MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 Spring 2005

WEEKLY QUIZ 9 SOLUTIONS

Quiz Date: Friday, April 8, 2005

FAMILY (LAST) NAME

GIVEN (FIRST) NAME

STUDENT ID NUMBER

Please check (✔) your class ☞

	L01	MTW 10:00	Walter Lewin
	L02	MTW 11:00	Walter Lewin
	L03	MTW 2:00	Min Chen
	L04	MTW 3:00	Min Chen

INSTRUCTIONS:

1. The FORMULA SHEET is in the back of this exam. You may tear it off. There is also an extra BLANK PAGE in case you need it.
2. This is a closed book exam. CALCULATORS, BOOKS, and NOTES are NOT ALLOWED.
3. Unless otherwise stated, to earn full credit you must show a valid DERIVATION and/or EXPLANATION of your answer, and you must express it in terms of the GIVEN VARIABLES.

Problem	**Maximum**	**Score**	**Grader**
1	30		
2	40		
3	30		
TOTAL	100		

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Problem 1: Basic concepts about the rotation of rigid bodies (*30 points*)

Mark your answer by circling it. For these short answer problems you need not show your work, and there will be no partial credit. *Warning: some problems may contain irrelevant information.*

(a) (*5 points*) A uniform solid cylinder and a uniform solid sphere have the same mass and radius, respectively M, and R. They roll without slipping down identical inclines. Which one reaches the bottom first? Briefly explain your answer.

The $\boxed{\textit{sphere}}$ *reaches the bottom first. Its smaller moment of inertia* ($\frac{2}{5}MR^2$ *instead of* $\frac{1}{2}MR^2$)*, associated with the fact that its mass is more closely concentrated near its center, implies that for a given speed of motion* v*, the kinetic energy of rotation* $\frac{1}{2}I\omega^2 = \frac{1}{2}Iv^2/R^2$ *is smaller.*

(b) (*5 points*) If the cylinder and the sphere start with the same angular velocity and they roll without slipping up identical slopes, which one reaches the higher height? Briefly explain your answer.

The $\boxed{\textit{cylinder}}$ *will reach the higher height. Its larger moment of inertia means that it has more rotational kinetic energy, all of which will be converted to potential energy when it reaches its maximum height.*

(c) (*5 points*) When is the angular momentum of a system constant? Choose one.

(i) When the total kinetic energy is constant.

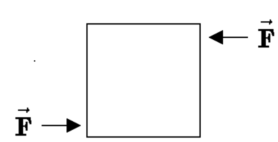
(ii) When no net external force acts on the system.

(iii) When the linear momentum and the energy are constant.

$\boxed{\text{(iv)}}$ When no external torque acts on the system.

(v) When the moment of inertia is constant.

The only other real candidate for an answer is (ii). However, the inclusion of the adjective "net," a synonym for "total," makes this false. If NO external force acted on the system, then there could be no external torque, and the angular momentum of the system would be conserved. However, if we know only that no NET external force acts, then there is the possibility, for example, that two equal but opposite external forces may act on the system, at different points. If the forces are not directed along the line that joins the two points, then they will create a net torque, even though the forces add to zero.

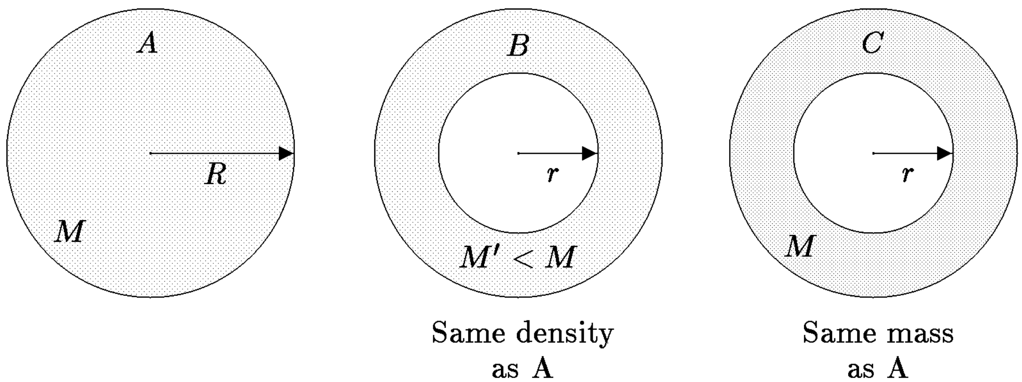


An example with zero net force but nonzero torque

(d) (*5 points*) A uniform cylinder of radius R and mass M rolls without slipping on a horizontal surface with a linear speed v. What is its kinetic energy?

