MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 Spring 2005

WEEKLY QUIZ 5 SOLUTIONS

Quiz Date: Friday, March 4, 2005

FAMILY (LAST) NAME

GIVEN (FIRST) NAME

STUDENT ID NUMBER

Please check (✔) your class ☞

	L01	MTW 10:00	Walter Lewin
	L02	MTW 11:00	Walter Lewin
	L03	MTW 2:00	Min Chen
	L04	MTW 3:00	Min Chen

INSTRUCTIONS:

1. The FORMULA SHEET is in the back of this exam. You may tear it off. There is also an extra BLANK PAGE in case you need it.
2. This is a closed book exam. CALCULATORS, BOOKS, and NOTES are NOT ALLOWED.
3. Unless otherwise stated, to earn full credit you must show a valid DERIVATION and/or EXPLANATION of your answer, and you must express it in terms of the GIVEN VARIABLES.

Problem	**Maximum**	**Score**	**Grader**
1	40		
2	30		
3	30		
TOTAL	100		

Name ____________________

Problem 1: Work, work, work, . . . (*40 points*)

In the following problems, you may assume that gravity acts downward with an acceleration which can be approximated as 10 m/s^2. *[Note: the explanations included below are for pedagogical purposes only. Students were expected only to circle the correct answers.]*

(a) (*3 points*) Tim lifts an 8 kg bag of groceries from the floor, and holds it at a height of 1 m above the floor. How much work was done on the bag by gravity?

(i) 0 J (ii) -80 erg (iii) 80 J (iv) 8.0 kJ **[(v) -80 J]**

Explanation: The force of gravity has magnitude $Mg = 80$ N*, pointing downward. The displacement is upward, so the work is negative:* $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}} = -80$ J.

(b) (*3 points*) How much work was done by Tim?

(i) 0 J (ii) -80 erg **[(iii) 80 J]** (iv) 8.0 kJ (v) -80 J

Explanation: The correct explanation here is more complicated than the answer. If Tim lifted the bag very slowly, then the force that he would apply would be almost exactly 80 N, just opposing gravity, and we would conclude that the work that he did was 80 J. (Note that in this case the force and the displacement are in the same direction, upward.) However, Tim might not have lifted the bag slowly. He might have started with a strong yank, with much more force than 80 N, to get the bag moving upward. He could then relax his pull at the end of the motion, allowing the bag to come to rest. Even for such a complicated motion, the work-energy theorem guarantees that the total work Jim does is 80 J. The reason is that the bag starts at rest and returns to rest at the end of the motion. The change in kinetic energy is therefore zero, and consequently the total work done on the bag is zero. Tim is the only contributor to the work besides gravity, and gravity does -80 J of work, so Tim must do 80 J of work, no matter how quickly or slowly he lifts the bag.

(c) (*3 points*) Tim holds the 8 kg bag stationary for 100 seconds. How much work has he done on the bag during this time?

[(i) 0 J] (ii) 8000 J (iii) 8000 N·s (iv) 800 J (v) 800 kJ (vi) -800 J

Explanation: If there is no displacement, there is no work. Tim might get tired, but only because his muscles waste energy when they are required to hold a position. No work is done, and a table can perform the same task with no input of energy.

— *Problem 1 Continues* —

(d) (*3 points*) Tim walks 10 m, holding the bag at a constant height above the floor. How much work has he done on the bag?

[(i) 0 J] (ii) -800 erg (iii) 800 J (iv) 80 kJ (v) -800 J

Explanation: If Tim moves the bag very slowly, then the only significant force he must apply is upward, to oppose gravity, and that force is perpendicular to the displacement, so the work $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}} = 0$. *If he moves the bag quickly he may apply significant horizontal forces, but as in (b) the work-energy theorem implies that the starting and stopping does not affect the total work done.*

(e) (*3 points*) A ball of mass 100 g is moving with a velocity $\vec{\mathbf{v}} = [2, 2, 2]$ m/s (or, equivalently $\vec{\mathbf{v}} = (2\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}} + 2\hat{\boldsymbol{k}})$ m/s). What is its kinetic energy?

