MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 Spring 2005

PROBLEM SET 1 SOLUTIONS

Problem 1: Y&F 1.4

Unit conversions are most reliably carried out by multiplying the original number by 1, where 1 is expressed as a ratio of two equal quantities expressed in different units. For example,

$$1 = \left(\frac{1\ \text{kg}}{1000\ \text{g}}\right) .$$

So

$$11.3\ \frac{\text{g}}{\text{cm}^3} = 11.3\ \frac{\cancel{\text{g}}}{\cancel{\text{cm}}^3} \times \left(\frac{1\ \text{kg}}{1000\ \cancel{\text{g}}}\right) \times \left(\frac{100\ \cancel{\text{cm}}}{1\ \text{m}}\right)^3 = \boxed{1.13 \times 10^4\ \frac{\text{kg}}{\text{m}^3}\ .}$$

Problem 2: Y&F 1.9

Using the same method as above,

$$15.0\ \frac{\text{km}}{\text{L}} = 15.0\ \frac{\text{km}}{\text{L}} \times \left(\frac{1\ \text{mi}}{1.609\ \text{km}}\right) \times \left(\frac{3.788\ \text{L}}{1\ \text{gal}}\right) = \boxed{35.3\ \frac{\text{mi}}{\text{gal}}\ .}$$

Problem 3:

SG 1A.1 An athlete runs at a uniform speed of 9.5 m/s. How long does it take him to run a distance of 200 m?

(a) 19 s; (b) 21 s; (c) 22 s; (d) none of these

Answer:

This is a straightforward application of $\Delta x = v\,\Delta t$, which implies that

$$\Delta t = \frac{\Delta x}{v} = \frac{200\ \text{m}}{9.5\ \text{m/s}} = 21.1\ \text{s}\ .$$

So the correct answer is $\boxed{\text{(b), 21 s.}}$

Problem 4:

SG 1A.3 An athlete runs 50 m along a straight track at a constant speed of 10 m/s. She then slows to 8 m/s for another 50 m.

(a) How long does it take her to run each segment?

(b) Plot (i) her position as a function of time; (ii) her velocity as a function of time; and (iii) her velocity as a function of distance.

(c) Over the complete 100 meters, what is her average velocity, averaged over time? What is her average velocity, averaged over distance?

Answer:

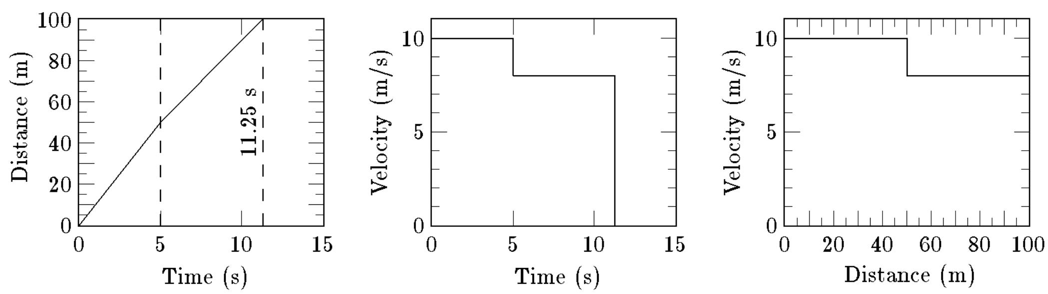
(a) For the first segment,

$$\Delta x_1 = 50 \text{ m}, v_1 = 10 \text{ m/s} \quad \Longrightarrow \quad \Delta t_1 = \frac{\Delta x_1}{v_1} = \boxed{5 \text{ s.}}$$

For the second segment,

$$\Delta x_2 = 50 \text{ m}, v_2 = 8 \text{ m/s} \quad \Longrightarrow \quad \Delta t_2 = \frac{\Delta x_2}{v_2} = \frac{25}{4} \text{ s} = \boxed{6.25 \text{ s.}}$$

