MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 Spring 2005

WEEKLY QUIZ 7 SOLUTIONS

Quiz Date: Friday, March 18, 2005

FAMILY (LAST) NAME

GIVEN (FIRST) NAME

STUDENT ID NUMBER

Please check (✔) your class ☞

	L01	MTW 10:00	Walter Lewin
	L02	MTW 11:00	Walter Lewin
	L03	MTW 2:00	Min Chen
	L04	MTW 3:00	Min Chen

INSTRUCTIONS:

1. The FORMULA SHEET is in the back of this exam. You may tear it off. There is also an extra BLANK PAGE in case you need it.
2. This is a closed book exam. CALCULATORS, BOOKS, and NOTES are NOT ALLOWED.
3. Unless otherwise stated, to earn full credit you must show a valid DERIVATION and/or EXPLANATION of your answer, and you must express it in terms of the GIVEN VARIABLES.

Problem	**Maximum**	**Score**	**Grader**
1	30		
2	20		
3	25		
4	25		
TOTAL	100		

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Problem 1: Basic concepts about momentum and systems of particles (*30 points*)

The following questions are to be answered either YES or NO. Circle the correct answer. You need not explain your answer. *[For pedagogical purposes, explanations are included with these solutions.]*

(a) (*3 points*) If I stand motionless on the ground, the force that my feet exert on the ground is equal in magnitude but opposite in direction to the force that the ground exerts on my feet. Are these two forces required to be equal and opposite by Newton's third law? [Yes] or No ?

Explanation: Whenever two bodies A and B exert forces on each other, the forces are equal in magnitude and opposite in direction, by Newton's third law.

(b) (*3 points*) Two clay balls moving in empty space, with no forces acting on them, collide and stick together. Is kinetic energy conserved in the collision? Yes or [No] ?

Explanation: Whenever two bodies stick together, the collision is inelastic. There is no way that they can stick together without losing kinetic energy. The lost kinetic energy goes into heat and the deformation of the bodies.

(c) (*3 points*) For the collision described in part (b), is the momentum of the balls conserved in the collision? That is, does the total momentum of the two particles after the collision have the same value as it had before the collision? [Yes] or No ?

Explanation: Since there are no external forces acting on the system of two balls, the total momentum of the two balls must be conserved.

(d) (*2 points*) A tennis ball is thrown against the wall in Room 26-100, and it bounces back. Is the momentum of the ball conserved in the collision? That is, does the momentum of the ball after the collision equal the value it had before the collision? Yes or [No] ?

Explanation: Since the tennis ball was going towards the wall before the collision and away from the wall afterward, the component of momentum perpendicular to the wall cannot be conserved. If the collision is elastic, this component of the momentum is reversed, while the tangential component is conserved.

A baseball bat and a soccer ball are tied together by a massless and inextensible string. The combination is thrown through the air so that the baseball bat tumbles in a complicated pattern. As the bat tumbles it sometimes pulls the string taut, while at other times the string is slack. Ignore air friction, but take into account the pull of gravity, in the negative y-direction.

(e) (*3 points*) Is the x-component of the total momentum of the baseball bat and soccer ball conserved? [Yes] or No ?

Explanation: The only external force acting on the system, gravity, has a vanishing x-component, so the x-component of momentum is not changed.

(f) (*3 points*) Is the y-component of the total momentum of the baseball bat and soccer ball conserved? Yes or **No** ?

Explanation: Since gravity acts in the y-direction, the y-component of the momentum changes with time according to $\mathrm{d}\vec{\mathbf{P}}_{\mathrm{tot}}/\mathrm{d}t = \vec{\mathbf{F}}^{\mathrm{ext}}_{\mathrm{tot}}$.

(g) (*3 points*) Is the total kinetic energy plus the gravitational potential energy of the baseball bat and soccer ball conserved? **Yes** or **No** ?

