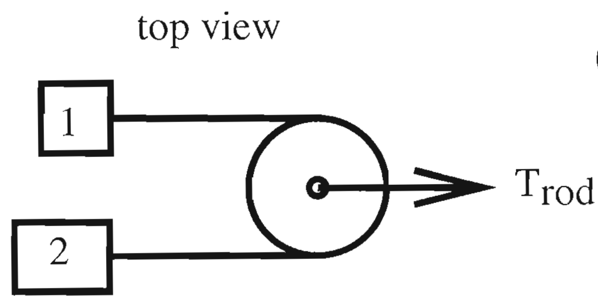
Problem 1. Two masses $m_2 > m_1$ slide on a horizontal table with coefficients of static and kinetic friction both equal to μ. They are attached to a massless string that loops around a massless pulley. The figure at right shows the view from above. This system is pulled to the right by a massless rod with tension T_{rod}.

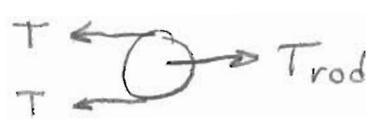


a) Explain in *words* why a certain minimum tension in the rod is required in order for *both* masses to accelerate. You may want to draw one or two free body diagrams.

Tension needs to be larger than friction. Static friction has a maximum value.

COMMENTS

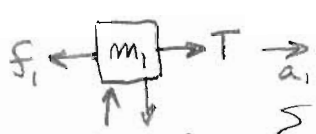
b) What is the minimum tension in the rod such that *both* masses accelerate?



Mass of pulley $= 0$

$\Rightarrow T_{rod} - 2T = 0$

$T_{rod} = 2T$



$\sum F_y = 0 \Rightarrow N_1 = m_1 g$

$f_1 \le \mu N_1 = \mu m_1 g$

$\sum F_x = T - f_1 = m_1 a_1$

$a_1 \neq 0$ if $T > \mu m_1 g$

Same picture for m_2:

$a_2 \neq 0$ if $T > \mu m_2 g$

$m_2 > m_1$ so both accelerate if

$\boxed{T_{rod} > 2\mu m_2 g}$

COMMENTS

$\sum \vec{F} = m\vec{a}$

Free body diagrams

Same tension on two sides of pulley

Static friction has an upper limit

Different values of T to accelerate each mass, which is bigger?

c) The acceleration of the pulley, a_p is not in general equal to either of the accelerations of the masses, a_1 and a_2. How *does* the acceleration of the pulley depend upon the accelerations of the two masses? Explain why your answer makes quantitative sense.

COMMENTS

If m_1 moves distance x_1 and m_2 moves distance x_2, there is a length x_1+x_2 of string available to move the pulley. But, need to add same amount of string on both sides of pulley. So, pulley moves $x_p = \frac{x_1+x_2}{2}$

Accel $= \frac{d^2x}{dt^2}$

$$\Rightarrow \boxed{a_p = \frac{a_1+a_2}{2}}$$

Physical constraint: Length of string is constant

Problem 2. In the figure at right mass M slides without friction on a horizontal surface. The coefficient of friction between m and M is μ. The spring, which has spring constant k, is stretched and then the two masses are released.

a) Explain in *words* why mass m will slip with respect to mass M if the spring is stretched too much.

On release, if slippage does not occur both masses have the same acceleration, which is proportional to the extension of the spring. To provide this acceleration to the top block the friction force must be equal to ma. However the friction force cannot exceed μmg. This limits a to μg and therefore limits the extension of the spring.

COMMENTS

b) What is the maximum amount by which the spring can be stretched such that mass m does not slide with respect to mass M?

$$(m+M)a = kX \quad (1)$$

$$m a_{max} = \mu mg$$

Since $a \leq a_{max}$

Thus $X < \dfrac{(m+M)a_{max}}{k}$

or $\boxed{X_{max} = \dfrac{\mu g (M+m)}{k}}$

COMMENTS

(1) Eqn of motion of 2 blocks together without slippage

(2) Eqn of motion of top block

c) What is the maximum speed the ensemble achieves when the spring is stretched the maximum distance found in part b)?

COMMENTS

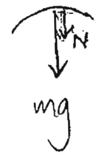
c) $\frac{1}{2} k x_m^2 = \frac{1}{2}(m+M) V_{max}^2$

$\therefore V_{max} = \mu g \sqrt{\frac{M+m}{k}}$

all potential energy becomes kinetic energy of both blocks.

