

PROBLEM SET 7 SOLUTIONS

March 17, 2005

Momentum, Energy, and Force:

Problem 1 (Y&F): Kinetic energy and momentum

8.3: a)
$$K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{1}{2}\frac{p^2}{m}.$$

b) From the result of part (a), for the same kinetic energy, $\frac{p_1^2}{m_1} = \frac{p_2^2}{m_2}$, so the larger mass

baseball has the greater momentum; $(p_{\text{bird}}/p_{\text{ball}}) = \sqrt{0.040/0.145} = 0.525$. From the result of part (b), for the same momentum $K_1m_1 = K_2m_2$, so $K_1w_1 = K_2w_2$; the woman, with the smaller weight, has the larger kinetic energy. $(K_{\text{man}}/K_{\text{woman}}) = 450/700 = 0.643$.

Problem 2 (Y&F): The momentum vector of a soccer ball

8.4: From Eq. (8.2),

$$p_x = mv_x = (0.420 \text{ kg})(4.50 \text{ m/s})\cos 20.0^\circ = 1.78 \text{ kg m/s}$$
$$p_y = mv_y = (0.420 \text{ kg})(4.50 \text{ m/s})\sin 20.0^\circ = 0.646 \text{ kg m/s}.$$

Problem 3 (Y&F): The force of a golf swing

8.7: $\frac{\Delta p}{\Delta t} = \frac{(0.0450 \text{ kg})(25.0 \text{ m/s})}{2.00 \times 10^{-3} \text{ s}} = 563 \text{ N}$. The weight of the ball is less than half a newton, so the weight is not significant while the ball and club are in contact.

Collisions and the Conservation of Momentum:

Problem 4 (SG): Elastic collisions in one and two dimensions

5B.3 (H) Two sliders on a linear air track are fitted with spring-loaded fenders so that their collisions will be perfectly elastic. If one has mass m and speed v and the other has mass M and is stationary, what will be the velocity of each one after the collision? Deduce from this what will be the result if the first slider collides elastically with an immovable wall.

Discuss what will happen if we extend this situation to two dimensions, so that an air puck on a frictionless table collides elastically with a wall. Assume the puck's initial velocity makes an angle θ with the wall.

Answer: In an elastic collision we can use conservation of both momentum and kinetic energy. The initial momentum before the collision is $P_i = mv$, while after the collision $P_f = mv' + MV'$, where v' and V' are the final velocities of the two particles. Momentum conservation ($P_i = P_f$) thus implies

$$v = v' + \frac{M}{m}V'.$$

Now we use kinetic energy conservation. The initial kinetic energy is $K_i = \frac{1}{2}mv^2$, and after the collision we have $K_f = \frac{1}{2}mv'^2 + \frac{1}{2}MV'^2$, so that $K_i = K_f$ implies

$$v^2 = v'^2 + \frac{M}{m}V'^2.$$

Squaring the momentum conservation equation yields

$$v^2 = v'^2 + 2\frac{M}{m}v'V' + \frac{M^2}{m^2}V'^2,$$

so that

$$2v' + \frac{M}{m}V' = V' \Rightarrow v' = \frac{1}{2} \left(1 - \frac{M}{m}\right) V'.$$

Inserting this back into the momentum conservation equation yields

$$v = \frac{1}{2} \left(1 + \frac{M}{m}\right) V' \Rightarrow V' = \frac{2mv}{M+m} \Rightarrow v' = -\frac{M-m}{M+m}v.$$

A collision with an immovable wall corresponds to $M \rightarrow \infty$, so that $V' \rightarrow 0$ and $v' \rightarrow -v$.

