MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 Spring 2005

PROBLEM SET 4 SOLUTIONS

February 24, 2005
Corrected Version, 2/25/05 12:30 am: μ_k changed to μ_s in Problem 7 (Y&F 5.95)

Frictional Basics:

Problem 1: Dragging crates in a warehouse

SG:6A.1 You are asked to drag a 45 kg crate across a warehouse floor. You find that in order to start the crate moving you have to apply a horizontal force of 250 N. Taking $g = 10$ m/s^2, what is the coefficient of static friction between the crate and the floor?

(a) 0.06; (b) 0.56; (c) 1.8; (d) 0.18.

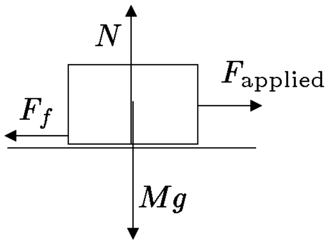
Your six-year-old cousin tries to help you by dragging the next crate, but she can only apply a force of 50 N. What frictional force opposes her efforts?

(a) 50 N; (b) 250 N; (c) 450 N; (d) none of these.

Answer:

(a) The force diagram for this situation is shown at the right. Since the crate cannot move in the vertical direction (because the floor is rigid), the vertical forces must sum to zero:

$$N - Mg = 0 \ .$$



When the horizontal applied force is just barely large enough to start the block moving, the acceleration must in principle be positive, but it can be arbitrarily small. The borderline case, therefore, is when the acceleration is zero. That is, any acceleration larger than zero, no matter how close to zero, will mean that the crate starts to move. For zero acceleration we must have zero horizontal force, so

$$F_{\text{applied}} - F_f = 0 \ ,$$

where F_{applied} is the magnitude of the force that you apply, and F_f is the magnitude of the force of friction. If the block is just about to slide, then the friction is static (there is no sliding yet), and the force of static friction must be at its maximum possible value, which is $F_f = \mu_s N$. So

$$F_{\text{applied}} = F_f = \mu_s N = \mu_s Mg \ ,$$

and hence

$$\mu_s = F_{\text{applied}}/(Mg) = 250 \text{ N} /(45 \text{ kg} \times 10 \text{ m/s}^2) = \boxed{0.56 \ .}$$

So the right choice is $\boxed{\text{(b).}}$

(b) The same force diagram applies, but the values of the forces are different. In this case the crate is ***NOT*** about to slide, so in this case the equation $F_f = \mu_s N$ does ***NOT*** hold. In this case $F_f < \mu_s N$, which is the situation when there is no relative motion of the two surfaces, and hence static friction, but when the surfaces are not about to slip. Since your cousin does not apply enough force to move the block, there is no motion and hence the net horizontal (as well as vertical) force must be zero. Thus,

$$F_{\text{applied}} - F_f = 0 \quad \Longrightarrow \quad F_f = F_{\text{applied}} = \boxed{50 \text{ N} ,}$$

so the correct choice is $\boxed{\text{(a).}}$

Circular Motion Basics:

Problem 2 (Y&F:5.114): What to do when your frictionless table has a hole in it

Since the hanging block does not move, the total vertical force on it must vanish. Thus

$$T - Mg = 0 ,$$

where T is the tension in the string. Since there is no friction where the string goes through the hole, the table acts like a frictionless pulley, and the tension is therefore the same on either side of the right-angle bend in the string. This tension, therefore, must be the force that holds the small block in uniform circular motion. The acceleration of an object in uniform circular motion has magnitude v^2/r, directed toward the center of the circle, so the radial component of $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ is

$$T = m\frac{v^2}{r} ,$$

so

$$v = \sqrt{\frac{Tr}{m}} = \boxed{\sqrt{\frac{Mgr}{m}} .}$$

More Difficult Friction Problems:

Problem 3: (S) Lugging a desk across a level floor

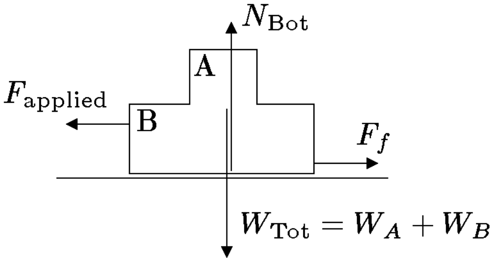
SG:6B.2 You have just moved into a new apartment, and you are attempting to shift a 60 kg desk from one side of a (fortunately uncarpeted) room to the other. Are you better off pulling horizontally, or at some angle to the horizontal? If the coefficient of kinetic friction between the desk and the floor is 0.45, what is the smallest force you can apply to move the desk at a constant speed?

