MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 Spring 2005

PROBLEM SET 3 SOLUTIONS

February 17, 2005

Fundamental concepts (Force, Mass, and Newton's Second Law):

Problem 1: Net force

SG:2A.1 An airplane is flying due west (relative to the ground) at a constant speed of 600 km/h. The mass of the plane is 8500 kg, and the engines are supplying a constant forward thrust of 5000 N. To the nearest 10 N, what is the magnitude of the net force acting on the plane?

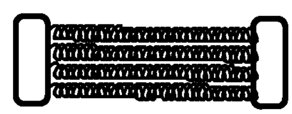
(a) 83300 N; (b) 5000 N; (c) 83450 N; (d) none of these

Answer:

We are told that the airplane is flying in a fixed direction at constant speed. That means that it has a constant velocity. If the velocity is constant there is no acceleration, and hence no net force acting. Therefore $\boxed{(d)}$ *is the correct answer.*

Problem 2: Combined effect of springs

SG:2B.3 (H) A body-building accessory consists of two handgrips joined by four identical springs. If each individual spring obeys Hooke's law with constant k, what is the spring constant of the whole device? What if the four springs had different constants?



Suppose you stretch the device by distance x.
Each spring, by Hooke's law, will exert a restoring force of magnitude kx.
therefore total restoring force has magnitude $4kx$
By definition, spring constant of whole device

$$= \left| \frac{\text{Total restoring force}}{\text{Total extension}} \right| = \frac{4kx}{x} = 4k$$

Similarly, if each spring had different constant

spring constant of whole device $= \dfrac{k_1x + k_2x + k_3x + k_4x}{x}$

$= k_1 + k_2 + k_3 + k_4$

$= \sum_{i=1}^{4} k_i$

Problem 3 (Y&F 4.17): Weight and mass on the Earth and on Io

(a) $\boxed{m = W/g = (44.0\text{ N})/(9.80\text{ m/s}^2) = 4.49\text{ kg}\ .}$

(b) The mass is the same, $\boxed{m = 4.49\text{ kg},}$ and the weight is

$\boxed{W = mg = (4.49\text{ kg})(1.18\text{ m/s}^2) = 8.13\text{ N}\ .}$

Problem 4: Comparison of electrostatic forces and gravity

SG:2B.6 (H) Calculate the electrostatic force exerted on an electron by a proton at a distance of 10^{-10} m. Compare this with the gravitational force between the two. In the light of your comparison, discuss why gravity, and not electromagnetism, is the fundamental force most apparent to us on a macroscopic scale. (The mass of the proton is 1.67×10^{-27} kg, that of the electron is 9.11×10^{-31} kg, and their charges are $\pm 1.60 \times 10^{-19}$ coulomb respectively. The numerical values (in SI units) of the relevant constants are $1/4\pi\epsilon_0 = 8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2$ for the electrostatic force and $G = 6.67 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2$ for the gravitational force.)

Answer:

From Coulomb's law, the force between an electron and a proton at a distance of 10^{-10} m has magnitude

$$\left|\vec{\mathbf{F}}_C\right| = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} = 8.99 \times 10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2} \times \frac{(1.60 \times 10^{-19}\,\text{C})^2}{(10^{-10}\text{ m})^2} = \boxed{2.30 \times 10^{-8}\text{ N}\ ,}$$

and is attractive. For comparison, the magnitude of the gravitational force between the same two particles is

$$\left|\vec{\mathbf{F}}_G\right| = G\frac{m_e\,m_p}{r^2} = 6.67 \times 10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\,\frac{(9.11 \times 10^{-31}\text{ kg})(1.67 \times 10^{-27}\text{ kg})}{(10^{-10}\text{ m})^2}$$

$$= \boxed{1.01 \times 10^{-47}\,\text{N}\ ,}$$

and is also attractive. The Coulomb force is therefore larger by a factor of

$$\frac{2.30 \times 10^{-8}\,\mathrm{N}}{1.01 \times 10^{-47}\,\mathrm{N}} \approx 2.27 \times 10^{39} \ .$$