(i) $\frac{1}{4}Mv^2$ (ii) $\frac{1}{2}Mv^2$ (iii) $\frac{1}{2}Mv^2 + MR^2v$ **(iv) $\frac{3}{4}Mv^2$** (v) $\frac{2}{5}Mv^2$ (vi) Mv^2

The moment of inertia is $I = \frac{1}{2}MR^2$, *so* $E_k = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2 + \frac{1}{2}I(v/R)^2 = \frac{3}{4}Mv^2$.



(e) (*5 points*) Consider three spheres, labeled A, B, and C, all with the same radius R. Sphere A is a uniform, solid sphere of mass M. Sphere B is made of the same material, and hence has the same density, but it has a hollow cavity of radius r, where $r < R$. Its mass M' is therefore less than M. Sphere C also has a hollow cavity of radius r, but it is made of a denser material so that its mass is M, the same as sphere A. Denote the moments of inertia of the three spheres as I_A, I_B, and I_C. List these moments of inertia from largest to smallest, noting if any or all of them are equal.

I_C, I_A, I_B . *Clearly* $I_A > I_B$, *since sphere A can be thought of as sphere B, but with extra matter included in the inner sphere (radius* $< r$*). This extra matter can only increase the moment of inertia.* $I_C > I_A$, *since both spheres have the same mass, but in sphere C the mass is pushed on average to a larger radius.*

(f) (*5 points*) Suppose that the three spheres in part (e) are all allowed to roll down the same incline, each starting from rest. Assume that they roll without slipping, but that all dissipative forces, such as air friction or rolling friction, are negligible. List the three spheres (A, B, and C) in the order in which they reach the bottom, again noting if any or all of them are equal.

A reaches the bottom first, followed by a tie between B and C. *B and C differ only by their density, which will have no effect on how fast they roll. The moment of* *inertia for C will be larger than that for B by the same ratio as the masses, M/M', so the energy conservation equation,*

$$E_{\text{mechanical}} = Mgy + \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = const\ ,$$

will be equivalent for the two cases— in the equation for C, every term will be larger than the corresponding term in the B-equation by the factor M/M'. To compare A with C, note that they have the same mass and radius, but that C has a larger moment of inertia. Therefore C will require more rotational energy for the same translational velocity, so there will be less energy available for translation, and it will move slower.

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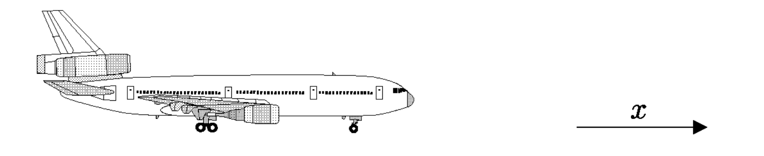
Problem 2: Spinning wheels on an aircraft as it lands (*40 points*)

An aircraft lands at a speed v_0. Before it touches down, its wheels are not rotating. After touching down, the wheels skid for a period of time, and afterwards roll without slipping. Assume that each wheel has radius R and moment of inertia I and supports a weight Mg (which includes the weight of the wheel), and that the pilot does not apply reverse thrust until the aircraft is no longer skidding. Assume further that the coefficient of friction between the wheels and the runway is μ_s for static friction and μ_k for kinetic friction. For purposes of describing directions, assume that we are standing alongside the runway, and that the plane is moving to the right, in the positive x-direction. For each part, your answer may include symbols that represent the answers to previous parts, whether or not you have correctly answered those previous parts.



(a) (*6 points*) Immediately after the plane touches the ground, what is the magnitude F and direction of the force of friction acting on each wheel?

(b) (*6 points*) At the same time as in (a), what is the torque τ, about the center of the wheel, due to friction acting on each wheel? Is this torque clockwise or counterclockwise as seen from our vantage point alongside the runway?

(c) (*6 points*) While the wheels are skidding, what is the velocity $v_x(t)$ of the plane? Take $t = 0$ as the instant when the wheels make contact with the ground.

(d) (*6 points*) During the same period as in (c), what is the angular velocity $\omega(t)$ of the wheels?

(e) (*6 points*) What is the speed v_f of the plane when it stops skidding?

(f) (*5 points*) Now suppose that as soon as the wheels touch the ground, the pilot applies a reverse thrust from the engines, resulting in a net force of magnitude F_d ($F_d > 0$) to the left. We will assume that F_d can be treated as a constant over the relevant time period, and we will assume that the engines are positioned so that the thrust exerts no torque about the center of mass of the plane. In this case, what would be the velocity $v'_x(t)$ of the plane during the period when the wheels are skidding. As in part (c), take $t = 0$ as the instant when the wheels make contact with the ground. Take M_{tot} to be the total mass of the aircraft, remembering that M denotes the mass per wheel.

(g) (*5 points*) During the same period as in (f), what is the angular velocity $\omega'(t)$ of the wheels?