Answer: See solution in the *Study Guide.*

Problem 4 (Y&F:5.62): A block on top of a block

(a) The most direct way to do part (a) is to consider the blocks as a unit, with total weight

$$\begin{aligned} W_{\text{Tot}} &= W_A + W_B \\ &= 1.20\text{ N } + 3.60\text{ N } = 4.80\text{ N } . \end{aligned}$$



Drawing one free body diagram for the two-block system, as shown at the right, one sees that the absence of a vertical acceleration implies that

$$N_{\text{Bot}} = W_{\text{Tot}} = W_A + W_B \ .$$

The bottom surface of the block is sliding relative to the floor, so there is kinetic friction of magnitude

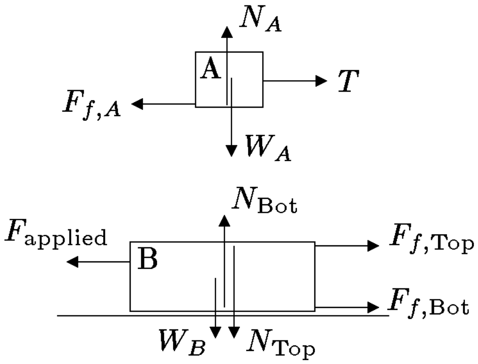
$$F_f = \mu_k N_{\text{Bot}} = \mu_k (W_A + W_B) \ .$$

If the block is being dragged at constant speed, then there is no horizontal acceleration and hence no horizontal force, so

$$F_{\text{applied}} = F_f = \boxed{\mu_k (W_A + W_B) = 0.30 \times 4.80\text{ N } = 1.44\text{ N } .}$$

(b) Now the two blocks move separately, so we draw a free body diagram for each. Block A is not moving and hence there is no net force acting on it, so

$$N_A = W_A \ .$$



By Newton's third law $N_{\text{Top}} = N_A$, since N_{Top} is the force that block A exerts on block B, and N_A is the force that block B exerts on block A. The top surface of B is sliding with respect to the bottom surface of A, so there is kinetic friction given by

$$F_{f,\text{Top}} = \mu_k N_{\text{Top}} = \mu_k N_A = \mu_k W_A \ .$$

Block B is not moving vertically, so the net vertical force must be zero, and hence

$$\begin{aligned} &N_{\text{Bot}} - W_B - N_{\text{Top}} = 0 \quad \Longrightarrow \\ &\quad N_{\text{Bot}} = N_{\text{Top}} + W_B = N_A + W_B = W_A + W_B \ . \end{aligned}$$

The bottom surface of Block B is sliding against the floor, so there is kinetic friction of magnitude

$$F_{f,\text{Bot}} = \mu_k N_{\text{Bot}} = \mu_k (W_A + W_B) \ .$$

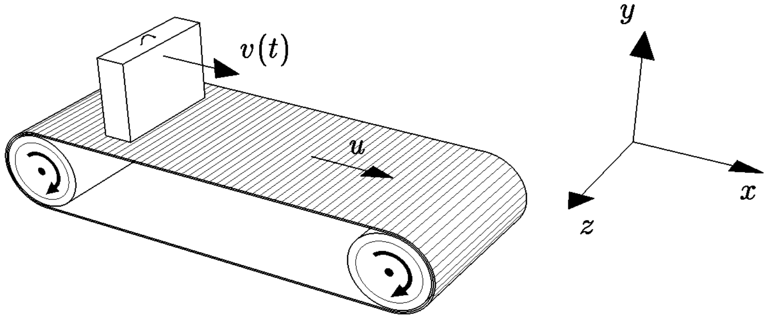
If block A is pulled at constant speed there is again no net force acting on it, so

$$F_{\text{applied}} = F_{f,\text{Top}} + F_{f,\text{Bot}} = \mu_k W_A + \mu_k (W_A + W_B)$$

$$= \boxed{\mu_k (2W_A + W_B) = 0.30 \times [2(1.20\text{ N}) + (3.60\text{ N})] = 1.80\text{ N}\ .}$$

Problem 5 (SG:7.8 revised): A suitcase placed on a conveyor belt

A suitcase of mass M is placed on a level conveyor belt at an airport. The coefficient of static friction between the suitcase and the conveyor belt is μ_s, and the coefficient of kinetic friction is μ_k, with $\mu_k < \mu_s$.



