s Mg$; (iv) $\mu_k Mg$; (v) $(\mu_s - \mu_k)Mg$;

(vi) Cannot be determined from this data

(b) (*2 points*) The same block is now being pulled across the floor by a horizontal rope at a constant velocity. What is the tension in the rope?

(i) 0; (ii) Mg; (iii) $\mu_s Mg$; $\boxed{\text{(iv) } \mu_k Mg;}$ (v) $(\mu_s - \mu_k)Mg$;

(vi) Cannot be determined from this data

(c) (*2 points*) The same block is now once again stationary on the floor. John is pulling on the rope with a force of magnitude F, which is not large enough to cause the block to move. What is the magnitude of the force of friction on the block?

(i) 0; (ii) Mg; (iii) $\mu_s Mg$; (iv) $\mu_k Mg$; (v) $(\mu_s - \mu_k)Mg$; $\boxed{\text{(vi) } F;}$

(vii) Cannot be determined from this data

(d) (*2 points*) Now suppose that John pulls harder and harder until the block starts to move. What is the magnitude of the force of friction on the block just before it starts to move?

(i) 0; (ii) Mg; $\boxed{\text{(iii) } \mu_s Mg;}$ (iv) $\mu_k Mg$; (v) $(\mu_s - \mu_k)Mg$; (vi) F;

(vii) Cannot be determined from this data

(e) (*2 points*) What is the magnitude of the force of friction on the block just after it starts to move?

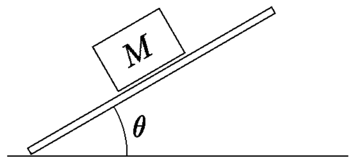
(i) 0; (ii) Mg; (iii) $\mu_s Mg$; $\boxed{\text{(iv) } \mu_k Mg;}$ (v) $(\mu_s - \mu_k)Mg$; (vi) F;

(vii) Cannot be determined from this data

— Problem 1 Continues —

Name ______________________________

A small block of mass M rests on a plank of wood, which is oriented at an angle θ with respect to horizontal. It remains stationary. The coefficients of friction between the block and the plank are μ_k for kinetic friction and μ_s for static friction.

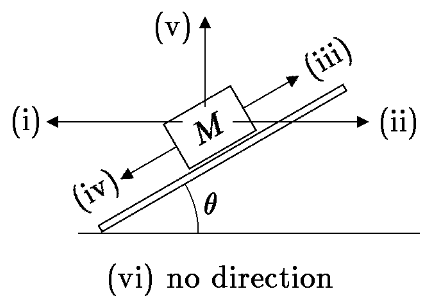


(f) (*2 points*) What is the magnitude of the force of friction acting on the block?

(i) 0; **[(ii) $Mg\sin\theta$;]** (iii) $Mg\cos\theta$; (iii) $\mu_s Mg\sin\theta$; (iv) $\mu_s Mg\cos\theta$; (v) $\mu_k Mg\cos\theta$;

(vi) $\mu_k Mg\sin\theta$; (vii) $(\mu_s - \mu_k)Mg\sin\theta$; (viii) Cannot be determined from this data

(g) (*2 points*) What is the direction of the force of friction? The possible answers are described both in words and on the diagram.



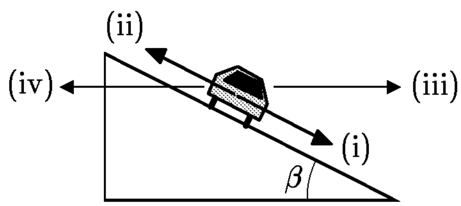
(i) horizontal to left; (ii) horizontal to right; **[(iii) upward along plank;]**

(iv) downward along plank; (v) vertical; (vi) no direction, because it vanishes

— Problem 1 Continues —

A circular road track is banked at just the right angle to optimize travel at 50 mph.

(h) (*2 points*) A car travels around the track at the ideal speed of 50 mph. What is the direction of the force of friction? The possible answers are described both in words and on the diagram to the right.