(b) Your graphs will presumably look more hand-drawn, but they should resemble the following:



(c) The time-averaged velocity is a weighted average over time, and it is always also equal to the total distance divided by the total time. (If there is more than one dimension involved, then the time-averaged velocity would be a vector, equal to the total displacement vector divided by the total time. But this problem is one-dimensional.) For the weighted average over time,

$$v_{\text{time-average}} = \frac{v_1 \,\Delta t_1 + v_2 \,\Delta t_2}{\Delta t_1 + \Delta t_2} = \frac{10 \,\frac{\text{m}}{\text{s}} \cdot 5 \text{ s} + 8 \,\frac{\text{m}}{\text{s}} \cdot \frac{25}{4} \text{ s}}{5 \text{ s} + \frac{25}{4} \text{ s}} = \frac{100 \text{ m}}{\frac{45}{4} \text{ s}} = \boxed{8.89 \,\frac{\text{m}}{\text{s}} \,.}$$

For the weighted average over distance,

$$\begin{aligned} v_{\text{distance-average}} &= \frac{v_1 \,\Delta x_1 + v_2 \,\Delta x_2}{\Delta x_1 + \Delta x_2} \\ &= \frac{10 \,\frac{\text{m}}{\text{s}} \cdot 50 \text{ m} + 8 \,\frac{\text{m}}{\text{s}} \cdot 50 \text{ m}}{50 \text{ m} + 50 \text{ m}} = \frac{900 \,\frac{\text{m}^2}{\text{s}}}{100 \text{ m}} = \boxed{9 \,\frac{\text{m}}{\text{s}} \,.} \end{aligned}$$

Problem 6: Y&F 2.5

Solutions are usually the clearest if they are worked first in symbols, with numbers plugged in at the end. If we let the two running speeds be $v_1 = 5.50$ m/s and $v_2 = 6.20$ m/s, and the circumference of the track be $L = 200$ m, then the faster runner will overtake the slower when she has run a distance L farther. This will happen at time t, where

$$v_2 t = v_1 t + L \ ,$$

so

$$t = \frac{L}{v_2 - v_1} \ .$$

Looking at this solution, one can see that it has the right dimensions, that it increases with L, and that it approaches infinity if $v_2 \to v_1$, all of which make it look right. Inserting numbers,

$$t = \frac{200 \text{ m}}{(6.20 - 5.50) \text{ m/s}} = \boxed{286 \text{ s} \ .}$$

The distance each runner has travelled is then

$$s_1 = v_1 t = 5.50 \text{ m/s} \times 286 \text{ s} = \boxed{1571 \text{ m.}}$$

$$s_2 = v_2 t = 6.20 \text{ m/s} \times 286 \text{ s} = \boxed{1771 \text{ m.}}$$

Problem 7: Y&F 2.9

(a) We want the average velocity between $t = 0$ and $t = 10$ s. Since $x(t) = bt^2 - ct^4$, we have $x(0) = 0$, and

$$x(t{=}10 \text{ s}) = 3.60 \ \frac{\text{m}}{\text{s}^2} \times (10 \text{ s})^2 - 0.0120 \ \frac{\text{m}}{\text{s}^4} \times (10 \text{ s})^4 = 360 \text{ m} - 120 \text{ m} = 240 \text{ m} \ .$$

The average velocity is then

$$v_{\text{average}} = \frac{\Delta x}{\Delta t} = \frac{240 \text{ m}}{10 \text{ s}} = \boxed{24 \text{ m/s} \ .}$$