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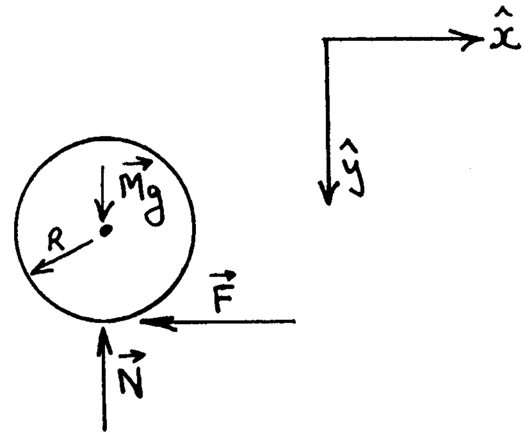
The force diagram for one wheel and the coordinate system we will use is shown on the right. Clockwise rotation of the wheel is in the $+\hat{z}$-direction, i.e into the plane of the paper.



while the wheel is skidding

Translational motion is given by: $[-F, Mg-N, 0] = M[a_x, 0, 0]$ ...(1)

Rotational motion about the axle ($\hat{z}$-axis): $\tau = I\alpha$; & $\tau = RF$ (clockwise) ...(2)

a) From (1) $Mg = N$ $\therefore$ $F = \mu_k Mg$ (since $F = \mu_k N$)

Friction opposes the direction of skidding, therefore F is in the $-\hat{x}$- direction, or $\vec{F} = [-\mu_k Mg, 0, 0]$

b) τ = RF clockwise

c) From (1) $a_x = -\frac{F}{M} = -\mu_k g$

This corresponds to one dimensional motion with constant acceleration $\therefore$ $v_x(t) = v_0 - \mu_k g t$ ....(3)

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d) Similarly from (2) $\omega(t) = \alpha t = \frac{RF}{I} t = \frac{R\mu_k M g}{I} t$ . . . . (4)

e) Skidding stops when rolling begins.

The condition for rolling is $v = R\omega$

i.e the wheels will start to roll at time t_f

given by $v(t_f) = R\omega(t_f)$ (or $v_f = R\omega_f$)

From (3) & (4) this condition corresponds to

$v_0 - \mu_k g t_f = \frac{R^2 \mu_k M g t_f}{I}$ or $t_f = \frac{v_0 I}{R^2 \mu_k M g + \mu_k g I}$

Using (3) again we find

$$v_f = v_0 - \frac{\mu_k g v_0 I}{R^2 \mu_k M g + \mu_k g I} = \frac{v_0 M R^2}{M R^2 + I}$$

(f) The extra drag force F_d adds another negative term to the equation for a_x, which becomes

$$a_x = -\mu_k g - \frac{F_d}{M_{\text{tot}}} ,$$

where M_{tot} is the total mass of the aircraft, equal to M times the number of wheels. No credit will be taken off for students who just use M, since the problem was not worded well, as it never defined M_{tot} or specified the number of wheels. Thus

$$\boxed{v'_x(t) = v_0 - \left[\mu_k g + \frac{F_d}{M_{\text{tot}}}\right] t .}$$

(g) The extra force does not change the torque on the wheels while they are skidding, since the normal force and hence the force of static friction will not be affected. So

$$\boxed{\omega'(t) = \frac{R\mu_k M g t}{I} .}$$

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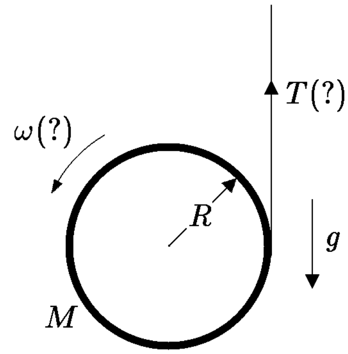
Problem 3: A cylinder on a string (*30 points*)

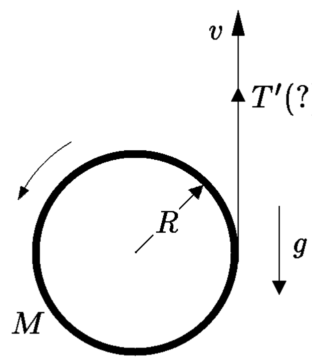
A hollow cylinder (i.e., a cylinder for which all the mass is concentrated in a wall of negligible thickness) of radius R and mass M is wrapped with an inextensible string of negligible mass. One end of the string is tied to the ceiling, and the cylinder is allowed to fall with its axis horizontal, as the string unrolls. Take the acceleration of gravity as g, downward, with $g > 0$.