Explanation: We decided to accept both answers, on the grounds that the question is ambiguous. In the absence of all dissipative forces, of which air friction is an example, mechanical energy would be conserved. If you were asked to solve a problem that was described in a manner similar to this question, you would be safe in assuming that mechanical energy is conserved. As a yes/no question, however, we decided to accept either answer. Although air friction has been ruled out, the two bodies can still lose mechanical energy by colliding inelastically with each other, or through friction involving the string. Since the question was not worded carefully enough to exclude these possibilities, we decided to give everyone credit for it.

(h) (*3 points*) Is the total momentum of the baseball bat and soccer ball equal to $M_{\mathrm{tot}}\vec{\mathbf{v}}_{\mathrm{cm}}$, where M_{tot} is the total mass of the two objects, and $\vec{\mathbf{v}}_{\mathrm{cm}}$ is the velocity of the center of mass of the two objects? **Yes** or No ?

Explanation: For ANY system of particles, $\vec{\mathbf{P}}_{\mathrm{tot}} = M_{\mathrm{tot}}\vec{\mathbf{v}}_{\mathrm{cm}}$.

(i) (*3 points*) Is the kinetic energy of the baseball bat and soccer ball equal to $\frac{1}{2}M_{\mathrm{tot}}v_{\mathrm{cm}}^2$, where M_{tot} is again the total mass of the system, and v_{cm} is the speed of the center of mass of the system? Yes or **No** ?

Explanation: For any system of particles one can write

$$K_{\mathrm{tot}} = \frac{1}{2}M_{\mathrm{tot}}v_{\mathrm{cm}}^2 + \sum_i \frac{1}{2}m_i\left(\vec{\mathbf{v}}_i - \vec{\mathbf{v}}_{\mathrm{cm}}\right)^2 \ ,$$

as shown on the formula sheet for this quiz. One can think of this as the division of kinetic energy between the motion of the center of mass and the motion about the center of mass. The second term is not zero, however, and could be equal to zero only if the baseball bat and soccer ball were both nonrotating and had a fixed relative position. The fact that the bat is tumbling, and the fact that the string is sometimes taut and sometimes slack, imply that in this case the second term is not zero.

(j) (*3 points*) Does the center of mass of the baseball bat / soccer ball system travel on a parabolic trajectory, as a single particle would? **[Yes]** or No ?

Explanation: The center of mass of any system moves as if it were a point particle, where the mass is the total mass of the system, and the force acting on the particle is the total force acting on the system. For a point particle with a constant force in the y-direction, the motion is a parabolic trajectory.

Problem 2: Force and potential energy in one dimension (reprise) (*20 points*)

A particle of mass m is constrained to move in one dimension, described by a coordinate y. The force on the particle depends on its position, and is given by

$$F_y = \begin{cases} 0 & \text{if } y \geq 0 \\ \frac{1}{2}ky^2 & \text{if } -a < y < 0 \\ \frac{1}{2}ka^2 & \text{if } y \leq -a \ , \end{cases}$$

where k and a are positive constants.

(a) (*5 points*) Is this force conservative? Give a brief explanation of your answer.

(b) (*5 points*) Choose the zero of potential energy $U(y)$ so that $U(0) = U_0$, where U_0 is a constant. What is $U(y)$ for $y > 0$?

(c) (*5 points*) What is $U(y)$ in the range $-a < y < 0$?

(d) (*5 points*) What is $U(y)$ in the range $y \leq -a$?

(a) $\boxed{\text{Yes.}}$ Since F_y depends only on y, and not on time or on the velocity, the work done to get from y_1 to y_2 will depend only on y_1 and y_2, and not on any other property of the trajectory.