Nonetheless, in everyday life we feel the force of gravity all the time, but the Coulomb force is very hard to detect. The reason is that gravity is always attractive, while electrostatic forces are usually cancelled almost exactly by the presence of equal or nearly equal amounts of positive and negative charge. When we feel the force of gravity holding ourselves and other objects to the floor, we should keep in mind that we are feeling the combined attraction caused by all $\sim 10^{52}$ *particles that make up the planet Earth.*

Newton's second law: force, mass, and acceleration:

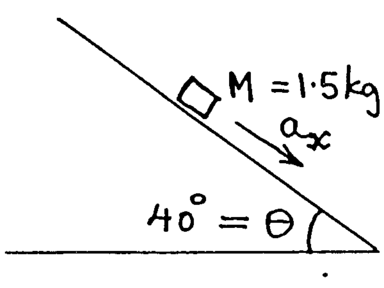
Problem 5: A block on an inclined plane

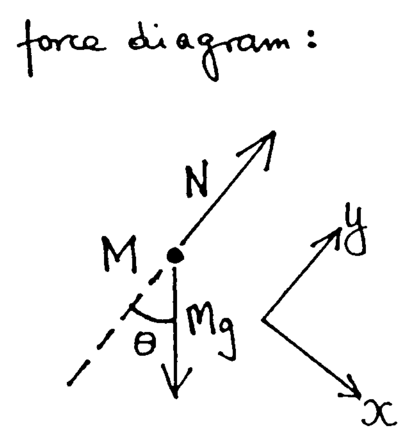
SG:2C.1 A block of mass 1.5 kg slides down a frictionless slope inclined at 40° to the horizontal. What is the magnitude of its acceleration down the slope?

(a) 9.8 m/s^2; (b) 6.3 m/s^2; (c) 7.5 m/s^2; (d) none of these

What is the magnitude of the force exerted on it by the slope?

(a) 6.3 N; (b) 7.5 N; (c) 11.3 N; (d) 9.4 N





(a) *Using the coordinate system shown on the force diagram, which is tilted so that the x-axis is parallel to the slope, the* $\vec{\mathbf{F}} = M\vec{\mathbf{a}}$ *equation becomes:*

$$[Mg\sin\theta, N - Mg\cos\theta, 0] = -M[a_x, 0, 0] \ ,$$

so

$$a_x = g\sin\theta = (9.80\ \mathrm{m/s^2})\sin 40^\circ$$
$$= \boxed{6.3\ \mathrm{m/s^2}\ .}$$

So the correct choice is $\boxed{(b).}$

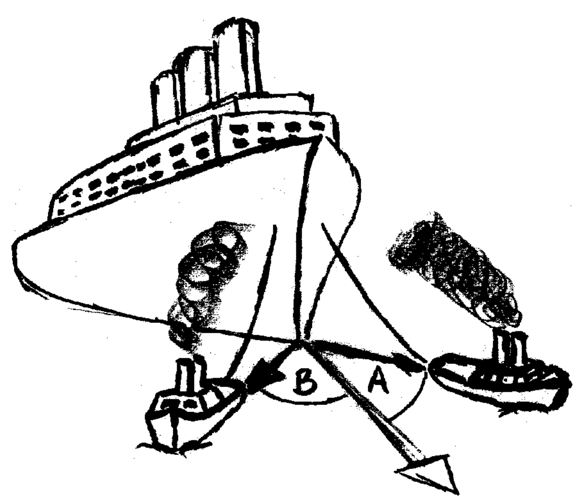
(b) *The force exerted on the block by the slope is N, the normal force, which from the y-component of the* $\vec{\mathbf{F}} = M\vec{\mathbf{a}}$ *equation is given by*