(b) The instantaneous velocity is obtained by differentiating,

$$v = \frac{\mathrm{d}x}{\mathrm{d}t} = 2bt - 4ct^3 \ ,$$

where we used the formula for differentiating a power of t:

$$\frac{\mathrm{d}}{\mathrm{d}t} t^n = n t^{n-1} \ .$$

So, at $t = 0$,

$$v = \boxed{0}\ ,$$

while at $t = 5.0$ s,

$$v = 2\left(3.60\ \frac{\text{m}}{\text{s}^2}\right)(5.0s) - 4\left(0.0120\ \frac{\text{m}}{\text{s}^4}\right)(5s)^3 = (36.0 - 6.0)\ \frac{\text{m}}{\text{s}} = \boxed{30.0\ \frac{\text{m}}{\text{s}}}\ ,$$

and at $t = 10$ s,

$$v = 2\left(3.60\ \frac{\text{m}}{\text{s}^2}\right)(10.0s) - 4\left(0.0120\ \frac{\text{m}}{\text{s}^4}\right)(10s)^3 = (72.0 - 48.0)\ \frac{\text{m}}{\text{s}} = \boxed{24.0\ \frac{\text{m}}{\text{s}}}\ .$$

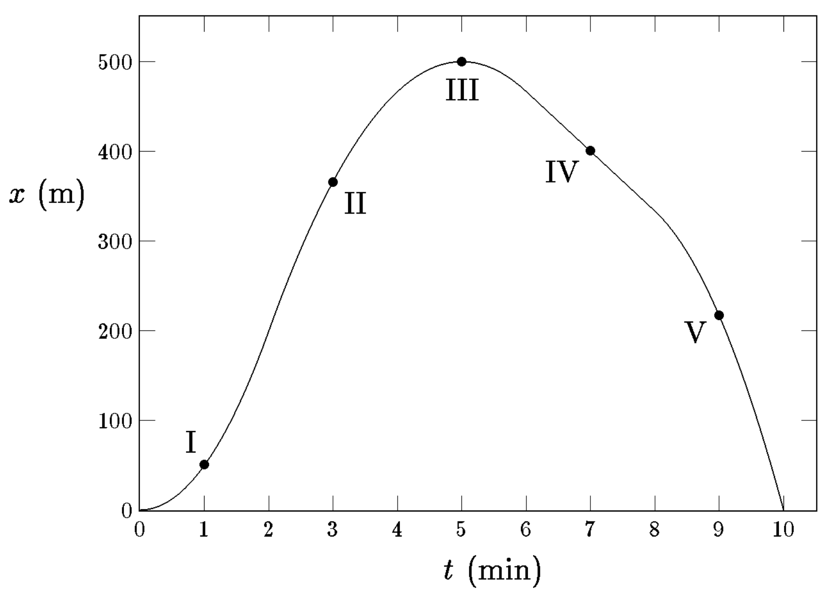
(c) The car is at rest when $v = 0$, which occurs when

