MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 Spring 2005

REVIEW PROBLEMS FOR FINAL EXAM

Monday, May 9, 2005

Due Date: NEVER!

Reading Assignment: Young and Freedman, Sections 37.6 – 37.9. Chapter 37 is available as a separate booklet, and it is now available as a PDF file on our website, restricted to the class: https://web.mit.edu/8.01/www/Spring05/37/YF37relativity.pdf.

Topics for the week: Continuation of Special Relativity. Further use of the Lorentz transformation, relativistic momentum, energy, and work, and the Doppler effect for electromagnetic radiation.

Instructions:

These problems are not to be handed in, but are given to you only to help you learn the material discussed in the last week of the term. Solutions will appear immediately. This material will be tested on the first 1/3 of the final exam, which will be treated as the equivalent of weekly quiz 13. The quiz 13 part of the final exam will include at least one problem that is at most a slight modification of one of the problems on this problem set.

1) Y&F:37.14 Inversion of the Lorentz Transformation
2) Y&F:37.20 Relativistic velocity addition
3) Y&F:37.25 Tell it to the Judge
4) Y&F:37.29 Relativistic momentum vs. Newtonian momentum
5) Y&F:37.37 Energy, momentum, and speed of a proton
6) Y&F:37.46 Energy of fusion
7) Moving perpendicular to a light wave. See text below.
8) Threshold for particle production. See text below.
9) A relativistic inelastic collision. See text below.

Text of Problems

Problem 1 (Y&F:37.14): Inversion of the Lorentz Transformation

Solve Eq. (37.21),

$$x' = \frac{1}{\sqrt{1-u^2/c^2}}(x-ut) = \gamma(x-ut)$$

$$y' = y; \qquad z' = z \tag{37.21}$$

$$t' = \frac{1}{\sqrt{1-u^2/c^2}}\left(t-\frac{ux}{c^2}\right) = \gamma\left(t-\frac{ux}{c^2}\right) ,$$

to obtain x and t in terms of x' and t', and show that the resulting transformation has the same form as the original one except for a change of sign for u.

Problem 2 (Y&F:37.20): Relativistic velocity addition

Two particles in a high-energy accelerator experiment are approaching each other head-on, each with a speed $0.9520c$ as measured in the laboratory. What is the magnitude of the velocity of one particle relative to the other?

Problem 3 (Y&F:37.25): Tell it to the Judge

(a) How fast must you be approaching a red traffic light ($\lambda = 675$ nm) for it to appear yellow ($\lambda = 575$ nm)? Express your answer in terms of the speed of light.

(b) If you used this as an excuse for not getting a ticket for running a red light, how much of a fine would you get for speeding? Assume that the fine is $1.00 for each kilometer per hour that your speed exceeds the posted limit of 90 km/h.

Problem 4 (Y&F:37.29): Relativistic momentum vs. Newtonian momentum

(a) At what speed is the momentum of a particle twice as great as the result obtained from the nonrelativistic expression mv? Express your answer in terms of the speed of light.

(b) A force is applied to a particle along its direction of motion. At what speed is the magnitude of force required to produce a given acceleration twice as great as the force required to produce the same acceleration when the particle is at rest? Express your answer in terms of the speed of light.

Problem 5 (Y&F:37.37): Energy, momentum, and speed of a proton

A proton (rest mass 1.67×10^{-27} kg) has total energy that is 4.00 times its rest energy. What is

(a) the kinetic energy of the proton?

(b) the magnitude of the momentum of the proton?

(c) the speed of the proton?

