MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 Spring 2005

PROBLEM SET 5 SOLUTIONS

March 3, 2005

Kinetic Energy:

Problem 1 (SG): Kinetic energy and velocity

4A.1 A particle of mass 0.3 kg moves with velocity [0.6, 1.2, - 0.5] m/s. What is its kinetic energy?

(a) 0.25 J; (b) 0.79 J; (c) 0.31 J; (d) 0.52 J.

Answer: The kinetic energy of a particle is $E_k = \frac{1}{2}m\left|\vec{\mathbf{v}}\right|^2$, and $\left|\vec{\mathbf{v}}\right|^2 = v_x^2 + v_y^2 + v_z^2$. So

$$E_k = \frac{1}{2}(0.3\text{ kg})\left[(0.6)^2 + (1.2)^2 + (0.5)^2\right]\ \text{m}^2/\text{s}^2 = 0.31\text{ J}\ .$$

So the right answer is $\boxed{\text{(c)}}$.

Problem 2 (SG): Kinetic energy of a projectile

4A.2 A projectile of mass m is launched over level ground with an initial speed of v_0 at an angle θ to the horizontal. Neglecting air resistance, what is its kinetic energy (a) just after launch, (b) at the top of its flight, (c) when it lands?

Answer:

(a) Just after launch the projectile has speed v_0. Therefore its kinetic energy is $\boxed{\frac{1}{2}mv_0^2.}$

(b) Neglecting air resistance, the horizontal component $v_0\cos\theta$ of the velocity of the projectile will be constant. At the top of its flight the projectile will have no vertical velocity, so its speed will be $v_0\cos\theta$. Its kinetic energy will therefore be $\boxed{\frac{1}{2}m\left(v_0\cos\theta\right)^2.}$

(c) By the work-energy theorem, the kinetic energy when the projectile lands will be equal to its initial kinetic energy, $\frac{1}{2}mv_0^2$, plus the total work done on the projectile. The only force acting on the projectile is gravity, and the work that it does while the particle is moving upward is $-mgh$, where h is the height above the ground that the projectile reaches. The work on the way down, however, is mgh, so the total work done by gravity is zero. The final kinetic energy is therefore $\boxed{\frac{1}{2}mv_0^2.}$

Work:

Problem 3 (SG): *STUDY:* Work done in hauling a barge

4C.2 (S) A horse tows a barge along a canal. The horse exerts a constant force of 750 N and the tow-rope makes an angle of 30° with the direction of motion of the barge. How much work is done by the horse on the barge, via the tow-rope, over a distance of 1 km? If the horse is traveling at 1 m/s, what power does it supply to the barge? Assume that this stretch of the canal is straight.

Answer: See complete solution in the *Study Guide*.

Problem 4 (Y&F): Work in compressing a spring

6.28: The intermediate calculation of the spring constant may be avoided by using Eq. (6.9) to see that the work is proportional to the square of the extension; the work needed to compress the spring 4.00 cm is $(12.0\,\text{J})\left(\frac{4.00\text{ cm}}{3.00\text{ cm}}\right)^2 = 21.3\,\text{J}$.

Problem 5 (SG): Vector description of work

4C.3 (H) A particle moves with constant velocity $[4, -2, 1]$ m/s. If its mass is 3 kg, what is its kinetic energy? One of the forces acting on it is given by $\vec{\mathbf{F}} = [-1, 2, 2]$ N. Find the work done by this force on the particle as it moves a distance of 3 m. What is the total work done on the particle, and why?

Answer: Its kinetic energy is

$$E_k = \frac{1}{2}m\left|\vec{\mathbf{v}}\right|^2 = \frac{1}{2}m\left(v_x^2 + v_y^2 + v_z^2\right) = \frac{1}{2}(3\text{ kg})\left(4^2 + 2^2 + 1^2\right)\ \text{m}^2/\text{s}^2 = \boxed{31.5\text{ J.}}$$

To find the work done by $\vec{\mathbf{F}}$ during a 3 m displacement, we need to calculate the displacement vector $\vec{\mathbf{d}}$. A unit vector in the direction of motion can be constructed from the velocity vector as

$$\hat{\boldsymbol{u}} = \frac{[4, \text{-}2, 1]\text{ m/s}}{\sqrt{4^2 + 2^2 + 1^2}\text{ m/s}} = \frac{1}{\sqrt{21}}[4, \text{-}2, 1]\ .$$