In the two-dimensional case we can separately consider the components of the momentum parallel and perpendicular to the wall. The wall does not exert a force on the puck in the parallel direction. Therefore, the corresponding component of the momentum of the puck is conserved, i.e.

$$p_i^{\text{parallel}} = p_f^{\text{parallel}}.$$

The situation for the component perpendicular to the wall corresponds to the one-dimensional case discussed before, i.e.

$$p_i^{\text{perpendicular}} = -p_f^{\text{perpendicular}}.$$

As a result, the angle between the momentum of the puck and the wall after the collision is equal and opposite to the one before the collision.

Problem 5 (Y&F): Completely inelastic collision of three spheres

8.64: a) $m_A v_{Ax} + m_B v_{Bx} + m_C v_{Cx} = m_{tot} v_x$, therefore

$$v_{Cx} = \frac{(0.100 \text{ kg})(0.50 \text{ m/s}) - (0.020 \text{ kg})(-1.50 \text{ m/s}) - (0.030 \text{ kg})(-0.50 \text{ m/s})\cos 60^\circ}{0.050 \text{ kg}}$$

$$v_{Cx} = 1.75 \text{ m/s}$$

Similarly,

$$v_{Cy} = \frac{(0.100 \text{ kg})(0 \text{ m/s}) - (0.020 \text{ kg})(0 \text{ m/s}) - (0.030 \text{ kg})(-0.50 \text{ m/s})\sin 60^\circ}{0.050 \text{ kg}}$$

$$v_{Cy} = 0.26 \text{ m/s}$$

b)

$$\begin{aligned} \Delta K &= \frac{1}{2}(0.100 \text{ kg})(0.5 \text{ m/s})^2 - \frac{1}{2}(0.020 \text{ kg})(1.50 \text{ m/s})^2 - \frac{1}{2}(0.030 \text{ kg})(-0.50 \text{ m/s})^2 \\ &\quad - \frac{1}{2}(0.050 \text{ kg}) \times [(1.75 \text{ m/s})^2 + (0.26 \text{ m/s})^2] = -0.092 \text{ J} \end{aligned}$$

Problem 6 (SG): A ballistic pendulum: firing a bullet into a block of wood

7.13 (H) A rifle of mass 10 kg fires a 10 g bullet into a 3 kg block of wood which is suspended by a thin wire, forming a **ballistic pendulum**. The bullet remains embedded in the block of wood, which is observed to swing so that it reaches a vertical height of 20 cm above its starting point. What was the speed of the bullet, and with what speed did the rifle recoil?

Ignore air resistance, friction in the suspension, and the vertical motion of the bullet, and assume that the block of wood is small relative to the length of the wire (we will see why this is necessary in Chapter 8 of the *Study Guide*).

Answer: Let $M = 10 \text{ kg}$ be the mass of the rifle, $m = 3 \text{ kg}$ the mass of the block of wood, $\mu = 10 \text{ g}$ the mass of the bullet, and $\Delta y = 20 \text{ cm}$ the vertical height that the block of wood rises. The events can be described in three stages:

- 1) The rifle fires, projecting the bullet forward at an unknown speed v_1 , and the rifle recoils backwards at a speed v_2 determined by the conservation of momentum:

$$Mv_2 = \mu v_1 .$$

- 2) The bullet lodges in the wood. This is an inelastic process, since two objects stick together. Kinetic energy is lost to heat, but momentum is still conserved. The speed v_3 of the bullet/block system after the bullet and block start to move together is determined by momentum conservation:

$$\mu v_1 = (m + \mu)v_3 .$$

- 3) As the pendulum swings, gravity and the vertical component of the tension of the wire exert vertical (y) forces on the bullet/block system. The horizontal component of the tension exerts a horizontal (x) force on the bullet/block system. Therefore both the vertical and horizontal components of the momentum will be changed, but the mechanical energy of the bullet/block system is conserved — tension does no work, since the motion of the block is perpendicular to the force of tension, and the work done by gravity can be described as a difference of potential energies. So

$$\frac{1}{2}(m + \mu)v_3^2 = (m + \mu)g \Delta y .$$

That is, all of the kinetic energy that remains after the bullet and block start to move together is converted to gravitational potential energy.