The conveyor belt moves with constant speed u, and at time $t = 0$ the suitcase is placed on the conveyor with speed $v = 0$. At $t = 0$, what is the total force $\vec{\mathbf{F}}$ acting on the suitcase? How long does the suitcase take to reach the speed of the conveyor belt (i.e. at what time t does $v(t) = u$)? Is the frictional force on the suitcase in the same direction or in the opposite direction of its motion? After the suitcase reaches the speed of the conveyor belt, what is the force of friction that acts on it?

Answer: The normal force upward cancels the force of gravity downward, so the total force acting on the suitcase at $t = 0$ is equal to the force of friction. Since the suitcase has zero velocity and the conveyor belt is moving, there is a nonzero relative velocity between the suitcase and the conveyor belt, and hence the friction is kinetic. So

$$\boxed{|\vec{\mathbf{F}}| = \mu_k N = \mu_k M g\ .}$$

The direction opposes the relative motion. Since the velocity of the suitcase relative to the conveyor belt is in the negative x-direction, the force on the suitcase is in the $\boxed{\text{positive } x\text{-direction}}$.

Since the total force is in the x-direction and is constant,

$$v_x(t) = \frac{F_x}{M} t\ .$$

Thus, the time t_1 at which the speed reaches u is given by

$$\frac{F_x}{M} t_1 = u \quad \Longrightarrow \quad t_1 = \frac{M}{F_x} u\ .$$

If one substitutes $F_x = \mu_k M g$, one finds that

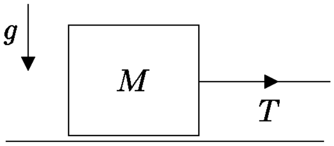
$$\boxed{t_1 = \frac{u}{\mu_k g}\ .}$$

Friction always acts to oppose the relative motion of two surfaces. In this case that means that friction causes the suitcase to accelerate in the direction of its motion.

Once the suitcase reaches the speed of the conveyor belt, there is no relative motion between the suitcase and the conveyor belt, so the relevant frictional force is that of static friction. Static friction will exert whatever force is necessary to prevent the surfaces from slipping, up to a maximum magnitude of $\mu_s|\vec{\mathbf{N}}|$, where $\vec{\mathbf{N}}$ is the normal force. In this case no horizontal force is necessary to keep the suitcase moving with the conveyor belt, so the force of friction will be $\boxed{\text{zero}}$.

Problem 6: (H) One or two blocks pulled by a rope

SG7.15 A block of mass M rests on a horizontal surface. The coefficient of kinetic friction between the block and the surface is μ_k, and the coefficient of static friction is μ_s, with $\mu_s > \mu_k$. The block is pulled horizontally by a massless inextensible rope, with a tension T that is gradually increased until the block starts to slide.

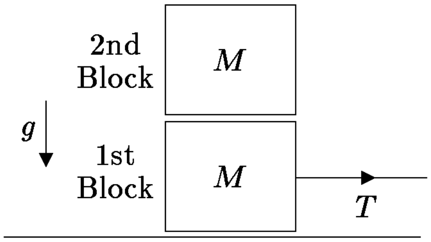


(a) What is the value of the tension T_1 at which the block begins to slide?

(b) When the tension was only $\frac{1}{2}T_1$, before the block began to slide, what was the magnitude and direction of the force of friction?

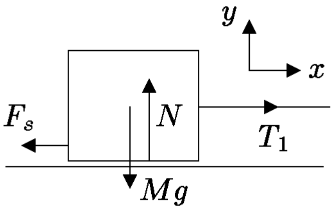
(c) If the tension is maintained at the value T_1, what is the acceleration of the block?