The displacement vector can then be written as

$$\vec{\mathbf{d}} = (3\text{ m})\hat{\boldsymbol{u}}\ ,$$

and the work done by $\vec{\mathbf{F}}$ is then

$$W_{\vec{\mathbf{F}}} = \vec{\mathbf{F}}\cdot\vec{\mathbf{d}} = (3\text{ m})\vec{\mathbf{F}}\cdot\hat{\boldsymbol{u}} = \frac{3\text{ m}}{\sqrt{21}}\left[(\text{-}1)(4) + (2)(\text{-}2) + (2)(1)\right]\ \text{N} = -\frac{18}{\sqrt{21}}\ \text{J} = \boxed{-3.93\text{ J.}}$$

The total work is $\boxed{\text{zero,}}$ since there is no change in the kinetic energy. Evidently there are other forces acting on the particle which cancel $\vec{\mathbf{F}}$, resulting in zero acceleration.

Problem 6 (SG): *STUDY:* Swinging a pail in a circle

4C.4 (S) A boy swings a pail of water on the end of a rope in a vertical circle. If the rope is 90 cm long, the mass of the pail plus water is 1.5 kg, and the rope is just taut at the top of the swing, what is the speed of the pail at the bottom of the swing, what work is done as the pail moves, and what force is responsible for the work? Assume that the boy's hand does not move during the period considered in this problem.

Answer: See complete solution in the *Study Guide*.

Work and Kinetic Energy:

Problem 7 (Y&F): Kicking a soccer ball

6.21:

$$s = \frac{\Delta K}{F} = \frac{\frac{1}{2}(0.420\,\mathrm{kg})((6.00\,\mathrm{m/s})^2 - (2.00\,\mathrm{m/s})^2)}{(40.0\,\mathrm{N})} = 16.8\,\mathrm{cm}$$

Problem 8 (Y&F): Work, kinetic energy, and inclined planes

6.15:

(a) Component of gravity parallel to incline:
Force component $= mg\sin\alpha$, down incline.
Displacement $= h/\sin\alpha$, down incline.
$W_{\|} = (mg\sin\alpha)(h/\sin\alpha) = mgh$.

Component of gravity perpendicular to incline:
Displacement $= 0$.
$W_{\perp} = 0$.

So, $W_{\text{gravity}} = W_{\|} + W_{\perp} = mgh$, which is the same as the work done by gravity when an object falls from height h in a straight line.

(b) The work-energy theorem says that the work is the change of the kinetic energy, or

$$W = K_f - K_i \ .$$

Here $K_i = 0$, so

$$K_f = \frac{1}{2}mv_f^2 = W = mgh \ ,$$

so

$$v_f = \sqrt{2gh} \ .$$

This is independent of α, and is the same as if the mass had been dropped from height h.

Intuitively, the work done by gravity is independent of the slope angle because the slope angle affects the force and the displacement in opposite ways. When the slope angle is small, there is a small force component in the direction of the displacement but a large displacement in this direction. When the slope angle is large, the force component in the direction of the displacement along the incline is larger but the displacement in this direction is smaller.

(c) $h = 15.0$ m, so $v = \sqrt{2gh} = \sqrt{2(9.8\text{ m/s}^2)\,(15.0\text{ m})} = 17.1$ m/s.

Problem 9 (Y&F): Maximum compression of a spring

6.81: a) At maximum compression, the spring (and hence the block) is not moving, so the block has no kinetic energy. Therefore, the work done *by* the block is equal to its initial kinetic energy, and the maximum compression is found from $\frac{1}{2}kX^2 = \frac{1}{2}mv^2$, or

$$X = \sqrt{\frac{m}{k}}v = \sqrt{\frac{5.00\text{ kg}}{500\text{ N/m}}}(6.00\text{ m/s}) = 0.600\text{ m}.$$

b) Solving for v in terms of a known X,

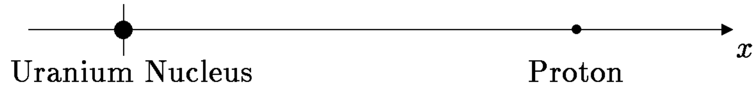
$$v = \sqrt{\frac{k}{m}}X = \sqrt{\frac{500\text{ N/m}}{5.00\text{ kg}}}(0.150\text{ m}) = 1.50\text{ m/s}.$$