(v) no direction

(i) downward along the bank of the track
(ii) upward along the bank of the track
(iii) directly inward (horizontally) toward the center of the circle of the track
(iv) directly outward (horizontally), away from the center of the circle of the track.
(v) **[no direction, because it vanishes]**

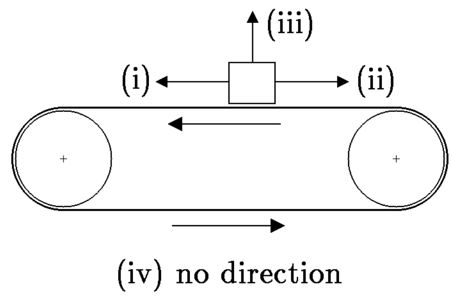
(i) (*2 points*) For the car described above, what is the direction of the acceleration?
(i) downward along the bank of the track
(ii) upward along the bank of the track
(iii) **[directly inward (horizontally) toward the center of the circle of the track]**
(iv) directly outward (horizontally), away from the center of the circle of the track.
(v) no direction, because it vanishes

(j) (*2 points*) The same car now travels around the track at 60 mph. What is the direction of the force of friction?
(i) **[downward along the bank of the track]**
(ii) upward along the bank of the track
(iii) directly inward (horizontally) toward the center of the circle of the track
(iv) directly outward (horizontally), away from the center of the circle of the track.
(v) no direction, because it vanishes

(k) (*2 points*) The same car now travels around the track at 40 mph. What is the direction of the force of friction?
(i) downward along the bank of the track
(ii) **[upward along the bank of the track]**
(iii) directly inward (horizontally) toward the center of the circle of the track
(iv) directly outward (horizontally), away from the center of the circle of the track.
(v) no direction, because it vanishes

— Problem 1 Continues —

(l) (*2 points*) A conveyor belt is used to transport machine parts in a horizontal direction, at a fixed speed. What is the direction of the force of friction acting on the parts? The possible answers are described both in words and on the diagram.



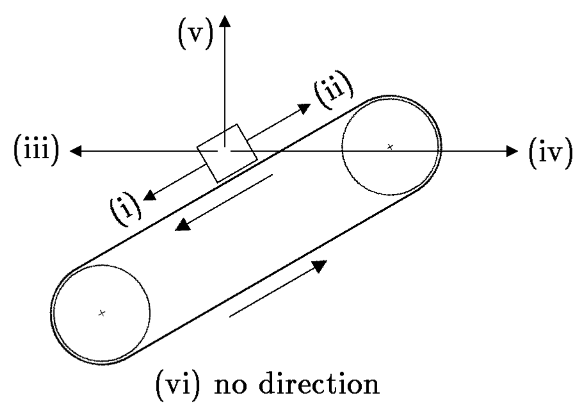
(i) forwards (ii) backwards (iii) upward **[(iv) no direction, because it vanishes]**

(m) (*2 points*) The conveyor belt is now speeded up. While the speed is increasing, what is the direction of the force of friction acting on the parts?

[(i) forwards] (ii) backwards (iii) upward (iv) no direction, because it vanishes

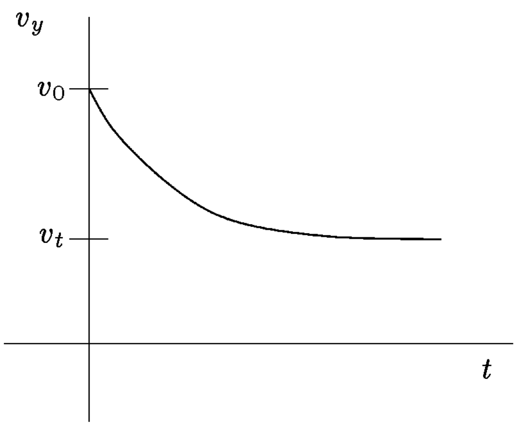
At the end of the horizontal stretch described above, the parts are transferred to another conveyor belt that transports them downward, at a fixed speed and fixed angle.