(i) 0 J (ii) 0.2 J **[(iii) 0.6 J]** (iv) 600 J (v) 200 J (vi) 600 erg

Explanation: $E_k = \frac{1}{2}M\left|\vec{\mathbf{v}}\right|^2 = \frac{1}{2}M\left(v_x^2 + v_y^2 + v_z^2\right) = \frac{1}{2}(0.1\text{ kg})(2^2 + 2^2 + 2^2)\text{ m}^2/s^2 = 0.6\text{ J}.$

(f) (*3 points*) A particle moves under the influence of a force $\vec{\mathbf{F}} = [1, -2, 0]$ N (or equivalently $\vec{\mathbf{F}} = (\hat{\boldsymbol{\imath}} - 2\hat{\boldsymbol{\jmath}})$ N). It moves from the origin of the coordinate system to the point $(x, y, z) = (2, 1, 3)$ m. What is the work done by the force $\vec{\mathbf{F}}$ on the particle?

[(i) 0 J] (ii) 4 J (iii) -4 J (iv) $\sqrt{70}$ J (v) $-\sqrt{70}$ J

(vi) -1 J (vii) 1 J (viii) $\sqrt{15}$ J (ix) $-\sqrt{15}$ J

Explanation: The displacement is $\vec{\mathbf{d}} = [2, 1, 3]$ m, *so the work is* $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}} = F_x d_x + F_y d_y + F_z d_z = [(1)(2) + (-2)(1) + (0)(3)]\text{ N}\cdot\text{m} = 0\text{ J}.$

(g) (*3 points*) The same particle as in part (f), still moving under the influence of the same force $\vec{\mathbf{F}} = [1, -2, 0]$ N, is now moved from the point $(x, y, z) = (2, 1, 3)$ m to the point $(x, y, z) = (3, 2, 4)$ m. How much work is done by the force $\vec{\mathbf{F}}$ during this time period?

(i) 0 J (ii) 4 J (iii) -4 J (iv) $\sqrt{70}$ J (v) $-\sqrt{70}$ J

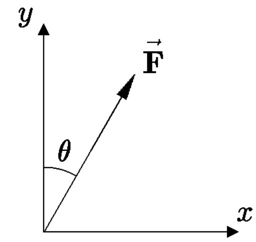
[(vi) -1 J] (vii) 1 J (viii) $\sqrt{15}$ J (ix) $-\sqrt{15}$ J

Explanation: This time the displacement is $\vec{\mathbf{d}} = ([3, 2, 4] - [2, 1, 3])\text{ m} = [1, 1, 1]\text{ m}$. *Then* $\vec{\mathbf{F}} \cdot \vec{\mathbf{d}} = [(1)(1) + (-2)(1) + (0)(1)]\text{ N}\cdot\text{m} = -1\text{ J}.$

— *Problem 1 Continues* —

Name ______________________

(h) (*3 points*) A particle moves along the x-axis a distance ℓ, while it is acted upon by a force of magnitude F, which acts in the x-y plane at an angle θ from the y-axis, as shown. What is the work done by the force on the particle?



(i) 0 (ii) $F\ell$ (iii) $F\ell\cos\theta$ **[(iv) $F\ell\sin\theta$]** (v) $F\ell\cos^2\theta$

Explanation: $W = \vec{\mathbf{F}}\cdot\vec{\mathbf{d}} = \left|\vec{\mathbf{F}}\right|\left|\vec{\mathbf{d}}\right|\cos\phi$, *where* ϕ *is the angle between* $\vec{\mathbf{F}}$ *and* $\vec{\mathbf{d}}$. *Here* $\phi = 90° - \theta$, *so* $W = F\ell\sin\theta$.

(i) (*3 points*) A particle moves along the y-axis of a coordinate system, with a force component $F_y = (2\ \mathrm{N/m^3})\,y^3$ acting on it. As the particle moves from the origin to $y = 3$ m, how much work is done on it by the force?

