MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 Spring 2005

PROBLEM SET 10

Sunday, April 10, 2005

Due Date: Thursday, April 14, 2005, 3:00 p.m.

Reading Assignment: Young and Freedman, Sections 10.7, 11.1–11.3; Busza, Cartwright, and Guth: review pp. 312–320 of Chapter 9.

Topics for the week: Rotation of Rigid Bodies in Three Dimensions. This will be our third and final week studying rotations. The problem set and quiz will review the material from the previous weeks, and introduce new material about gyroscopes, rotational equilibrium, and the center of gravity. This week you will not only be expected to understand the vector definitions of angular velocity, torque, and angular momentum, but you will be expected to apply them to fully three-dimensional situations, such as the gyroscope.

Instructions:

If a problem is marked ***DO***, you should write a solution to hand in to be graded. The graders will read your answers to one or two questions on each problem set, and they will check whether the other problems have at least been handed in.

The quiz on this material, to be given at 10:05 am on Friday, April 15, will include at least one problem that is at most a slight modification of one of the problems (***DO*** or ***STUDY***) on this problem set. In addition, to encourage you to review Quizzes 8 and 9, at least 20 points of Quiz 10 will be a revised version of one or more questions from these two quizzes. By "revised version," I mean that the problem will be sufficiently similar so that if you really understand the solution to the earlier problem, the solution to the Quiz 10 problem will be straightforward.

Your written solutions are due by 3:00 pm in room 4-339B on Thursday, April 14. Please indicate the number, instructor, and time of your recitation section, and be sure to submit your paper to the correct bin. Solutions will be made available on the 8.01 website shortly afterward, so that you will be able to use them in studying for the quiz.

Rotation About a Fixed Axis

1) ***DO:*** Quiz 8, Problem 3, slightly revised: A pivoted disk with attached weights. The text appears at the end of this listing. You may look at the posted solution, but note that the location of the marble and the disk of radius $R/2$ has been changed. The radii of both disks, however, are unchanged.

Rotation about a Moving Axis

2) ***DO:***		Quiz 9, Problem 3, slightly revised: A cylinder on a string. The text appears at the end of this listing. Again you may look at the posted solution, but note that the hollow cylinder has been replaced by a solid cylinder.
3) ***DO:***		Striking a billiard ball with a cue (described at the end of this listing)

Rotation, Angular Momentum, and Torque in Three Dimensions

4) ***STUDY:***	SG:9C.2	(S) Verifying that torque is the rate of change of angular momentum
5) ***STUDY:***	SG:9D.1	(S) A skewed rod on a turntable
6) ***DO:***	SG:9D.9	Rotating meteoroid

Gyroscopes

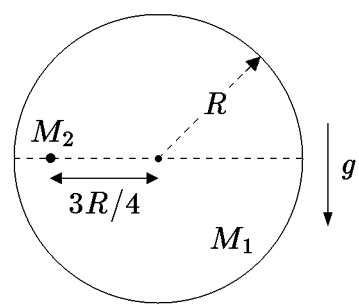
7) ***STUDY:***	SG:9D.4	Motion of a gyroscope
8) ***DO:***	Y&F:10.48	Precession of a gyroscope and the force on its pivot

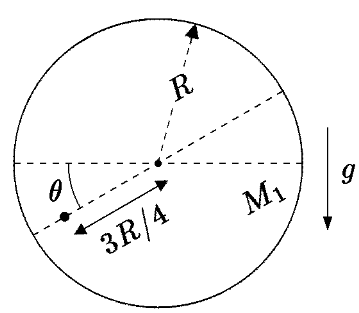
Rotational Equilibrium

9) ***DO:***	Y&F:11.12	When will the beam tip?
10) ***DO:***	Y&F:11.46	Balancing pieces of steel on the edge of a table
11) ***DO:***	SG:9A.6	(H) Torques on a non-uniform rod
12) ***DO:***	SG:9A.7	(H) Will the ladder slip?

Problem 1: Revised version of Quiz 8, Problem 3: A pivoted disk with attached weights.