(d) A second block, identical to the first, is placed directly on top of the first block while both are at rest. The coefficients of friction between the two blocks are $\bar{\mu}_k$ for kinetic friction and $\bar{\mu}_s$ for static friction. As before, a horizontal rope is attached to the first (lower) block, and the tension in the rope is increased gradually from zero. At some value of the tension the two blocks begin to move, but there is initially no relative velocity between the two. As the steady increase in the tension is maintained, they accelerate faster and faster. At what value of the tension will the second block begin to slip relative to the first block? In what direction will it slip, relative to the block below?



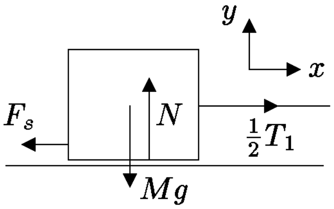
Answer:

(a) The block begins to slide when the tension T_1 equals the maximal static friction force, which is given by $\mu_s N$. There is no acceleration in the vertical direction, so the normal force must cancel the force of gravity, implying that $N = Mg$. Thus,



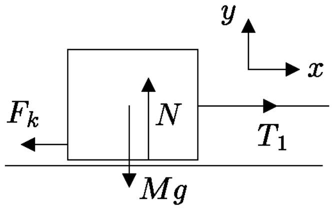
$$\boxed{T_1 = \mu_s N = \mu_s Mg\ .}$$

(b) The tension force $\frac{1}{2}T_1$ is insufficient to set the block in motion. Hence, the friction force will exactly cancel the tension force, and the block will remain at rest. $\boxed{\text{Its direction will oppose the tension force}}$ and its magnitude is given by



$$\boxed{F_s = \frac{1}{2}T_1 = \frac{1}{2}\mu_s Mg\ .}$$

(c) The acceleration a is determined by the tension force T_1 and the kinetic friction force $F_k = \mu_k N = \mu_k Mg$ opposing it. The total force — and hence the acceleration — is $\boxed{\text{in the direction of the tension force,}}$ and the magnitude of the acceleration is given by



$$Ma = T_1 - F_k = \mu_s Mg - \mu_k Mg$$

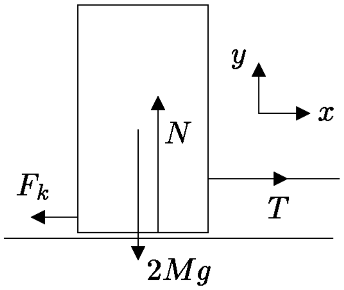
$$\Longrightarrow \quad \boxed{a = (\mu_s - \mu_k)g\ .}$$

Alternatively, in vector notation one could write

$$\boxed{\vec{\mathbf{a}} = [(\mu_s - \mu_k)g,\, 0,\, 0]\ .}$$

(d) Before the upper block begins to slip, it is moving together with the lower block due to static friction between them. We can consider the two blocks as one system.

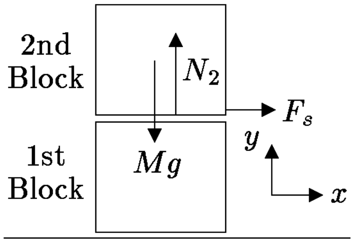
The two blocks have total mass $2M$ and the ground thus exerts a normal force $N = 2Mg$ on them. The normal force determines the kinetic friction force $F_k = \mu_k N = 2\mu_k Mg$ which opposes the applied tension force T_2. The total external force acting on the two blocks causes their acceleration a, so



$$2Ma = T_2 - F_k = T_2 - 2\mu_k Mg \ .$$

Note that the total mass that is accelerated is $2M$.

Now to find when the upper block will slip, we must consider it individually as a system. The only force that acts directly on the upper block is static friction, so its acceleration is due to that force. The magnitude of the static friction force on the upper block is therefore $F_s = Ma = (T_2 - 2\mu_k Mg)/2$. The absence of acceleration in the y-direction implies that the normal force acting on the upper block is $N_2 = Mg$. The upper block begins to slip when F_s reaches its maximal value $\bar{\mu}_s N = \bar{\mu}_s Mg$, i.e. when



$$\frac{T_2 - 2\mu_k Mg}{2} = \bar{\mu}_s Mg \ .$$

Solving for T_2, this gives

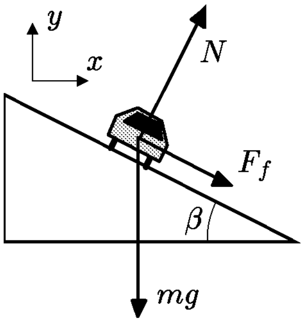
$$\boxed{T_2 = 2(\bar{\mu}_s + \mu_k)Mg \ .}$$