(i) 0 J **[(ii) 40.5 J]** (iii) -40.5 J (iv) 162 J (v) 81 J

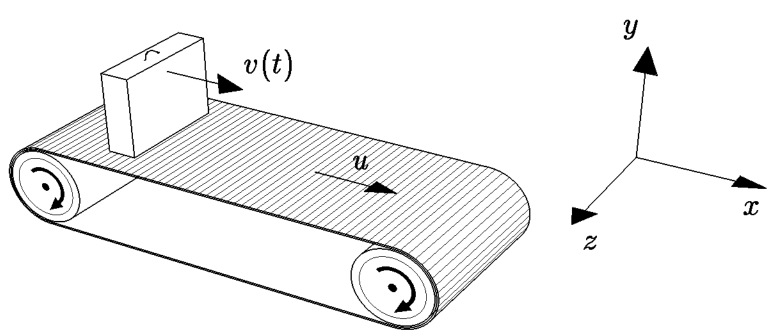
Explanation: For a varying force, we must integrate: $W = \int\vec{\mathbf{F}}\cdot d\vec{\mathbf{r}} = \int_0^{3\mathrm{m}} F_y\,dy = (2\ \mathrm{N/m^3})\int_0^{3\mathrm{m}} y^3 dy = (2\ \mathrm{N/m^3})\ \left(\frac{1}{4}y^4\right)\Big|_0^{3\mathrm{m}} = \frac{81}{2}\ \mathrm{N{\cdot}m} = 40.5\ \mathrm{J}.$

(j) (*3 points*) A cannon ball of mass m is fired with an initial velocity $\vec{\mathbf{v}}_0 = v_{0,x}\hat{\boldsymbol{\imath}} + v_{0,y}\hat{\boldsymbol{\jmath}}$, which makes an angle $\theta = \arctan(v_{0,y}/v_{0,x})$ with respect to the horizontal. What is the kinetic energy of the cannon ball when it is at the peak (i.e., highest elevation) of its trajectory?

(i) 0 J (ii) $\frac{1}{2}mv_{0,y}^2\cos^2\theta$ (iii) $\frac{1}{2}mv_{0,x}^2\cos^2\theta$ (iv) $\frac{1}{2}mv_{0,y}^2$ **[(v) $\frac{1}{2}mv_{0,x}^2$]**

Explanation: At the highest elevation v_y *vanishes, since the cannon ball has been going upward and has stopped its vertical motion as it is about to start downward.* v_x *is constant during the motion, so* $E_k = \frac{1}{2}m\left|\vec{\mathbf{v}}\right|^2 = \frac{1}{2}mv_{0,x}^2$.

A suitcase of mass M is placed on a level conveyor belt at an airport. The coefficient of static friction between the suitcase and the conveyor belt is μ_s, and the coefficient of kinetic friction is μ_k, with $\mu_k < \mu_s$. The conveyor belt moves with constant speed u, and at time $t = 0$ the suitcase is placed



on the conveyor with speed $v = 0$. At a time t_f, after moving a distance ℓ, the suitcase catches up with the conveyor belt, and starts to move at speed u with the conveyor belt. Gravity acts downward with acceleration $g > 0$. Work can depend on one's frame of reference, so be sure to answer the following three parts in the frame of reference of the airport.

(k) (*3 points*) How much work does gravity do on the suitcase, from $t = 0$ to $t = t_f$?

[(i) 0] (ii) $\mu_k Mg$ (iii) $\frac{1}{2}Mu^2$ (iv) $-\frac{1}{2}Mu^2$ (v) $Mg\ell$ (vi) $\mu_s Mg\ell$

Explanation: The force of gravity is perpendicular to the displacement, so the work done by gravity is zero.

(l) (*3 points*) How much work does friction do on the suitcase during this period?