(b) From the formula sheet,

$$U(x_p) = U_0 - \int_{x_0}^{x_p} F \, \mathrm{d}x \ ,$$

so we can replace x by y, and take $y_p > 0$ and $y_0 = 0$, with $U_0 = U(y_0) = U_0$. Then

$$U(y_p) = U_0 - \int_0^{y_p} 0 \, \mathrm{d}y = \boxed{U_0 \ .}$$

(c) Applying the same reasoning for $-a < y_p < 0$, everything is the same except that F_y in the region of integration will be given by $\frac{1}{2}ky^2$. So

$$U(y_p) = U_0 - \frac{1}{2}k \int_0^{y_p} y^2 \, \mathrm{d}y = \boxed{U_0 - \frac{1}{6}\, k y_p^3 \ .}$$

(d) The easiest thing is to use $y = -a$ as y_0. From (c), we know that $U(-a) = U_0 + \frac{1}{6}ka^3$. It is important to realize that the answer in part (c) for $U(y_p)$ holds for $y_p = -a$, and not just for $-a < y_p < 0$. Because the potential energy function is defined as an integral, it is always a continuous function. It follows that its value at $y = -a$ is necessarily equal to the limiting value as $y \to -a$ from above.

So

$$U(y_p) = U_0 + \frac{1}{6}ka^3 - \int_{-a}^{y_p} F_y \, \mathrm{d}y = U_0 + \frac{1}{6}ka^3 - \frac{1}{2}ka^2 \int_{-a}^{y_p} \mathrm{d}y$$

$$= \boxed{U_0 + \frac{1}{6}ka^3 - \frac{1}{2}ka^2(y_p + a)}$$

$$= \boxed{U_0 - \frac{1}{3}ka^3 - \frac{1}{2}ka^2 y_p \ .}$$

Some of you may wonder what happens if one tries to apply the general formula from the formula sheet directly, using $y_0 = 0$ as we did in parts (b) and (c). We will get the same answer as above, as long as we remember that to integrate a function that changes form depending on y, as $F_y(y)$ does, one must break up the range of integration into different segments. For $y_p < -a$,

$$\begin{aligned} U(y_p) &= U_0 - \int_0^{y_p} F_y(y)\,\mathrm{d}y \\ &= U_0 - \frac{1}{2}k\int_0^{-a} y^2\,\mathrm{d}y - \frac{1}{2}ka^2\int_{-a}^{y_p} \mathrm{d}y \\ &= U_0 + \frac{1}{6}ka^3 - \frac{1}{2}ka^2(y_p + a)\ , \end{aligned}$$

as expected.

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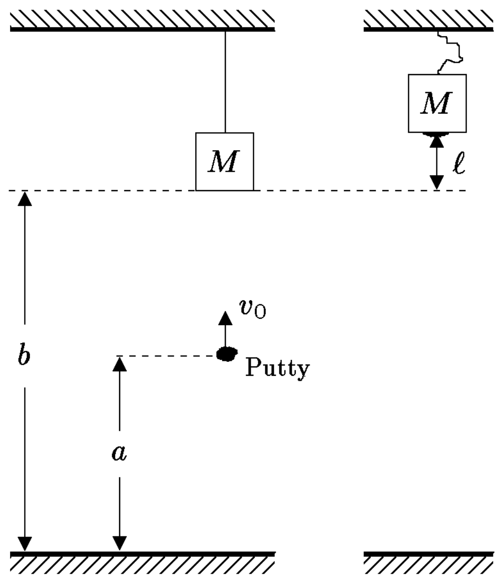
Problem 3: The vertical ballistic pendulum (*25 points*):

A block of mass M hangs from the ceiling on a string, at a height b above the floor. Tommy stands directly below the block, and throws a blob of putty directly upward at it. The putty has mass m, and starts at height a above the floor with an initial upward speed v_0.



(a) (*8 points*) What is the upward speed v_1 of the putty when it reaches height b, but just before it hits the block?

(b) (*9 points*) The putty sticks to the block, so after the collision the putty and block move upward together. (You may assume that the deformation of the putty takes negligible time.) What is the upward speed v_2 of the putty-block system immediately after the two stick together? Your answer may depend on the variable v_1, the answer to part (a), whether or not you solved part (a).

(c) (*8 points*) Measured from the initial position of the block, by what distance ℓ does the putty-block system move upward before gravity pulls it back down? Your answer may depend on v_1, v_2, or any of the given variables, whether or not v_1 and v_2 have been found.