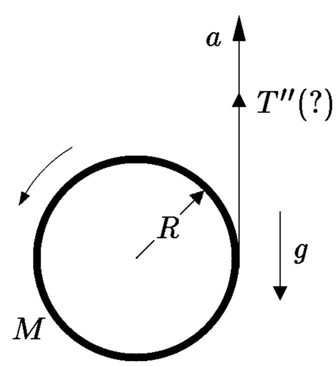
(a) (*10 points*) Find the angular velocity ω of the cylinder after it falls a distance ℓ, starting from rest with the string taut.

(b) (*10 points*) Find the tension T in the string, as the cylinder is falling.

(c) (*5 points*) Now suppose that instead of the string being tied to the ceiling, it is being held by a person who pulls the end upward with a constant speed v. What is the tension T' in the string in this case?



(d) (*5 points*) Now suppose that instead of pulling the string upward with a constant speed, the person pulls the end of the string upward with a constant acceleration a. What is the tension T'' of the string in this case?



(a) This can be solved most easily by using conservation of energy. If we define the gravitational potential energy to be zero at the bottom of the fall, then the initial energy is all potential energy,

$$E_{\text{initial}} = U(\ell) = Mg\ell \ .$$

After falling a distance ℓ the cylinder will be moving at speed v and angular velocity ω, so its energy will be

$$F_{\text{final}} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 \ .$$

The string is being unwound from a circle of radius R, so v will be related to ω by $v = R\omega$. The moment of inertia for the cylindrical shell is given in the table as $I = MR^2$ So, setting the initial and final energies equal,

$$Mg\ell = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}(MR^2)\omega^2 = MR^2\omega^2 \ ,$$

so

$$\boxed{\omega = \frac{\sqrt{g\ell}}{R} \ .}$$

(b) METHOD 1: The tension can be found from the previous answer, adding a little information. The final vertical velocity v_y, after falling the distance ℓ, is given by $v_y = -R\omega = -\sqrt{g\ell}$. Here I am defining v_y as being positive when upward, so its value for this problem is negative. For uniform acceleration we know that $v^2 = 2a\ell$, so the cylinder is undergoing uniform acceleration with $a_y = -g/2$. The $\vec{\mathbf{F}} = M\vec{\mathbf{a}}$ equation for motion in the vertical (y) direction is $F_y = T - Mg = Ma_y$, so $a_y = -g/2$ implies that

$$\boxed{T = \frac{1}{2}Mg \ .}$$

METHOD 2: Alternatively, we can use the torque and force equations to find the tension directly. Once again, the $\vec{\mathbf{F}} = M\vec{\mathbf{a}}$ equation for vertical motion is

$$F_y = T - Mg = Ma_y \ .$$

The torque is given by $\boldsymbol{\tau} = TR$, and it is positive (counterclockwise). So

$$\boldsymbol{\tau} = TR = I\alpha \ .$$

The rolling constraint for these sign conventions is $v_y = -R\omega$, since a positive (counterclockwise) rotation corresponds to motion of the cylinder downward (negative v_y).

Differentiation of the rolling constraint implies that $a_y = -R\alpha$, so the torque equation can be rewritten as

$$TR = -Ia_y/R = -MRa_y \ ,$$

where I used $I = MR^2$. The torque equation then becomes $T = -Ma_y$, which can be substituted into the first equation to give

$$T - Mg = -T \ ,$$

leading immediately to $\boxed{T = \frac{1}{2}Mg \ .}$

(c) The answer is the same as part (b), $\boxed{T' = \frac{1}{2}Mg \ .}$ One way to see this is to work the problem in the frame of reference that is moving upwards at speed v relative to the ground, and then the problem becomes identical to the previous one. If the problem is attacked in the frame of reference of the ground, then one can use Method 2 above, and the only change is in the rolling constraint. Now if ω were zero the cylinder would be moving upward with speed v, so the rolling constraint becomes

$$v_y = -R\omega + v \ .$$

Since v is a constant, differentiation gives $a_y = -R\alpha$ as before, so the derivation goes through without any change.

(d) This time the rolling constraint can be written as

$$v_y = -R\omega + v(t) \ ,$$

where $v(t)$ is the instantaneous velocity of the hand that is pulling the string upwards. Now when we differentiate with respect to time,

$$a_y = -R\alpha + a \ ,$$

where $\mathrm{d}v/\mathrm{d}t = a$, where a is the upward acceleration of the person's hand, as specified in the problem. We still have

$$T''R = I\alpha \ ,$$

so

$$T''R = -I(a_y - a)/R = -MR(a_y - a) \ .$$

Then $Ma_y = Ma - T''$, which can be inserted into the $\vec{\mathbf{F}} = M\vec{\mathbf{a}}$ equation to give

$$T'' - Mg = Ma - T'' \ ,$$

so

$$\boxed{T'' = \frac{1}{2}M(g + a) \ .}$$