(i) 0 (ii) $\mu_k Mg$ **[(iii) $\frac{1}{2}Mu^2$]** (iv) $-\frac{1}{2}Mu^2$ (v) $Mg\ell$ (vi) $\mu_s Mg\ell$

Explanation: Friction is the only horizontal force. Since the displacement is horizontal, friction is the only contribution to the work. The total work has to be the change in kinetic energy (work-energy theorem), $\frac{1}{2}Mu^2$, so this must be the work done by friction. Note that in this case friction does positive work, although in most circumstances friction slows things down and does negative work. Note also that the work done by friction is equal to the force times the distance, and hence $\mu_k Mg\ell$, but this was not one of the choices.

(m) (*4 points*) How much work does the force of friction from the suitcase do on the belt, during this time period?

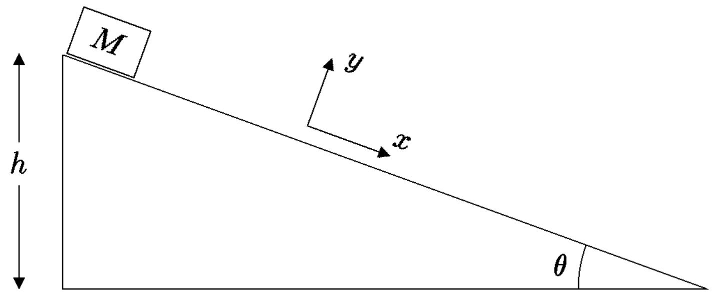
(i) 0 (ii) $\mu_k Mg\ell$ (iii) $-\mu_k Mg\ell$ (iv) $\mu_k Mgut_f$ **[(v) $-\mu_k Mgut_f$]**

Explanation: Tricky, tricky. The suitcase is slipping during this time, but the work on the belt is determined by the force acting on the belt and the displacement of the belt, as measured in the reference frame of the airport. The belt is moving at constant speed u in this frame, and the time interval is t_f. So the displacement of the belt is ut_f, in the x-direction. The force of friction that the suitcase exerts on the belt is the negative of the force of friction that the belt exerts on the suitcase (Newton's 3rd law), so it has magnitude $\mu_k Mg$, in the negative x direction. The work is then $W = F_x\, d_x = (-\mu_k Mg)(ut_f)$.

— End of Problem 1 —

Name

Problem 2: A block sliding on an inclined plane (*30 points*)



A block of mass M is placed on a plank of wood, oriented at an angle θ with respect to the horizontal, and the block then slides down until it meets the floor. The plank does not move. The initial height of the block is h, measured from the floor, and the size of the block is small compared to h. The coefficients of static and kinetic friction between the block and the plank are denoted by μ_s and μ_k, respectively. We will describe the system using the x-y coordinate system shown, with the x-axis oriented along the plane of the wood, and the y-axis oriented perpendicular to it. Assume that gravity acts downward, with acceleration $g > 0$. For the following questions, be careful that the answer you give has the correct sign.

(a) (*5 points*) As the block slides down the plank (from height h to height zero), how much work W_y is done on the block by the y-component of the gravitational force?

(b) (*5 points*) As the block slides down the plank, how much work W_x is done on the block by the x-component of the gravitational force?

(c) (*5 points*) As the block slides down the plank, how much work W_f is done on the block by friction?

(d) (*5 points*) As the block slides down the plank, how much work $W_{f,\text{plank}}$ is done by friction on the ***plank***?

(e) (*5 points*) As the block slides down the plank, how much work is done by the normal force acting on the block from the plank?

In addition to the forces of gravity and friction, there is an electrostatic force acting on the block. We are told that as the block slides down the plank, the electrostatic force does work W_{elec} on the block.

(f) (*5 points*) What is the speed of the block when it reaches the floor?