So,

$$v_3 = \sqrt{2g \Delta y} ,$$

and then the speed of the bullet is

$$\begin{aligned} v_1 &= \frac{m + \mu}{\mu} v_3 = \frac{m + \mu}{\mu} \sqrt{2g \Delta y} \\ &= \frac{(3 + 0.01) \text{ kg}}{0.01 \text{ kg}} \sqrt{2 \times (9.8 \text{ m/s}^2) \times 0.20 \text{ m}} = \boxed{596 \text{ m/s}} . \end{aligned}$$

The recoil speed of the rifle is then

$$v_2 = \frac{\mu}{M} v_1 = \frac{0.01 \text{ kg}}{10 \text{ kg}} 596 \text{ m/s} = \boxed{0.596 \text{ m/s}} .$$

Problem 7 (Y&F): A head-on collision of two cars

8.32: (a) Momentum conservation tells us that both cars have the same change in momentum, but the smaller car has a greater velocity change because it has a smaller mass.

$$\begin{aligned} M\Delta V &= m\Delta v \\ \Delta v \text{ (small car)} &= \frac{M}{m} \Delta V \text{ (large car)} \\ &= \frac{3000 \text{ kg}}{1200 \text{ kg}} \Delta V = 2.5\Delta V \text{ (large car)} \end{aligned}$$

(b) The occupants of the small car experience 2.5 times the velocity change of those in the large car, so they also experience 2.5 times the acceleration. Therefore they feel 2.5 times the force, which causes whiplash and other serious injuries.

[Note by Alan Guth: The above solution, and many of the solutions to the Young and Freedman problems, was taken from the “Instructor Solutions Manual” written by Prof. A. Lewis Ford. These solutions have a different font from the others, so you can tell them apart if you want to. When Prof. Ford concluded that the small car would undergo 2.5 times the acceleration, he was tacitly assuming that the interiors of both cars would undergo their velocity change in the same amount of time. This is no doubt approximately true, but there can be differences caused by the way in which the cars crumble under pressure.]

Problem 8 (Y&F): The beta decay of ^{210}Bi and the antineutrino.

8.89: The “missing momentum” is

$$5.60 \times 10^{-22} \text{ kg} \cdot \text{m/s} - (3.50 \times 10^{-25} \text{ kg})(1.14 \times 10^3 \text{ m/s}) = 1.61 \times 10^{-22} \text{ kg} \cdot \text{m/s} .$$

Since the electron has momentum to the right, the neutrino’s momentum must be to the left.

Problem 8-alternate (Y&F): The decay of ^{232}Th : Recoil kinetic energy. (This problem was mistakenly assigned in the original version of the problem set, so I am including the solution for those students who did this problem instead of Y&F 8.89.)

8.88: The ratios that appear in Eq. (8.42) are $\frac{0.0176}{1.0176}$ and $\frac{1}{1.0176}$, so the kinetic energies are

$$\text{a) } \frac{0.0176}{1.0176} (6.54 \times 10^{-13} \text{ J}) = 1.13 \times 10^{-14} \text{ J} \text{ and b) } \frac{1}{1.0176} (6.54 \times 10^{-13} \text{ J}) = 6.43 \times 10^{-13} \text{ J}.$$

Note that the energies do not add to $6.54 \times 10^{-13} \text{ J}$ exactly, due to roundoff.

Center of Mass:

Problem 9 (Y&F): The center of mass of the Sun-Jupiter system

8.45: Measured from the center of the sun,

$$\frac{(1.99 \times 10^{30} \text{ kg})(0) + (1.90 \times 10^{27} \text{ kg})(7.78 \times 10^{11} \text{ m})}{1.99 \times 10^{30} \text{ kg} + 1.90 \times 10^{27} \text{ kg}} = 7.42 \times 10^8 \text{ m}.$$

The center of mass of the system lies outside the sun.

