MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 Spring 2005

PROBLEM SET 12 (THE LAST!)

Saturday, April 30, 2005

Due Date: Thursday, May 5, 2005, 3:00 p.m.

Reading Assignment: Young and Freedman, Sections 37.1 – 37.5. Chapter 37 is available as a separate booklet. If you bought your book at the Coop either this term or last, it should have been included. From what we have been told, the Coop does not have copies of the chapter now. If you cannot obtain a copy of this chapter, please send email to Alan Guth (guth@ctp.mit.edu), and we'll try to figure out what to do.

Topics for the week: Special Relativity. The postulates of relativity, relativity of simultaneity, time dilation, length contraction, and the Lorentz transformation.

Instructions:

If a problem is marked ***DO***, you should write a solution to hand in to be graded. (This time all the problems are marked ***DO***.) The graders will read your answers to one or two questions on each problem set, and they will check whether the other problems have at least been handed in.

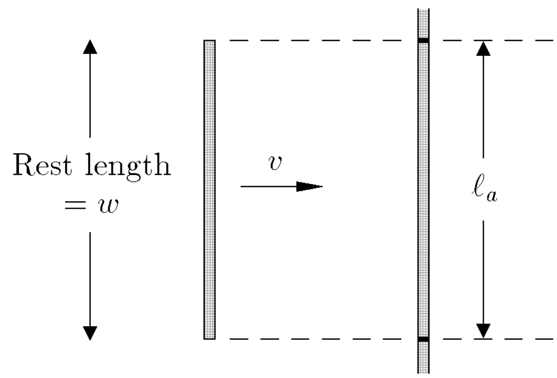
The quiz on this material, to be given at 10:05 am on Friday, May 6, will include at least one problem that is at most a slight modification of one of the problems on this problem set.

Your written solutions are due by 3:00 pm in room 4-339B on Thursday, May 5. Please indicate the number, instructor, and time of your recitation section, and be sure to submit your paper to the correct bin. Solutions will be made available on the 8.01 website shortly afterward, so that you will be able to use them in studying for the quiz.

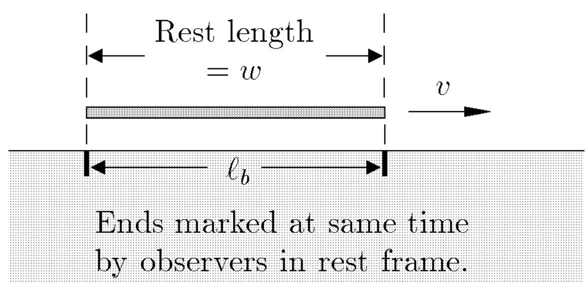
1) ***DO:***		Moving rods and clocks. The text of this problem is stated below.
2) ***DO:***	Y&F:37.2	The lifetime of a relativistic muon. The text of this problem is reproduced below.
3) ***DO:***		Time as measured in relativistic spacecraft. The text of this problem, which is based on Y&F:37.6, is stated below.
4) ***DO:***	Y&F:37.10	An unstable particle produced in the upper atmosphere. A slight rewording of this problem, intended to improve its clarity, is stated below.
5) ***DO:***		A quantitative treatment of the relativity of simultaneity. See the text below.
6) ***DO:***		A high speed space chase. See the text below.
7) ***DO:***		Dynamic Duo jumps off a fast (relativistic) train. See the text below.

Problem 1: Moving rods and clocks

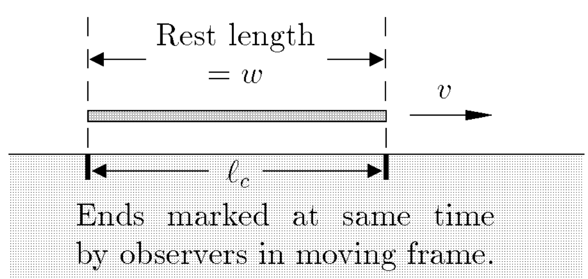
(a) A rod, with rest length w, moves with speed v in a direction perpendicular to its length. It passes a second, longer rod, which is at rest and is oriented parallel to the first rod. As it passes, observers who are at rest on the second rod mark the location of each end of the first rod. What is the distance l_a between these two marks, as measured in the stationary frame?