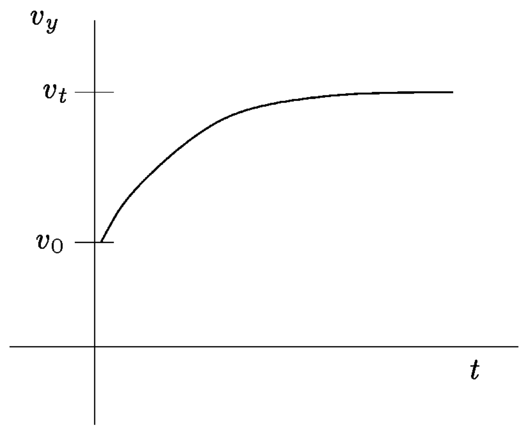
(n) (*2 points*) What is the direction of the force of friction acting on the parts? The possible answers are described both in words and on the diagram.



(i) forward along the belt
(ii) **[backward along the belt]**
(iii) forward horizontally
(iv) backward horizontally
(v) upward
(vi) no direction, because it vanishes

(o) (*2 points*) A pebble is dropped from some height into a deep pond. Once the pebble is under water, there is a drag force of magnitude $\left|\vec{\mathbf{F}}\right| = kv$, where k is a constant and v is the speed of the pebble. Trying the experiment a number of times, Timmy has made graphs of the speed of the pebble vs. time, starting from when the pebble entered the water. The only problem is that Timmy cannot be trusted. He shows you the following two graphs:





What is the most reasonable explanation of these graphs?

(i) The first graph shows what happens when the pebble is dropped from a small height, and the second shows what happens when it is dropped from a large height.

(ii) The first graph shows what happens when the pebble is dropped from a large height, and the second shows what happens when it is dropped from a small height.

(iii) The first graph is the correct qualitative shape for dropping the pebble from any height, but the second is not possible for water, although it could happen for a more viscous liquid like oil.

(iv) The second graph is the correct qualitative shape for dropping the pebble from any height, but the first is not possible for water, although it could happen for a more viscous liquid like oil.

— *End of Problem 1* —

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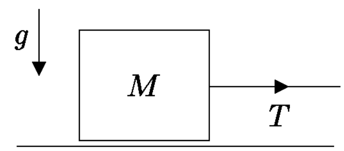
Problem 2: One or two blocks pulled by a rope (*40 points*)



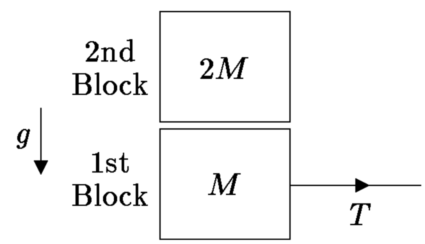
A block of mass M rests on a horizontal surface. The coefficient of kinetic friction between the block and the surface is μ_k, and the coefficient of static friction is μ_s, with $\mu_s > \mu_k$. The block is pulled horizontally by a massless inextensible rope, with a tension T that is gradually increased until the block starts to slide.

(a) (*7 points*) What is the value of the tension T_1 at which the block begins to slide?

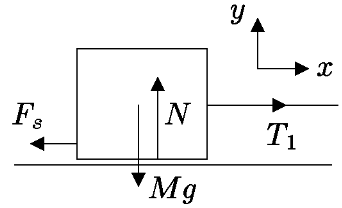
(b) (*8 points*) When the tension was only $\frac{1}{3}T_1$, before the block began to slide, what was the magnitude and direction of the force of friction?

(c) (*10 points*) If the tension is maintained at the value T_1, what is the acceleration of the block?

(d) (*15 points*) A second block, with twice the mass of the first, is placed directly on top of the first block while both are at rest. The lower surface of the second block is rough, so coefficients of friction between the two blocks are $2\mu_k$ for kinetic friction and $2\mu_s$ for static friction. (Recall that μ_k and μ_s denote the coefficients of kinetic and static friction, respectively, for the interface between the first block and the horizontal surface below.) As before, a horizontal rope is attached to the first (lower) block, and the tension in the rope is increased gradually from zero. At some value of the tension the two blocks begin to move, but there is initially no relative velocity between the two. As the steady increase in the tension is maintained, they accelerate faster and faster. At what value of the tension will the second block begin to slip relative to the first block? In what direction will it slip, relative to the block below?