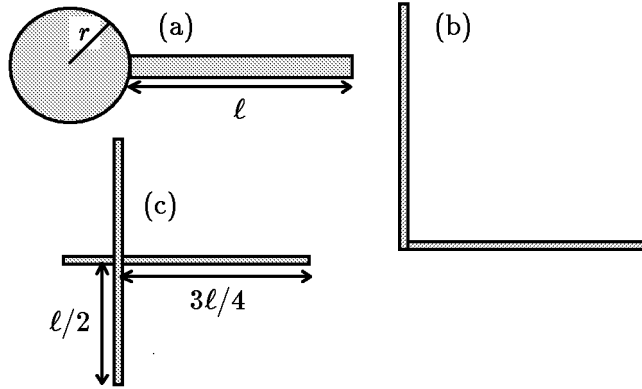
Problem 10 (SG) STUDY: Two skaters pulling themselves together with a rope

5C.2 (S) A girl is teaching her younger brother to skate by towing him around on a rope. They finish their practice session by hauling in on the rope from each end until they meet. If her mass is 40 kg, his is 30 kg, and the rope is 5 m long, how far from her original position will they end up? Assume that they were stationary to begin with, that the rope has negligible mass, and that there is no friction.

Answer: See complete solution in the *Study Guide*.

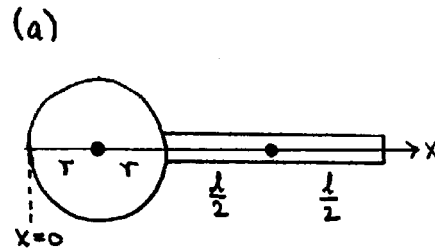
Problem 11 (SG): Center of mass of combinations of shapes

5C.4 (H) Find the position of the center of mass of (a) a sphere of mass m and radius r attached to a rod of mass m and length ℓ ; (b) two rods of mass m and length ℓ joined at right angles; (c) two rods of mass m and length ℓ crossed as shown in the diagram.



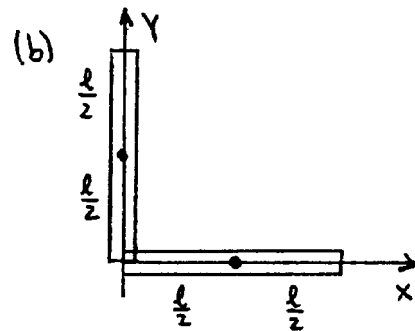
Answer: (a) The center of mass of the sphere is on the x-axis at $x = r$, and the center of mass of the rod is at $x = 2r + \ell/2$. The center of mass of the whole system therefore is at

$$x_{cm} = \frac{Mr + M(2r + \ell/2)}{M + M} = \frac{3}{2}r + \frac{\ell}{4}.$$



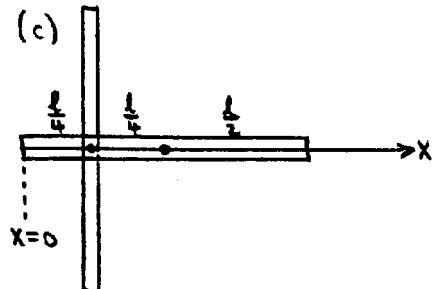
(b) The center of mass of rod 1 is at $\vec{r}_1 = [x_1, y_1] = [\ell/2, 0]$, and the center of mass of rod 2 is at $\vec{r}_2 = [x_2, y_2] = [0, \ell/2]$. The center of mass of the whole system therefore is at

$$\begin{aligned} \vec{r}_{cm} &= \frac{M\vec{r}_1 + M\vec{r}_2}{M + M} = \frac{1}{2}(\vec{r}_1 + \vec{r}_2) \\ &= [\ell/4, \ell/4]. \end{aligned}$$



(c) The center of mass of rod 1 is on the x-axis at $x = \ell/2$, and the center of mass of rod 2 is on the x-axis at $x = \ell/4$. Hence, the center of mass of the whole system is at

$$x_{cm} = \frac{M\ell/2 + M\ell/4}{M + M} = \frac{3\ell}{8}.$$



Problem 12 (SG): Two balls, a string, and a table

- 7.19 (H) Two identical small balls, A and B, are connected by a string of length ℓ and negligible mass. Ball A is placed on a frictionless table (with a frictionless edge), and B is held a distance $\ell/2$ from the edge of the table so that the string is horizontal and just taut.