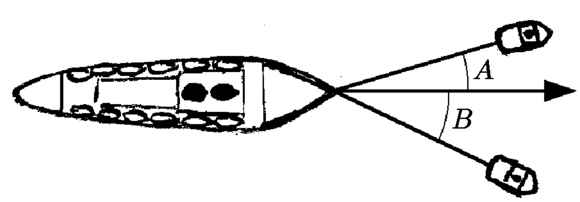
$$N - Mg\cos\theta = 0 \quad \Longrightarrow \quad N = Mg\cos\theta = \boxed{(1.5\ \mathrm{kg})\,(9.80\ \mathrm{m/s^2})\ \cos 40^\circ = 11.3\ \mathrm{N}\ .}$$

So the correct choice is $\boxed{(c).}$

Problem 8: Tugboats pulling a liner

SG:2C.4 (H) Two tugboats are towing a liner out of harbor. Their lines are arranged as in the diagram. If tug A exerts a force of magnitude F_A, derive an expression (in terms of F_A and the angles A and B) for the magnitude F_B of the force that tug B must exert if the net acceleration of the liner is to be straight ahead. Calculate F_B if $F_A = 3.1 \times 10^5$ N, with angle $A = 15^\circ$ and angle $B = 18^\circ$.





Force diagram for liner:

$\vec{F} = M\vec{a}$ gives: $[F_A\cos A + F_B\cos B,\ F_A\sin A - F_B\sin B,\ 0] = M[a, 0, 0]$

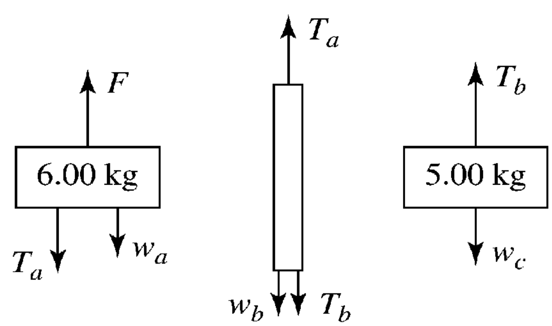
$\therefore\ F_A\sin A - F_B\sin B = 0$

$\therefore\ \frac{F_B}{F_A} = \frac{\sin A}{\sin B}$ or $F_B = F_A \frac{\sin A}{\sin B}$

$= 3.1\times10^5 \frac{\sin 15}{\sin 18} = \underline{\underline{2.6\times10^5\ \text{N}}}$

Problem 9 (Y&F 4.49): Blocks, ropes, and tension

a)



b) The net force on the system is $200\ \text{N} - (15.00\ \text{kg})(9.80\ \text{m/s}^2) = 53.0\ \text{N}$ (keeping three figures), and so the acceleration is $(53.0\ \text{N})/(15.0\ \text{kg}) = 3.53\ \text{m/s}^2$, up.

c) The net force on the 6-kg block is $(6.00\,\text{kg})(3.53\,\text{m/s}^2) = 21.2\,\text{N}$, so the tension is found from

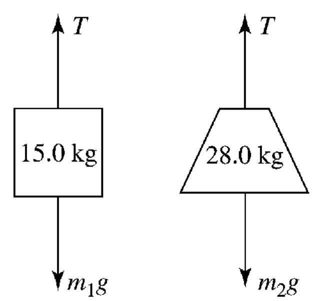
$$F - T - mg = 21.2\,\text{N}, \text{ or } T = (200\,\text{N}) - (6.00\,\text{kg})(9.80\,\text{m/s}^2) - 21.2\,\text{N} = 120\,\text{N}.$$

Equivalently, the tension at the top of the rope causes the upward acceleration of the rope and the bottom block, so $T - (9.00\,\text{kg})g = (9.00\,\text{kg})a$, which also gives $T = 120\,\text{N}$.

d) The same analysis of part (c) is applicable, but using $6.00\,\text{kg} + 2.00\,\text{kg}$ instead of the mass of the top block, or 7.00 kg instead of the mass of the bottom block. Either way gives $T = 93.3\,\text{N}$.