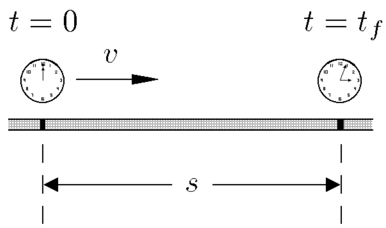
b) The same rod again moves with speed v, but this time it is moving parallel to its length, moving along the surface of a stationary table. At a certain prearranged time, observers in the rest frame of the table mark the instantaneous locations of each end of the rod. What is the distance ℓ_b between the two marks, as measured in the stationary frame?



c) The rod is still moving along the table, but this time there are travelers on the moving rod who have synchronized their clocks in the frame of the rod. At a prearranged time, travelers at each end of the rod reach out and make a mark on the table. What is the distance ℓ_c between these two marks, as measured in the stationary frame (i.e., the frame of the table)?



d) A clock moves along a track at speed v. The track is at rest, and has length s. The clock is set to zero at the start of the track. What time t_f does it read when it reaches the end of the track?



Problem 2: The lifetime of a relativistic muon (Y&F:37.2)

The positive muon (μ^+), an unstable particle, lives on average 2.20×10^{-6} s (measured in its own frame of reference) before decaying.

(a) If such a particle is moving, with respect to the laboratory, with a speed of $0.900c$, what average lifetime is measured in the laboratory?

(b) What average distance, measured in the laboratory, does the particle move before decaying?

Problem 3: Time as measured in relativistic spacecraft (based on Y&F:37.6)

While you are on a space station in deep space, far away from any gravitational fields, a race pilot flies past you in her space racer at a constant speed of $0.800c$ relative to you. At the instant the space racer passes you, both of you start timers at zero.

(a) There is a space buoy at rest relative to you, at a distance of 1.20×10^8 m (as measured in your reference frame). If you looked at your timer at the same time (as measured in your coordinate system) that the space racer passes the buoy, what time would it read?

(b) When the space racer passes this buoy, what does the pilot read on her timer?

(c) When the race pilot passes the buoy, how far has she traveled since passing you, as measured in her reference frame?

(d) At the instant which, as judged by the space racer, is simultaneous to her passing the buoy, what is the reading on your timer?

Problem 4: An unstable particle produced in the upper atmosphere (Y&F:37.10, slightly reworded)

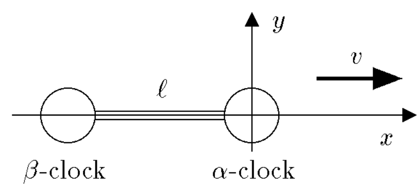
An unstable particle is created in the upper atmosphere from a cosmic ray and travels straight down toward the surface of the Earth with a speed of $0.99540c$ relative to the Earth. A scientist at rest on the Earth'S surface measures that the particle is created at an altitude of 45.0 km.

(a) As measured by the scientist, how much time does it take the particle to travel the 45.0 km to the surface of the Earth?

(b) Use the length contraction formula to calculate the distance from where the particle is created to the surface of the Earth as measured in the particle's frame.

(c) In the particle's frame, how much time elapses from when the particle is created to when it strikes the surface of the Earth? Calculate this time both by the time dilation formula and also from the distance calculated in part (b). Do the two results agree?

Problem 5: A quantitative treatment of the relativity of simultaneity



Consider a device consisting of two clocks and a rod between them, as shown at the right. The rod is oriented along the x axis of a coordinate system, which we will refer to as the "laboratory frame." The whole device is moving to the right (positive x direction), relative to the laboratory frame, with speed v. We will call the leading clock the α-clock, and the rear clock the β-clock. The length of the rod in the rest frame of the device, is ℓ_0, but in the laboratory frame is has length ℓ.

(a) In terms of ℓ_0, v, and the speed of light c, what is the length ℓ, as measured in the laboratory frame?

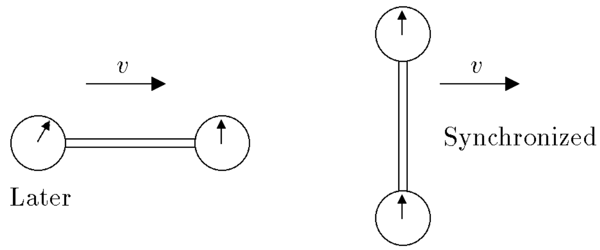
(b) Two observers ride on the moving device, one located at each clock, and they synchronize the two clocks using the definition of simultaneity appropriate to their frame of reference. There are many (equivalent) ways that they could do this, but we will suppose that at the instant when the α-clock passes the origin of our coordinate system, the observer at that clock sets the clock to zero, and emits a light pulse. When the light pulse reaches the β-clock, the observer on that clock sets it to ℓ_0/c, allowing for the light-travel time between the two clocks. In the laboratory frame, at what time t_f does the light pulse reach the β-clock? At what location x_f does this happen?