7-6) 7.46

(a) At point B, the car experiences gravity straight down and (possibly) a normal force pushing down.



$a = \frac{v^2}{R}$ mg

For minimum energy, the normal force goes to zero,

so $mg = \frac{mv_B^2}{R}$ —(1)

From conservation of energy from point A to B,

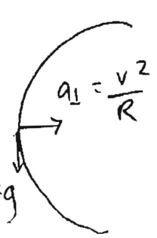
$$mgh_A = mgh_B + KE_B$$
$$= mg(2R) + \frac{1}{2}mv_B^2$$

use (1):
$$= mg\,2R + \frac{1}{2}mRg = \frac{5}{2}mRg$$
$$\Rightarrow h_A = \frac{5}{2}R$$

Goal: Use energy conservation on a loop-the-loop.

Since the normal force always pushes out against the car at the point of contact, at the top of the loop it has to push down.

(b)



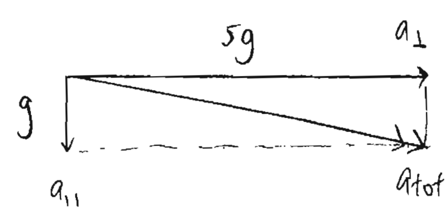
$a_\perp = \frac{v^2}{R}$

$a_\parallel = g$

$$\Delta KE = \Delta U$$
$$\frac{1}{2}mv^2 = (h-R)mg$$
$$\frac{1}{2}v^2 = 2.5Rg$$
$$v = (5Rg)^{1/2} =$$

radial acceleration $= \frac{v^2}{R}$
$= 5g = 49\,ms^{-2}$

tangential acceleration $= g = 9.8\,ms^{-2}$



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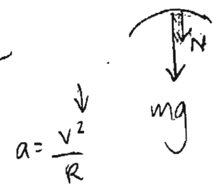

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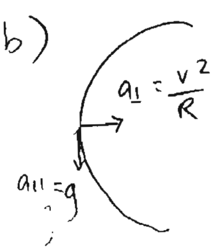
(b)



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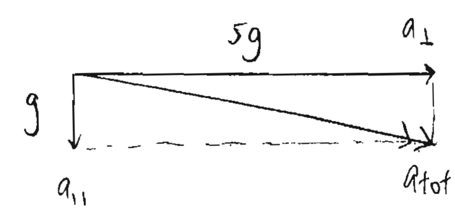


Problem 4. An object moving under the influence of a conservative force has potential energy as a function of postion given by $U(x) = C(x-2a)^2(x-4a)^2$ where a is a constant measured in meters and C is a constant. The derivative of the potential with respect to position is given by
$dU/dx = 4C(x-2a)(x-3a)(x-4a)$.

a) What are the units of C?

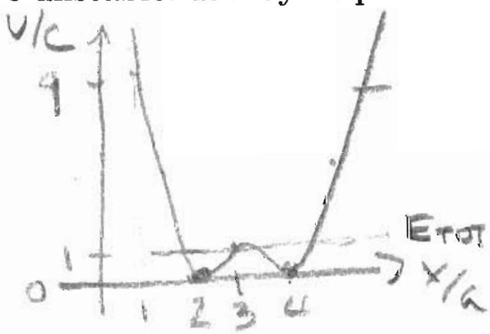
$[U] = \frac{kg\,m^2}{s^2}$ $\quad [C] = \frac{[U]}{[a^4]} = \frac{kg\,m^2/s^2}{m^4}$

$[C] = \frac{kg}{m^2 s^2}$

COMMENTS

b) Find the equilibrium positions of the object, and indicate which are stable and which are unstable. It may help to sketch the potential energy.

U/c	x/a
64	0
9	1
0	2
1	3
0	4
9	5
64	6



$dU/dx = 0$

$4C(x-2a)(x-3a)(x-4a) = 0$

Zeroes at $x=2a$, $x=3a$ & $x=4a$

equilibria

$x=2a$ & $x=4a$ are minima

∴ stable

$x=3a$ a maximum

∴ unstable

COMMENTS

equilibria when $\frac{dU}{dx} = 0$

On either side of 2a force $-\frac{dU}{dx}$ points back to 2a.

On either side of 3a force points away from 3a

c) What is the minimum *total* energy the object must have such that if it starts at $x < 2a$ it will at some later time be at $x > 4a$?

COMMENTS

If $E_{TOT} < Ca^4$, a particle on the left has too little energy to reach the maximum at $x = 3a$