If mass B is now released, will it hit the side of the table before A falls off the edge, or vice versa?

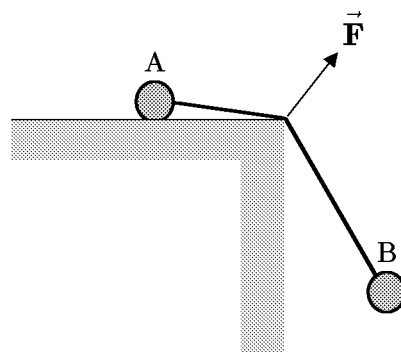
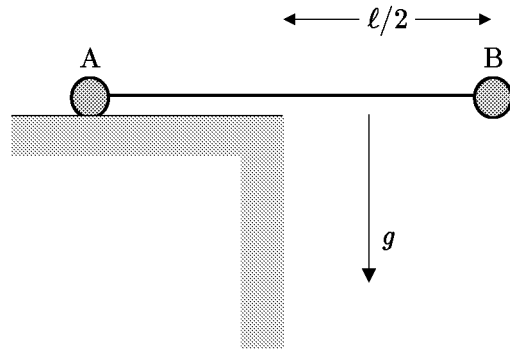
Explain your reasoning clearly!

Note: This problem can be solved by reasoning. It is *not* necessary to solve the equations of motion explicitly.

Answer: The reasoning is simple if one focuses attention on the center of mass, remembering that

$$M_{\text{tot}} \frac{d^2 \vec{r}_{\text{cm}}}{dt^2} = \vec{F}_{\text{tot}}^{\text{ext}}.$$

That is, the center of mass moves as if it were a point particle with a mass equal to the total mass, acted upon by the total force. We focus on the system that includes the two balls and the string that connects them. Since A and B have the same mass, and the string has negligible mass, the center of mass is located exactly midway between the two balls. The center of mass starts out, therefore, right at the edge of the table, at rest. Gravity and the normal force from the top surface of the table are both vertical forces. The only horizontal force on the system comes from the edge of the table. Once ball B starts to fall, so that the string makes an angle at the edge, the normal force at the edge will include a horizontal component to the right. The center of mass, therefore, will move to the right, and will therefore always be to the right of the edge of the table. So when ball B hits the side of the table, ball A must be to the right of the table, because otherwise the center of mass would not be to the right of the edge of the table. Conclusion: ball A falls off the edge of the table **BEFORE** ball B strikes the side of the table.

**Impulse:****Problem 13 (Y&F): Orbital maneuvering system on the space shuttle**

8.10: a) $\vec{F} = (1.04 \times 10^5 \text{ kg} \cdot \text{m/s}) \hat{j}$. b) $(1.04 \times 10^5 \text{ kg} \cdot \text{m/s}) \hat{j}$.

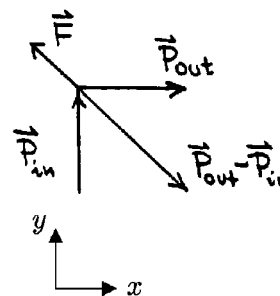
c) $\frac{(1.04 \times 10^5 \text{ kg} \cdot \text{m/s})}{(95,000 \text{ kg})} \hat{j} = (1.10 \text{ m/s}) \hat{j}$. d) The initial velocity of the shuttle is not known; the change in the square of the speed is not the square of the change of the speed.