Once the upper block starts slipping, there is kinetic friction between the two blocks which is less than static friction. As a result, the upper block accelerates by a smaller amount than the lower block and thus $\boxed{\text{slips in the direction opposite the direction of motion.}}$

More Difficult Circular Motion Problems:

Problem 7 (Y&F:5.95): Maximum and minimum speeds on a banked road

(a) If the car is traveling at its maximum speed, then friction is needed to prevent the car from sliding up the hill. Hence friction points down the hill, as shown in the diagram at the right. It is static friction, because when a wheel rolls without slipping, the relative velocity of the tire with respect to the road, at the point of contact, is zero. The vertical acceleration of the car is zero, while the horizontal acceleration has magnitude v^2/R, where v is the speed of the car, since it is in uniform circular motion about a circle of radius R. The acceleration is directed toward the center of the circle, which in the diagram is toward the right. Us-



ing the coordinate system shown, $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ can be written as

$$\begin{aligned} F_x &= N\sin\beta + F_f\cos\beta = m\frac{v^2}{R} \\ F_y &= N\cos\beta - F_f\sin\beta - mg = 0 \ . \end{aligned}$$

If the car is going at the maximum speed that it can go without slipping, then $F_f = \mu_s N$. Replacing F_f in the two equations above by $\mu_s N$, we have two equations for two unknowns (N and v), so they can be solved. The easiest way to find v is to write

$$\begin{aligned} N\sin\beta + \mu_s N\cos\beta &= m\frac{v^2}{R} \\ N\cos\beta - \mu_s N\sin\beta &= mg \ , \end{aligned}$$

and then one can divide the top equation by the bottom, eliminating N:

$$\frac{\sin\beta + \mu_s\cos\beta}{\cos\beta - \mu_s\sin\beta} = \frac{v^2}{Rg} \ ,$$

so

$$\boxed{v = \sqrt{\frac{\sin\beta + \mu_s\cos\beta}{\cos\beta - \mu_s\sin\beta}Rg} \ .}$$

For $\mu_s = 0.30$, $\beta = 25°$, $R = 50$ m, and $g = 9.8$ m/s^2, one finds $\boxed{v = 20.9 \text{ m/s.}}$

(b) The same analysis applies, except that this time the force of friction is preventing the car from sliding down the embankment, so it is directed up the bank. Thus,

$$\begin{aligned} F_x &= N\sin\beta - F_f\cos\beta = m\frac{v^2}{R} \\ F_y &= N\cos\beta + F_f\sin\beta - mg = 0 \ , \end{aligned}$$

where again $F_f = \mu_s N$. Proceeding as before,

$$\boxed{v = \sqrt{\frac{\sin\beta - \mu_s\cos\beta}{\cos\beta + \mu_s\sin\beta}Rg} \ .}$$

Substituting the same numerical values as before, one finds $\boxed{v = 8.5 \text{ m/s.}}$

Problem 8 (Y&F:5.115): A bead on a rotating circular hoop.

5.115: a) The analysis is the same as that for the conical pendulum of Example 5.22, and so

$$\beta = \arccos\left(\frac{gT^2}{4\pi^2 L}\right) = \arccos\left(\frac{(9.80\,\mathrm{m/s^2})(1/4.00\,\mathrm{s})^2}{4\pi^2(0.100\,\mathrm{m})}\right) = 81.0°.$$

b) For the bead to be at the same elevation as the center of the hoop, $\beta = 90°$ and $\cos\beta = 0$, which would mean $T = 0$, the speed of the bead would be infinite, and this is not possible. c) The expression for $\cos\beta$ gives $\cos\beta = 2.48$, which is not possible. In deriving the expression for $\cos\beta$, a factor of $\sin\beta$ was canceled, precluding the possibility that $\beta = 0$. For this situation, $\beta = 0$ is the only physical possibility.

Problems with Air Drag:

Problem 9 (Y&F:5.14): An airplane moving in a straight line

5.14: a) In level flight, the thrust and drag are horizontal, and the lift and weight are vertical. At constant speed, the net force is zero, and so $F = f$ and $w = L$. b) When the plane attains the new constant speed, it is again in equilibrium and so the new values of the thrust and drag, F' and f', are related by $F' = f'$; if $F' = 2F$, $f' = 2f$. c) In order to increase the magnitude of the drag force by a factor of 2, the speed must increase by a factor of $\sqrt{2}$.