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This problem is an exercise in multiple conservation laws, essentially identical to the ballistic pendulum problem.

(a) As the putty rises from height a to height b, mechanical energy is conserved. Since v_1 is the the speed of the putty when it reaches height b,

$$\frac{1}{2}mv_0^2 + mga = \frac{1}{2}mv_1^2 + mgb \ ,$$

so

$$\boxed{v_1 = \sqrt{v_0^2 - 2g(b-a)} \ .}$$

(b) Since the putty and block stick together, the collision is inelastic. Momentum, however, is conserved. Even though there is a force acting (i.e., gravity), the negligible time interval implies that the momentum imparted by gravity during this time interval, $\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}\,\Delta t$, is negligible. Since v_2 be the speed of the putty-block system just after the collision, conservation of momentum implies

$$mv_1 = (M+m)v_2 \quad \Longrightarrow \quad v_2 = \boxed{\frac{m}{M+m}v_1} = \boxed{\frac{m}{M+m}\sqrt{v_0^2 - 2g(b-a)} \ .}$$

(c) As the putty-block system coasts upward under the influence of only gravity, mechanical energy is conserved. At the top, all the energy is potential energy, so

$$\frac{1}{2}(M+m)v_2^2 + (M+m)gb = (M+m)g(b+\ell) \ .$$

$$\therefore \quad \ell = \boxed{\frac{v_2^2}{2g}} = \boxed{\left(\frac{m}{M+m}\right)^2 \frac{v_0^2 - 2g(b-a)}{2g}} \ .$$

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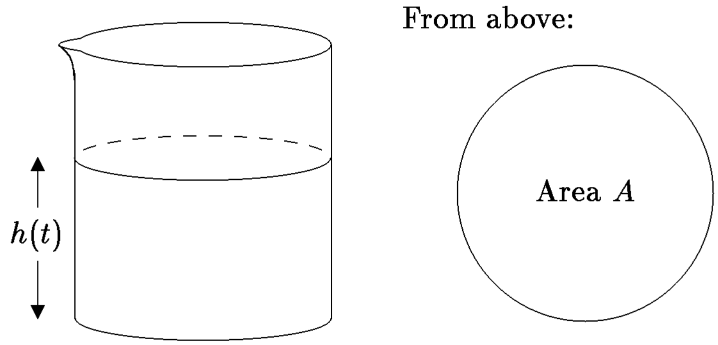
Problem 4: A beaker in the rain (*25 points*)

An initially empty beaker, in the shape of a cylinder with cross sectional area A, is left out in the rain. The raindrops hit the beaker vertically downward with speed v. The rain continues at a constant rate, so the height of the water in the beaker $h(t)$ increases with time t at a rate $\mathrm{d}h/\mathrm{d}t = w$, where w is negligible compared to v. The raindrops quickly come to rest inside the beaker, so we can neglect any kinetic energy of the water that has collected in the beaker. Let ρ denote the density of water (i.e., the mass per unit volume).



(a) (*5 points*) What is the rate at which the mass of the water in the beaker increases with time?

(b) (*5 points*) Let $y_{\mathrm{cm}}(t)$ denote the height of the center of mass of all the water that has collected in the beaker by time t. What is the rate $\mathrm{d}y_{\mathrm{cm}}/\mathrm{d}t$ at which this height increases?

(c) (*5 points*) The total momentum of any system of particles $\vec{\mathbf{P}}_{\mathrm{tot}}$ is equal to the total mass M_{tot} times the velocity $\vec{\mathbf{v}}_{\mathrm{cm}}$ of the center of mass. Should we conclude, therefore, that the water in the beaker has a vertical momentum equal to its mass times the value of $\mathrm{d}y_{\mathrm{cm}}/\mathrm{d}t$ as described in part (b)? Explain your answer in one or two sentences.