Name

(a) The block moves only in the x-direction, so the displacement in the y direction is zero. So

$$\boxed{W_y = 0\ .}$$

(b) The x-component of the gravitational force on the block is $F_x = Mg\sin\theta$. The distance traveled ℓ can be found from $h = \ell\sin\theta$, so $\ell = h/\sin\theta$. The work is then

$$W_x = F_x\,\ell = (Mg\sin\theta)\left(\frac{h}{\sin\theta}\right) = \boxed{Mgh\ .}$$

(c) The frictional force is kinetic friction, $F_x = -\mu_k N = -\mu_k Mg\cos\theta$. So

$$W_f = F_x\ell = (-\mu_k Mg\cos\theta)\left(\frac{h}{\sin\theta}\right) = \boxed{-\mu_k Mgh\cot\theta\ .}$$

(d) Since the plank does not move, the work done on it is $\boxed{W_{f,\text{plank}} = 0\ .}$

(e) The normal force acts in the y-direction, and there is no displacement in the y-direction. So the work done by the normal force is $\boxed{\text{zero.}}$

(f) The work-energy theorem guarantees that

$$\frac{1}{2}Mv_f^2 - \frac{1}{2}Mv_i^2 = Mgh - \mu_k Mgh\cot\theta + W_{\text{elec}}\ ,$$

where the initial speed v_i is zero. So the final speed is

$$\boxed{v_f = \sqrt{2gh(1-\mu_k\cot\theta) + \frac{2W_{\text{elec}}}{M}}\ .}$$

Name ______________________________

Problem 3: A proton and a uranium nucleus (*30 points*)

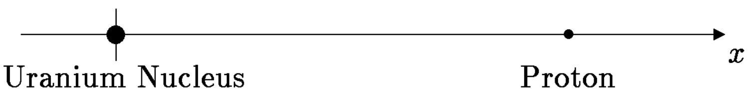
A proton with mass m is propelled at an initial speed of v_0 directly towards a uranium nucleus from a distance x_0 away. The proton is repelled by the uranium nucleus with a force of magnitude $F = \alpha/x^2$, where x is the separation between the two objects and α is a positive constant. Assume that the uranium nucleus remains at rest. Do not assume that x_0 is large, so do not assume $1/x_0$ is negligible.

(a) (*10 points*) As the proton approaches the uranium nucleus, the repulsive force slows down the proton until it comes momentarily to rest, at a distance x_{min}, after which the proton moves away from the uranium nucleus. What is this minimum separation x_{min}?

(b) (*10 points*) Consider some separation x_1 in the range $x_{\text{min}} < x_1 < x_0$. This is, x_1 represents a separation that is smaller than the initial separation between the proton and uranium nucleus, but still large enough so that it is reached during the subsequent motion of the proton. What is the speed of the proton when it is at a distance x_1 from the nucleus?

(c) (*10 points*) What is the speed of the proton when it is again a distance x_0 from the uranium nucleus?

(a) We can define an x-coordinate by the diagram



The work done by the force on the proton, as it travels from x_0 to $x_{\rm min}$, is then given by

$$W = \int_{x_0}^{x_{\rm min}} F_x \, {\rm d}x = \int_{x_0}^{x_{\rm min}} \frac{\alpha}{x^2} \, {\rm d}x = - \left. \frac{\alpha}{x} \right|_{x_0}^{x_{\rm min}} = -\alpha \left(\frac{1}{x_{\rm min}} - \frac{1}{x_0} \right) \, .$$

The work-energy theorem implies that

$$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 = W = -\alpha \left(\frac{1}{x_{\rm min}} - \frac{1}{x_0} \right) \, ,$$

where $v_f = 0$. So

$$\boxed{x_{\rm min} = \frac{\alpha}{\frac{\alpha}{x_0} + \frac{1}{2} m v_0^2} \, .}$$

(b) This is really the same situation as before, but this time $v_f \neq 0$, and the final value of x is called x_1. So the work-energy theorem becomes

$$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 = W = -\alpha \left(\frac{1}{x_1} - \frac{1}{x_0} \right) \, ,$$

which implies

$$\boxed{v_f = \sqrt{v_0^2 - \frac{2\alpha}{m} \left(\frac{1}{x_1} - \frac{1}{x_0} \right)} \, .}$$

(c) When the proton returns to x_0, the total work done is zero, since the work done on the way out is exactly the negative of the work done on the way in. Since the net work done is zero, the kinetic energy does not change, so $\boxed{v = v_0 \, .}$