Problem 6 (Y&F:37.46): Energy of fusion

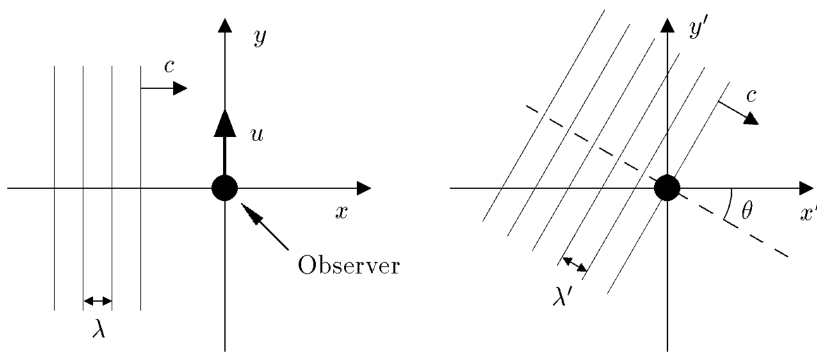
In a hypothetical nuclear-fusion reactor, two deuterium nuclei combine or "fuse" to form one helium nucleus. The mass of a deuterium nucleus, expressed in atomic mass units (u), is 2.0136 u; that of a helium nucleus is 4.0015 u (1 u $= 1.6605402 \times 10^{-27}$ kg)

(a) How much energy is released when 1.0 kg of deuterium undergoes fusion?

(b) The annual consumption of electrical energy in the United States is of the order of 1.0×10^{19} J. How much deuterium must react to produce this much energy?

Problem 7: Moving Perpendicular to a Light Wave

A planar light wave propagates along the x-axis, as shown on the diagram on the left. The wave moves of course at speed c, and it has a wavelength λ. An observer moves at speed u along the y-axis, as shown.



(a) Introduce a primed frame of reference which is at rest relative to the moving observer. Suppose, as we usually assume, that the origins of the two systems coincide at $t = t' = 0$. Write down the equations which give the primed coordinates in terms of the unprimed coordinates.

(b) In the primed reference frame, the wave propagates at an angle θ with respect to the x'-axis. Find θ.

(c) To the moving observer, the wavelength will appear Doppler shifted. Determine the wavelength λ' as seen by the moving observer. *(Caution: since the motion is perpendicular to the direction of wave propagation, the formulas in Young and Freedman do not apply. You must work from first principles.)*

Problem 8: Threshold for Particle Production

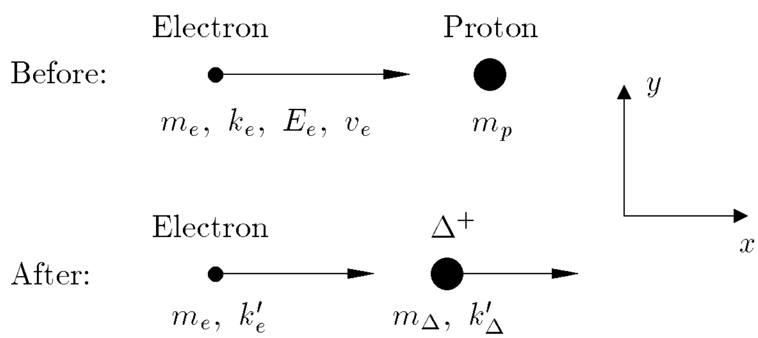
An electron of energy E (in the laboratory frame) collides with a proton at rest. Note that E denotes the ***total*** energy of the electron, which means kinetic energy plus rest energy. Let m_e denote the mass of the electron and let m_p denote the mass of the proton. In terms of m_e, m_p, and c, what is the minimum value of E that would allow the reaction

$$e^- + p \to e^- + p + p + \bar{p}$$

to take place? Here $\bar{p}$ denotes an antiproton, which also has a mass m_p. *(Hint: when E is at its minimum, the four particles of the final state will all move together, as if they were one particle of mass $m_e + 3m_p$. Any motion of these particles relative to each other would require a larger value of E.)*

Problem 9: A Relativistic Inelastic Collision

An electron of mass m_e and momentum k_e is incident on a stationary proton, of mass m_p. The electron moves along the x-axis, in the positive direction.



(a) What is the total energy E_e of the electron?

(b) What is the speed v_e of the electron?

(c) In the course of the collision the proton is excited, becoming a very short-lived particle (lifetime $\approx 10^{-23}$ sec.) called a Δ^+, with a mass m_Δ. Assume for simplicity that the final electron and Δ^+ continue to move along the x-axis, so it remains a one-dimensional problem. Denote the final momentum of the electron by k'_e, and the final momentum of the Δ by k'_Δ. Write down two conservation equations that would allow one to solve for the two unknowns, k'_e and k'_Δ. You need not solve these equations.