(d) (*10 points*) If the beaker is placed on a scale, while the beaker is still in the rain, the impact of the raindrops on the beaker will cause the reading on the scale to be larger than the weight of the beaker and the water it contains. By how much is the reading on the scale increased by the impact of the raindrops? (Neglect the effect of raindrops that hit the scale directly.)

(a) The mass in the beaker is given by the product of the density ρ and the volume $V = Ah$ that is filled, i.e.

$$m = \rho A h \ .$$

When the height changes at a rate $\mathrm{d}h/\mathrm{d}t = w$, the mass changes at the rate

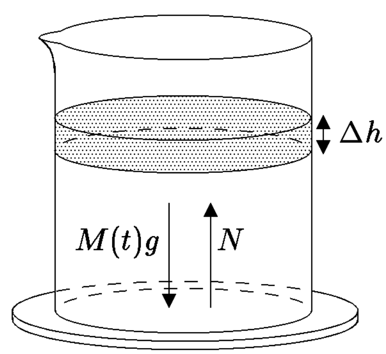
$$\frac{\mathrm{d}m}{\mathrm{d}t} = \rho A \frac{\mathrm{d}h}{\mathrm{d}t} = \boxed{\rho A w \ .}$$

(b) The height of the center of mass of the water in the beaker is at $y_{\mathrm{cm}} = h/2$ (at least if we assume an incompressible fluid, i.e. a constant density, which is a very good approximation for water). The rate at which the height of the center of mass increases is

$$\boxed{\frac{\mathrm{d}y_{\mathrm{cm}}}{\mathrm{d}t} = \frac{1}{2}\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{w}{2} \ .}$$

(c) **No, the water has no vertical momentum.** The center of mass is moving upward because more water is added to the beaker; i.e. as time evolves we are talking about a whole sequence of physical systems, not about the center of mass of a fixed given system. At any given time, the water in the beaker is at rest, and hence its momentum vanishes.

(d) The raindrops enter the beaker with a speed v and then quickly come to rest inside the beaker. Consider a short time interval Δt, and consider the system that consists of the beaker, all the water that has landed in the beaker by the beginning of the time interval, plus the water that will come to rest inside the beaker during the time interval. During the time interval Δt the height of the water in the beaker will increase by $\Delta h = w\,\Delta t$, so the volume of water that will come to rest during the time interval is $\Delta V = A\,\Delta h = Aw\,\Delta t$, shown shaded in the diagram on the right. The mass of this water is $\Delta M = \rho \Delta V = \rho A w\,\Delta t$. Taking the vertical direction as the y-direction, the momentum of this water at the beginning of the interval is $\vec{\mathbf{P}}_i = [0,\, -v\Delta M,\, 0] = [0,\, -\rho A w v\,\Delta t,\, 0]$. The final momentum is zero, after the raindrops come to rest, so $\Delta\vec{\mathbf{P}} \equiv \vec{\mathbf{P}}_f - \vec{\mathbf{P}}_i = -\vec{\mathbf{P}}_i$. The total external force applied to the system is equal to the rate of change of its momentum, so



$$\vec{\mathbf{F}}^{\mathrm{ext}}_{\mathrm{tot}} = \frac{\Delta\vec{\mathbf{P}}}{\Delta t} = \frac{-\vec{\mathbf{P}}_i}{\Delta t} = [0,\, \rho A w v,\, 0] \ .$$

From the diagram one can see that $F^{\text{ext}}_{\text{tot},y} = N - M(t)g$, where $M(t)$ is the total mass of the beaker and the water in it at time t. So,

$$N = M(t)g + \rho A w v \ .$$

By Newton's third law the beaker exerts a force of equal magnitude on the scale, which determines its reading. The first term is just the weight of the system, while the question asks by how much the reading is increased by the impact of the raindrops. Thus the second term in the above expression is the answer to the question:

$$\boxed{\Delta N = \rho A w v \ .}$$

Since scales often read in units of mass rather than force, it would be equally correct to say that the mass reading would be increased by

$$\boxed{\Delta M = \frac{\rho A w v}{g} \ .}$$