Problem 10 (Y&F:5.105): Fluid resistance and terminal speed

(a) Newton's second law for the vertical component of the velocity, defining positive as downward, gives

$$m\frac{\mathrm{d}v_y}{\mathrm{d}t} = mg - kv_y \ .$$

The terminal velocity v_t is that velocity for which $\mathrm{d}v_y/\mathrm{d}t = 0$, so

$$v_t = \frac{mg}{k} \ .$$

The equation of motion can then be rewritten in terms of v_t, giving

$$m\frac{\mathrm{d}v_y}{\mathrm{d}t} = -k(v_y - v_t) \ .$$

Bringing everything that depends on v_y to the left, and everything that depends on t to the right, one can rewrite the equation as

$$\frac{\mathrm{d}v_y}{v_y - v_t} = -\frac{k}{m}\,\mathrm{d}t \ ,$$

which can be integrated. To understand the limits of integration, note that v_0 is the speed at $t = 0$, and v_y is the speed at some arbitrary time t. So

$$\int_{v_0}^{v_y} \frac{\mathrm{d}v_y}{v_y - v_t} = -\frac{k}{m} \int_0^t \mathrm{d}t \ .$$

Evaluating,

$$\ln(v_y - v_t) - \ln(v_0 - v_t) = -\frac{k}{m} t \ ,$$

so

$$\ln\left(\frac{v_y - v_t}{v_0 - v_t}\right) = -\frac{k}{m} t \ ,$$

and exponentiation gives

$$\frac{v_y - v_t}{v_0 - v_t} = e^{-kt/m} \ ,$$

which can be solved for v_y to give

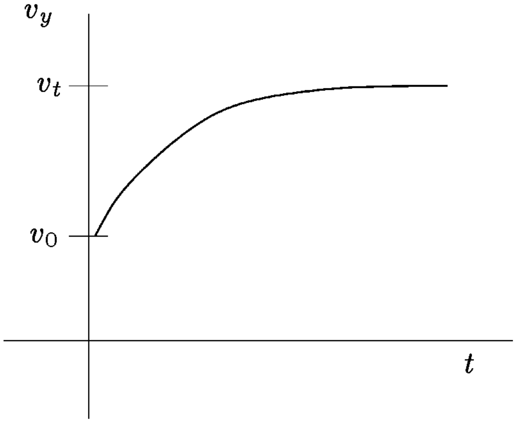
$$\boxed{v_y = v_0 e^{-kt/m} + v_t\left(1 - e^{-kt/m}\right) \ .}$$

Note that this equation give $v_y(t{=}0) = v_0$, as it must, and that it gives

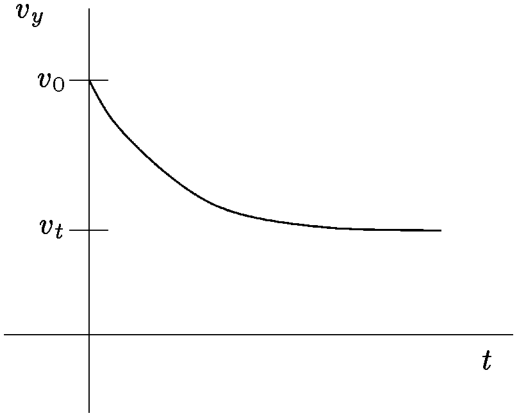
$$\lim_{t\to\infty} v_y(t) = v_t \ ,$$

as we expected.

(b) The downward gravity force is larger than the upward fluid resistance force so the acceleration is downward, until the fluid resistance force approaches that of gravity as the terminal speed is approached. The object speeds up until v_y asymptotically approaches v_t. With the positive direction defined as downward, the graph looks like



(c) This time the upward resistance force is larger than the downward gravity force so the acceleration is upward and the object slows down, until the fluid resistance force approaches that of gravity as the terminal speed is approached. The graph looks like



(d) When $v_0 = v_t$ the acceleration at $t = 0$ is zero and remains zero; the velocity is constant and equal to the terminal velocity.

Solutions written by Alan Guth.