A uniform disk of mass M_1 and radius R is pivoted on a frictionless horizontal axle through its center. A small marble of mass M_2 is attached to the disk at radius $3R/4$, at the same height as the axle. Assume that the marble is small enough to be treated as a point mass. The acceleration of gravity is downward, with magnitude g.



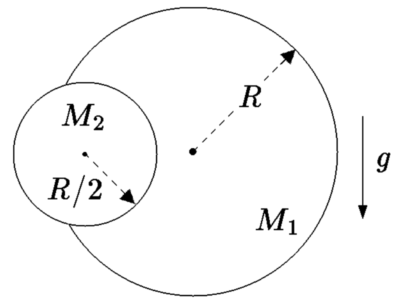


(a) If this system is released from rest to rotate about the pivot, what will be the angular acceleration α_0 of the disk immediately after it is released?

(b) After the disk has rotated through an angle θ, what will be the angular acceleration α_1?

(c) What will be the maximum angular velocity $\omega_{\max}$ that the disk will reach in its subsequent motion?

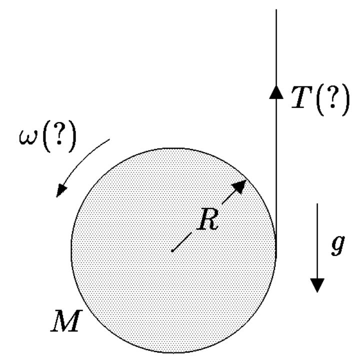
(d) Now consider the situation if the marble is replaced by a disk of radius $R/2$, with the same mass M_2, located with its center at the same place where the marble was located in part (a). (When calculating the torque on this disk, you can use the fact that the torque caused by gravity can be calculated as if the force of gravity were a single force acting at the center of mass of the object.) For this case, find the angular acceleration α_0' immediately after the system is released from rest.



(e) For the case described in part (d), what will be the maximum angular velocity ω_{max}' that the disk will reach in its subsequent motion?

Problem 2: Revised version of Quiz 9, Problem 3: A cylinder on a string.

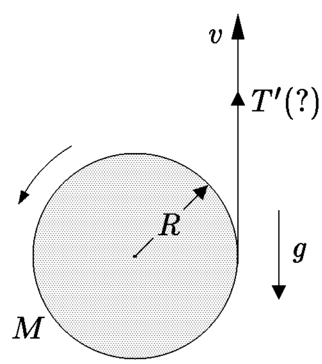
A solid cylinder of radius R and mass M is wrapped with an inextensible string of negligible mass. One end of the string is tied to the ceiling, and the cylinder is allowed to fall with its axis horizontal, as the string unrolls. Take the acceleration of gravity as g, downward, with $g > 0$.



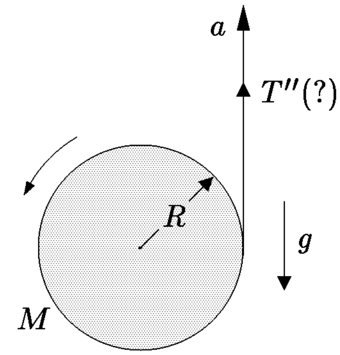
(a) Find the angular velocity ω of the cylinder after it falls a distance ℓ, starting from rest with the string taut.

(b) Find the tension T in the string, as the cylinder is falling.

(c) Now suppose that instead of the string being tied to the ceiling, it is being held by a person who pulls the end upward with a constant speed v. What is the tension T' in the string in this case?



(d) Now suppose that instead of pulling the string upward with a constant speed, the person pulls the end of the string upward with a constant acceleration a. What is the tension T'' of the string in this case?



Problem 3: Striking a billiard ball with a cue

A billiard ball, which we treat as a solid ball of mass M and radius R, is at rest on a frictionless table. The ball is hit sharply by a cue, which imparts a horizontal impulse of momentum at a height b above the center of the ball. The strike is centered, which means that the impulse of momentum lies in the vertical plane that includes the center of the sphere and the point of impact.

Even though the surface is frictionless, if the ball is hit at just the right height, it will roll without slipping immediately after it is hit. For what value of b will this happen?