Rockets and Other Momentum Exchange Problems:**Problem 14 (SG): Reaction force of the water on a fire hydrant**

5A.4 (H) A fire hydrant delivers water at a volume flow rate (i.e. volume/time) L . The water travels vertically upwards through the hydrant at speed v and then does a 90° turn to emerge horizontally at the same speed v . Assuming the pipe and nozzle have uniform cross-sections throughout, obtain an expression for the force exerted by the water on the corner of the hydrant. If the rate of delivery is $L = 800$ liters per minute at $v = 25$ m/s, what is the magnitude of the force that the structure of the hydrant has to withstand? What is the direction of that force? (One liter of water has a mass of one kilogram.)

Answer: The force is given by the rate of change of momentum. As water flows through the hydrant, its momentum changes its direction by 90 degrees but not its magnitude. The force that the hydrant exerts on the water is the rate of change of the momentum of the water per time, and the force $\vec{\mathbf{F}}$ that the water exerts on the hydrant is equal in magnitude and opposite in direction to that. Consider a time interval Δt , and let $\vec{\mathbf{P}}_{\text{in}}$ and $\vec{\mathbf{P}}_{\text{out}}$ denote the momentum of the water that enters and leaves the hydrant during this time, respectively. Thus,

$$\vec{\mathbf{F}} = -\frac{\vec{\mathbf{P}}_{\text{out}} - \vec{\mathbf{P}}_{\text{in}}}{\Delta t},$$



where the minus sign is included because $\vec{\mathbf{F}}$ is the force on the hydrant, not on the water. If the volume per unit time of the water flow is L ($L = 800$ liters/minute), and the density of the water (mass per unit volume) is ρ ($\rho = 1$ kg/liter), then the mass of the water passing in a time Δt is $\Delta M = \rho L \Delta t$. Taking the speed of the water as v ($v = 25$ m/s),

$$\vec{\mathbf{P}}_{\text{in}} = [0, \Delta M v, 0] = [0, \rho L v \Delta t, 0]$$

$$\vec{\mathbf{P}}_{\text{out}} = [\Delta M v, 0, 0] = [\rho L v \Delta t, 0, 0]$$

Thus,

$$\vec{\mathbf{F}} = -\frac{[\rho L v \Delta t, 0, 0] - [0, \rho L v \Delta t, 0]}{\Delta t} = [-\rho L v, \rho L v, 0].$$

The magnitude of the force is therefore

$$|\vec{\mathbf{F}}| = \sqrt{\rho^2 L^2 v^2 + \rho^2 L^2 v^2} = \sqrt{2} \rho L v.$$

The direction, as can be seen from the diagram, is at an angle of 45° with respect to the horizontal, where the horizontal component is opposite the direction in which the water is being ejected. Numerically,

$$|\vec{\mathbf{F}}| = \sqrt{2} \times \frac{1 \text{ kg}}{\text{liter}} \times 800 \frac{\text{liter}}{\text{minute}} \times \frac{1 \text{ minute}}{60 \text{ s}} \times 25 \frac{\text{m}}{\text{s}} = 470 \text{ N},$$

where the answer has been rounded to 2 significant figures.

Problem 15 (Y&F): A rocket near the surface of the Earth

8.104: a) Including the extra force, Eq. (8.37) becomes

$$m \frac{dv}{dt} = -v_{\text{ex}} \frac{dm}{dt} - mg,$$

where the positive direction is taken upwards (usually a sign of good planning). b) Dividing by a factor of the mass m ,

$$a = \frac{dv}{dt} = -\frac{v_{\text{ex}}}{m} \frac{dm}{dt} - g.$$

c) $20 \text{ m/s}^2 - 9.80 \text{ m/s}^2 = 10.2 \text{ m/s}^2$. d) $3327 \text{ m/s} - (9.80 \text{ m/s}^2)(90) = 2.45 \text{ km/s}$, which is about three-fourths the speed found in Example 8.17.