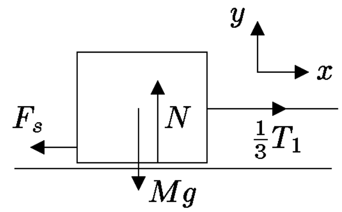


(a) The block begins to slide when the tension T_1 equals the maximal static friction force, which is given by $\mu_s N$. There is no acceleration in the vertical direction, so the normal force must cancel the force of gravity, implying that $N = Mg$. Thus,



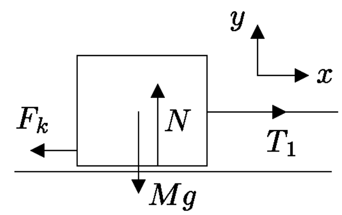
$$\boxed{T_1 = \mu_s N = \mu_s Mg \ .}$$

(b) The tension force $\frac{1}{3}T_1$ is insufficient to set the block in motion. Hence, the friction force will exactly cancel the tension force, and the block will remain at rest. Its direction will oppose the tension force and its magnitude is given by



$$\boxed{F_s = \frac{1}{3}T_1 = \frac{1}{3}\mu_s Mg \ .}$$

(c) The acceleration a is determined by the tension force T_1 and the kinetic friction force $F_k = \mu_k N = \mu_k Mg$ opposing it. The total force — and hence the acceleration — is in the direction of the tension force, and the magnitude of the acceleration is given by



$$Ma = T_1 - F_k = \mu_s Mg - \mu_k Mg$$

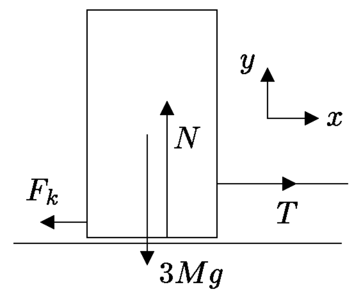
$$\Longrightarrow \quad \boxed{a = (\mu_s - \mu_k)g \ .}$$

Alternatively, in vector notation one could write

$$\boxed{\vec{\mathbf{a}} = [(\mu_s - \mu_k)g,\ 0,\ 0] \ .}$$

(d) Before the upper block begins to slip, it is moving together with the lower block due to static friction between them. We can consider the two blocks as one system.

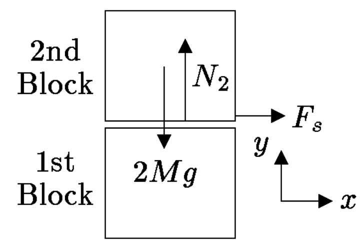
The two blocks have total mass $3M$ and the ground thus exerts a normal force $N = 3Mg$ on them. The normal force determines the kinetic friction force $F_k = \mu_k N = 3\mu_k Mg$ which opposes the applied tension force T. The total external force acting on the two blocks causes their acceleration a, so



$$3Ma = T - F_k = T - 3\mu_k Mg \ .$$

Note that the total mass that is accelerated is $3M$.

Now to find when the upper block will slip, we must consider it individually as a system. The only force that acts directly on the upper block is static friction, so its acceleration is due to that force. The magnitude of the static friction force on the upper block is therefore $F_s = 2Ma = 2(T - 3\mu_k Mg)/3$. The absence of acceleration in the y-direction implies that the normal force acting on the upper block is $N_2 = 2Mg$. The upper block begins to slip when F_s reaches its maximal value $2\mu_s N_2 = 4\mu_s Mg$, i.e. when



$$\frac{2}{3}(T - 3\mu_k Mg) = 4\mu_s Mg \ .$$

Solving for T, this gives

$$\boxed{T = 3(\mu_k + 2\mu_s)Mg \ .}$$

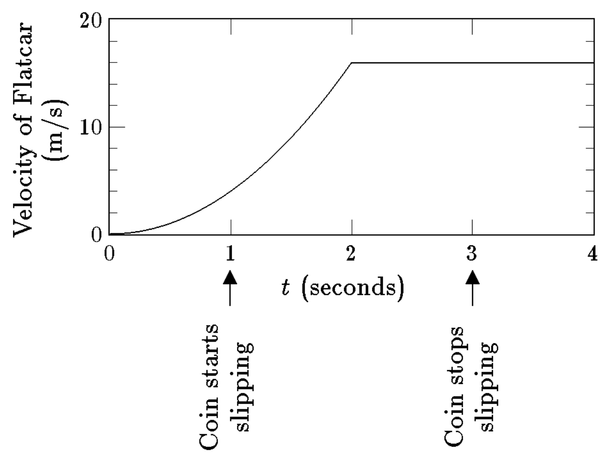
Once the upper block starts slipping, there is kinetic friction between the two blocks which is less than static friction. As a result, the upper block accelerates by a smaller amount than the lower block and thus **slips in the direction opposite the direction of motion.**