(c) In the laboratory frame of reference, what is the reading t_α on the α-clock at time t_f, the same instant when the light pulse reaches the β-clock?

(d) We let $t_\beta = \ell_0/c$ denote the time to which the β-clock is set when the light pulse reaches it, and we continue to use t_α to denote the reading on the α-clock at the same time, as measured in the laboratory frame. Then the quantity $t_\beta - t_\alpha$ indicates the error in synchronization of the two clocks, as seen in the laboratory frame. Show that this difference is given by

$$t_\beta - t_\alpha = \frac{v\ell_0}{c^2} .$$

That is, whenever a system that is moving relative to an observer contains clocks which are synchronized in the frame of the system and which are separated from each other in the direction of the motion, then in the observer's reference frame the trailing clock will read a later time than the leading clock by an offset $v\ell_0/c^2$. On the other hand, if the clocks are separated in a direction perpendicular to the motion, they will appear to be synchronized, as illustrated below:

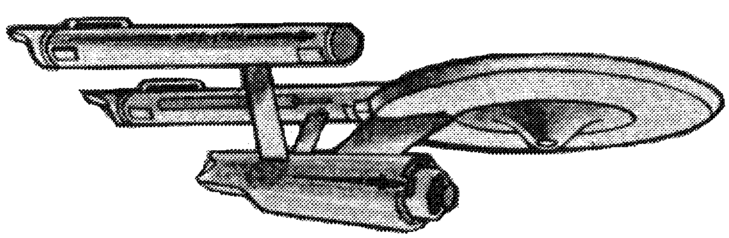


The above statement, when combined with statements about time dilation and length contraction, give a full description of the kinematical consequences of special relativity.

Problem 6: A high speed space chase

A badly programmed computer in the control system of the starship Excalibur goes berserk, launching the fully armed spacecraft from the planet Gamma Trianguli VI. The Excalibur moves at speed v_X along a straight line leading toward one of the most populated areas in the galaxy.

(a) When the Excalibur leaves Gamma Trianguli VI, the clocks on the planet read t_1. After traveling for a time t_2 as measured on its own clock, the computer transmits a message to Gamma Trianguli VI, threatening to destroy the planet Eminiar VII. If the message travels at the speed of light, what is the time t_3 on the clocks of Gamma Trianguli VI when the message is received?

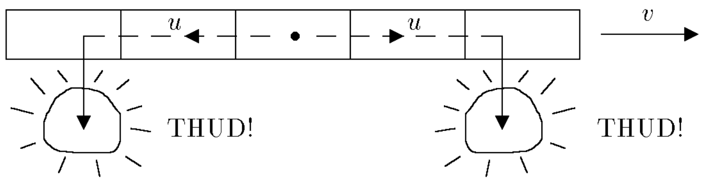


(b) The starship Enterprise arrives at Gamma Trianguli VI at time t_E, measured on the planet's clocks. It immediately takes off after Excalibur at speed v_E, where fortunately $v_E > v_X$. It rapidly overtakes Excalibur. At the moment of interception, what is the time t_4, as measured in the coordinate system of Gamma Trianguli VI?

(c) As measured on the Excalibur clocks, how much time Δt_X elapses between its take-off from Gamma Trianguli VI and the moment of interception?

Problem 7: Dynamic Duo jumps off a fast train

Batman and Robin are standing next to each other on the roof of a train, which is moving at a steady velocity of magnitude v. (Since this is a relativity problem, v is of course not negligible compared to c.) The superheroes simultaneously set their stop-watches to zero, and then by prearrangement they start running along the train in opposite directions at speed u, measured in the frame of reference of the train. They each run for a time Δt, as measured on their own stop-watches, and then they each jump from the train. They hit the ground with a sharp thud, and come instantly to a stop. Miraculously neither member of the Dynamic Duo is hurt.



(a) As measured on the ground, how far apart are the two points at which the pair hit the ground?

(b) As seen from the ground, do the two superheroes hit the ground simultaneously? If not, calculate the time difference.