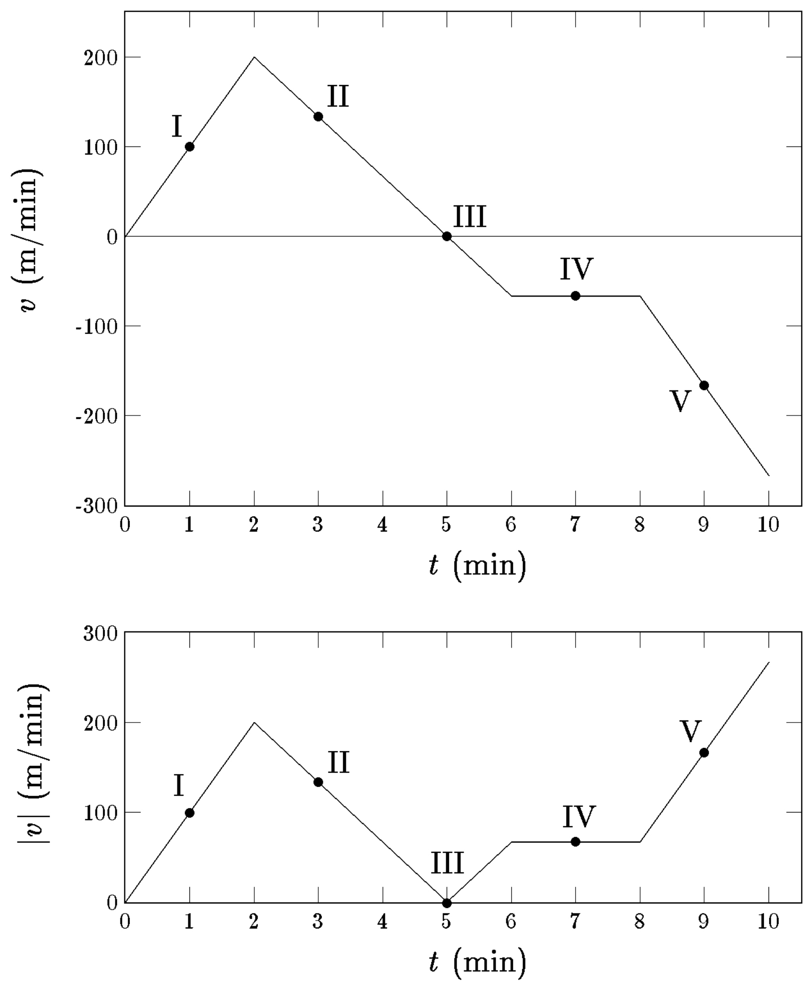
$$2bt - 4ct^3 = 0\ ,$$

or

$$t = \sqrt{\frac{b}{2c}} = \sqrt{\frac{3.60\ \text{m/s}^2}{2 \cdot 0.0120\ \text{m/s}^4}} = \boxed{12.2\ \text{s}}\ .$$

Problem 8: Y&F 2.10





Shown above are the graphs of the velocity vs. time, and the magnitude of the velocity vs. time, which can be extracted from the graph of x vs. time by differentiating, or drawing its slope. The graphs of v vs. t are in fact built from straight lines, but there is no way to know this without making very careful measurements on the original x vs. t curve, which you were not expected to do. But you should be able to understand where v is positive, where it is negative, and where it is increasing or decreasing. The answers to the specific questions are as follows:

(a) III: The $x(t)$ curve is horizontal, and the $v(t)$ curve therefore goes through zero; this corresponds to the time when she stops.

(b) There are no points where the velocity is constant and positive. At II the velocity is almost constant, but if you hold a ruler against the $x(t)$ curve you can see that it is not straight, but is curving slightly downward, so the velocity is decreasing at II, as can be seen very clearly in the graph of $v(t)$. It is hard to see in the graph of $x(t)$, however, so I would not deduct points for students who answered II.

(c) IV: Here the $x(t)$ curve is plainly straight, tilted downward (negative velocity).

(d) I and V: At I the $x(t)$ curve has a positive slope that is increasing. At V the curve has a negative slope that is becoming more negative, and hence is increasing in magnitude. In the graph of $|v(t)|$, it is obvious that $|v(t)|$ is increasing at these points.

(e) II: The $x(t)$ curve is tilted upward (positive slope and hence positive velocity), but becoming less so. Again in the graph of $|v(t)|$, it is obvious that $|v(t)|$ is decreasing.

Note that the behavior at III is ambiguous, since $|v(t)|$ has a cusp, meaning that it has a discontinuity in its slope. $|v(t)|$ is decreasing for t approaching 5 min from earlier times, but for times just after $t = 5$ min $|v(t)|$ is increasing. The slope of $|v(t)|$ at $t = 5$ min is mathematically undefined.

Problem 9: Y&F 1.24

With a pulse rate of a bit more than one beat per second, a heart will beat 10^5 times per day. With 365 days in a year and a lifespan of about 70 years, the number of beats in a lifetime is about $\boxed{3 \times 10^9.}$ With $\frac{1}{20}$ L (50 cm^3) per beat, and about $\frac{1}{4}$ gallon per liter, this comes to about $\boxed{4 \times 10^7 \text{ gallons.}}$

Solutions written by Alan Guth.