Problem 3: A rocket-propelled railroad flatcar (*30 points*)

A rocket-propelled railroad flatcar begins at rest at time $t = 0$, and then accelerates along a straight track with a speed given by

$$v(t) = (4\ \text{m/s}^3)\, t^2$$

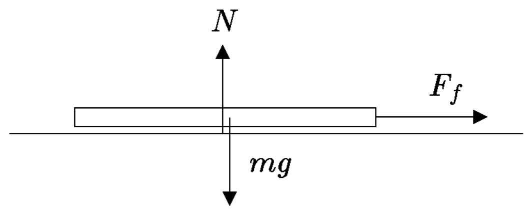
for $0 < t < 2$ s. Then the acceleration ends, and the flatcar continues at a constant speed of 16 m/s, as shown on the graph below. A coin is initially at rest on the floor of the flatcar. At $t = 1$ s the coin begins to slip, and it stops slipping at $t = 3$ s. Take $g = 10$ m/s^2.



a) (*15 points*) What is the coefficient of static friction between the coin and the floor?

b) (*15 points*) What is the coefficient of kinetic friction between the coin and the floor? *(Hint: Note that between $t = 1$ s and $t = 3$ s, the coin has a constant acceleration. Can you determine this acceleration from the given information?)*

(a) As the flatcar begins to accelerate, the coin is held at a fixed position relative to the car by static friction. The coefficient of static friction can be computed from the time at which it starts to slip, $t = 1$ s, which is when static friction has reached its maximal strength, $\mu_s N$. The acceleration along the track during the period $0 \le t \le 2$ s is given by



$$a = \frac{dv}{dt} = \frac{d}{dt}(4 \text{ m/s}^3)\, t^2 = (8 \text{ m/s}^3)\, t \ .$$

It slips when

$$ma = \mu_s N \ ,$$

where m is the mass of the coin and $N = mg$ is the normal force. The factors of m cancel out, giving

$$\mu_s = \frac{a}{g} = \frac{(8 \text{ m/s}^3)(1 \text{ s})}{10 \text{ m/s}^2} = \boxed{0.8 \ .}$$

(b) As the hint suggests, the coefficient of kinetic friction is found from the acceleration during the time interval $1 \text{ s} < t < 3 \text{ s}$, while the coin is slipping. It is subtle, but one must realize that even though the flatcar is moving in a complicated way during this time, the coin itself is slipping on the floor of the flatcar, and is therefore subject only to the force of kinetic friction. The magnitude of the force of kinetic friction is $F_f = \mu_k mg$, independent of the velocity of the flatcar underneath the coin. So the force on the coin during this period is constant, and it therefore undergoes uniform acceleration with magnitude

$$a = \frac{F_f}{m} = \frac{\mu_k mg}{m} = \mu_k g \ ,$$

so $\mu_k = a/g$. Knowing that the acceleration is constant, we can express its value in terms of the initial and final velocities:

$$a = \frac{\Delta v}{\Delta t} = \frac{v(3\,\text{s}) - v(1\,\text{s})}{2 \text{ s}} = \frac{(16 \text{ m/s}) - (4 \text{ m/s}^3)(1 \text{ s})}{2 \text{ s}} = 6 \text{ m/s}^2 \ .$$

(At both times t=1 s and t=3 s the coin is moving at the same speed as the flatcar, so we know what values to use for $v(1\,\text{s})$ and $v(3\,\text{s})$.) Then finally

$$\mu_k = \frac{a}{g} = \frac{6 \text{ m/s}^2}{10 \text{ m/s}^2} = \boxed{0.6 \ .}$$