Problem 10 (Y&F 5.15): Atwood's machine

a)



b) The tension is related to the masses and accelerations by

$$T - m_1 g = m_1 a_1$$
$$T - m_2 g = m_2 a_2.$$

For the bricks accelerating upward, let $a_1 = -a_2 = a$ (the counterweight will accelerate down). Then, subtracting the two equations to eliminate the tension gives

$$(m_2 - m_1)g = (m_1 + m_2)a, \text{ or}$$

$$a = g\frac{m_2 - m_1}{m_2 + m_1} = 9.80\,\text{m/s}^2\left(\frac{28.0\,\text{kg} - 15.0\,\text{kg}}{28.0\,\text{kg} + 15.0\text{kg}}\right) = 2.96\,\text{m/s}^2.$$

c) The result of part (b) may be substituted into either of the above expressions to find the tension $T = 191\,\text{N}$. As an alternative, the expressions may be manipulated to eliminate a algebraically by multiplying the first by m_2 and the second by m_1 and adding (with $a_2 = -a_1$) to give

$$T(m_1 + m_2) - 2m_1m_2g = 0, \quad \text{or}$$

$$T = \frac{2m_1m_2g}{m_1 + m_2} = \frac{2(15.0\,\text{kg})\,(28.0\,\text{kg})\,(9.80\,\text{m/s}^2)}{(15.0\,\text{kg} + 28.0\,\text{kg})} = 191\,\text{N}.$$

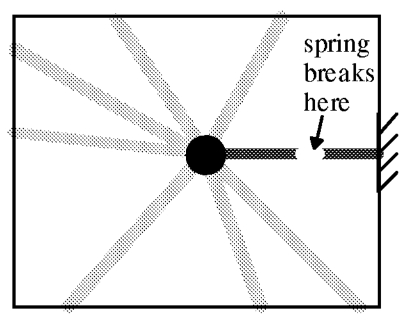
In terms of the weights, the tension is

$$T = w_1 \frac{2m_2}{m_1 + m_2} = w_2 \frac{2m_1}{m_1 + m_2}.$$

If, as in this case, $m_2 > m_1, 2m_2 > m_1 + m_2$ and $2m_1 < m_1 + m_2$, so the tension is greater than w_1 and less than w_2; this must be the case, since the load of bricks rises and the counterweight drops.

Problem 11: What happens when a spring breaks?

SG:3.15 A mass M is held stationary by many light springs as shown. If the spring on the right breaks, what is the acceleration of the mass immediately afterwards? Explain your reasoning clearly. Assume that the spring in question obeys Hooke's law with a force constant k, and that before breaking it was extended by an amount x; also assume that the total mass of all the springs is negligible compared to M.



Answer:

Before the spring breaks, it is stretched by an amount x, and therefore exerts a force on the mass M of magnitude kx, directed toward the right. Since the mass was held stationary, the total force acting on it must have been zero. Thus, the total force acting on M from all of the other springs must have had magnitude kx, directed to the left. When the spring breaks, the force it applies drops rapidly to zero, so the net force acting on M is kx acting to the left. Since the mass of the springs themselves is negligible, this force acts on the mass M, producing

an acceleration of magnitude $\left|\vec{\mathbf{F}}\right|/M = kx/M$, acting to the left.

Newton's third law: identifying 3rd law force pairs:

Problem 12 (Y&F 4.22): Forces on an elevator passenger

Answer:

The reaction to the upward normal force on the passenger is the downward normal force, also of magnitude 620 N, that the passenger exerts on the floor. The reaction to the passenger's weight is the gravitational force that the passenger exerts on the earth, upward and also of magnitude 650 N.

The passenger's mass is 650 N/g. If we adopt a coordinate system for which the vertical direction is the y-direction, then the passenger's acceleration is

$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{F}}_{\text{total}}}{m} = \frac{(620\ N)\,\hat{\boldsymbol{\jmath}} - (650\ N)\,\hat{\boldsymbol{\jmath}}}{650\ N/(9.80\ m/s^2)} = \boxed{-(0.452\ m/s^2)\,\hat{\boldsymbol{\jmath}}\ .}$$

So the acceleration has magnitude 0.452 m/s^2, downward.