Problem 10 (Y&F): A proton and a uranium nucleus

(a) As the proton moves from $x_1 = 5.00$ m to some closer distance x_2, the work done by the repulsive force is given by

$$W = \int_{x_1}^{x_2} F_x\,\mathrm{d}x = \int_{x_1}^{x_2} \frac{\alpha}{x^2}\,\mathrm{d}x = \alpha\left(\frac{1}{x_1} - \frac{1}{x_2}\right) .$$

The signs can be confusing. x measures the separation between the proton and the nucleus, so we can draw it as



A repulsive force on the proton is then a force to the right, in the $+x$ direction, and is therefore described by a force component that is positive, like

$$F_x = \frac{\alpha}{x^2} ,$$

for $\alpha > 0$. If x_2 is smaller than x_1, then $1/x_2$ is larger than $1/x_1$, so the expression for the work W is negative. This is correct, because the repulsive force slows the motion of the proton, and therefore does negative work.

For $x_2 = 8.00 \times 10^{-10}$ m, the work done by the force is

$$W = \alpha\left(\frac{1}{x_1} - \frac{1}{x_2}\right) = 2.12 \times 10^{-26}\text{ N}\cdot\text{m}^2\left(\frac{1}{5\text{ m}} - \frac{1}{8.00 \times 10^{-10}\text{ m}}\right)$$
$$= -2.65 \times 10^{-17}\text{ J}.$$

Note that x_1 is so large compared to x_2 that the term $1/x_1$ is negligible. The velocity can then be found from the work-energy theorem:

$$W = K_f - K_i = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \quad\Longrightarrow\quad v_2 = \sqrt{v_1^2 + \frac{2W}{m}} .$$

Numerically,

$$v_2 = \sqrt{(3.00 \times 10^5\,\text{m/s})^2 + \frac{2(-2.65 \times 10^{-17}\,\text{J})}{1.67 \times 10^{-27}\,\text{kg}}} = \boxed{2.41 \times 10^5 \text{ m/s}\,.}$$

(b) If the proton comes to rest then $K_f = 0$, so the work-energy theorem implies that $K_i = \frac{1}{2}mv_1^2 = -W$. Recognizing that $1/x_1$ is again negligible compared to $1/x_2$, one can write $W = -\alpha/x_2$, and then one has

$$x_2 = \frac{\alpha}{K_i} = \frac{2\alpha}{mv_1^2} = \frac{2\,(2.12 \times 10^{-26}\,\text{N} \cdot \text{m}^2)}{(1.67 \times 10^{-27}\,\text{kg})\,(3.00 \times 10^5\,\text{m/s})^2} = \boxed{2.82 \times 10^{-10}\,\text{m}\,.}$$

(c) The repulsive force has done no net work, so the kinetic energy and hence the speed of the proton have their original values, and the speed is $3.00 \times 10^5\,\text{m/s}$.

Power:

Problem 11 (Y&F): Light bulbs and running: what is a joule?

Energy = (power)(time), so in an hour a 100 W light bulb uses (100 W)(3600 s) = 3.6×10^5 J. For a 70 kg person to have this much kinetic energy, his/her speed can be found from

$$K = \frac{1}{2}mv^2 \text{ , so } v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\,(3.6 \times 10^5\,\text{J})}{70\text{ kg}}} = \boxed{101 \text{ m/s}\ .}$$

Problem 12 (Y&F): Electrical power consumption and solar power

6.46: a) $$\frac{(1.0\times10^{19}\text{ J/yr})}{(3.16\times10^{7}\text{ s/yr})} = 3.2\times10^{11}\text{ W.}$$

b) $$\frac{3.2\times10^{11}\text{ W}}{2.6\times10^{8}\text{ folks}} = 1.2\,\text{kW/person.}$$

c) $$\frac{3.2\times10^{11}\text{ W}}{(0.40)1.0\times10^{3}\text{ W/m}^2} = 8.0\times10^{8}\,\text{m}^2 = 800\,\text{km}^2.$$

Solutions written by Alan Guth.