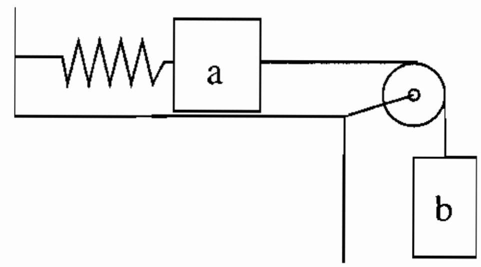
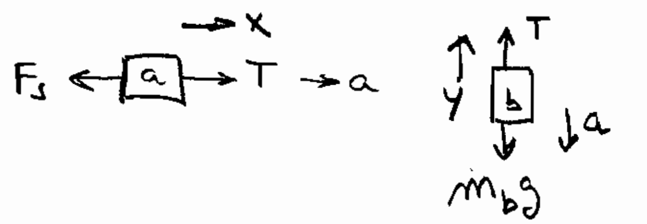
Problem 1. Two masses, m_a and m_b are connected by a massless string looped over a massless pulley. Mass m_a slides along a frictionless surface and is also connected to a massless spring with constant k. Mass m_a is held in place at point where the spring is unstretched and is then suddenly released.



a) Find the tension in the string just after the mass is released.



$$T - F_s = m_a a \qquad T - m_b g = -m_b a$$

$$a = \frac{T - F_s}{m_a} \qquad T - m_b g = -m_b\left(\frac{T - F_s}{m_a}\right)$$

$$T\left(1 + \frac{m_b}{m_a}\right) = m_b\left(g + \frac{F_s}{m_a}\right)$$

$$T\left(\frac{m_a + m_b}{m_a}\right) = m_b\left(g + \frac{F_s}{m_a}\right)$$

$$T = \frac{m_a m_b}{m_a + m_b}\left(g + \frac{F_s}{m_a}\right)$$

In this case, spring is unstretched so $F_s = 0$

$$T = \frac{m_a m_b g}{m_a + m_b}$$

COMMENTS

Free body diagrams

$\vec{F} = m\vec{a}$

Solve one eq for a
Plug into other eq to find T

Limits: $m_a \to \infty$ no accel and $T = m_b g$

$m_b \to \infty$ it falls with accel equal to g and $T = m_a g$

If either mass is zero $T \to 0$

b) Mass m_b falls and the spring stretches a maximum distance D at which point the direction of motion reverses. Find the tension in the string at the instant that the motion reverses.

Now $F_s = kD$

$$T = \frac{m_a m_b}{m_a + m_b}\left(g + \frac{kD}{m_b}\right)$$

Note that you could find D from Work & Energy

$PE_I = m_b g D \quad PE_F = \frac{1}{2} k D^2$

$KE_I = KE_F = 0 \quad W_{ext} = 0$

$\frac{1}{2} k D^2 = m_b g D \quad D = \frac{2 m_b g}{k}$

$$T = \frac{m_a m_b g + 2 m_b^2 g}{m_a + m_b}$$

COMMENTS

Use solution to part (a) but now with spring force

Limits: $k \to 0$ spring has no effect so the answer is the same as part (a)

Limits: $m_a \to 0$ $T \to 2 m_b g$ which is just the spring force

$m_b \to 0$ no acceleration and $T \to 0$

c) Suppose that the position of mass m_a is gently adjusted until the system is in its equilibrium configuration. By how much is the spring stretched?

$T - F_s = 0 \quad T - m_b g = 0$

$F_s = kx = m_b g$

$$x = \frac{m_b g}{k}$$

Alternative: Equilibrium point must be ½ way between release and lowest point so

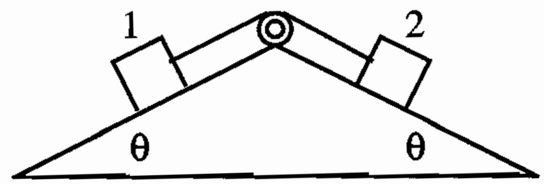
$$x = \frac{D}{2} = \left(\frac{2 m_b g}{k}\right)\frac{1}{2} = \frac{m_b g}{k}$$

COMMENTS

No acceleration at equilibrium point

solve for X

Problem 2. Two blocks, m_1 and m_2 are connected by a massless string looped over a massless pulley. They slide along a double incline whose two sides make angle θ to the horizontal. The coefficient of friction between each of the blocks and the incline is μ. The mass of block m_2 is greater than that of m_1. You may take the coefficients of sliding and static friction to be equal.

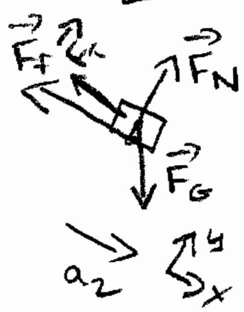


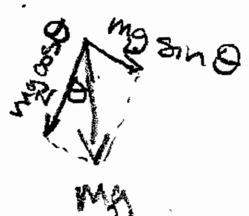
a) Assuming that the blocks *do* accelerate, find the accleration of m_2

M_2

$|\vec{F}_T| = T \quad |\vec{F}_G| = m_2 g$

$|\vec{F}_f| = f \quad |\vec{F}_N| = N$





Newton's 2nd

y component $m_2 a_y = N - m_2 g\cos\theta$ ($a_y = 0$)

$N = m_2 g \cos\theta$

$f = \mu N$ if sliding

$= \mu m g \cos\theta$

x component

both uphill

$m_2 a_2 = m_2 g \sin\theta - T - \mu m g\cos\theta$

$m_1 a_1 = T - m_1 g\sin\theta - \mu m_1 g \cos\theta$ ($a_1 = a_2$)

both downhill

$(m_2 + m_1) a_2 = (m_2 - m_1) g \sin\theta - \mu (m_2 + m_1) g\cos\theta$

$$a_2 = \frac{(m_2 - m_1)\sin\theta - \mu(m_2 + m_1)\cos\theta}{m_1 + m_2}\, g$$

dimensionless

acceleration of gravity

COMMENTS

Find: a_2

Use: Newton's 2nd law

Draw: Free body diagram

Choose coordinates:

take a_2 downhill

take a_1 uphill

constraint: $a_1 = a_2$

decompose $\vec{F}_g$ into components

if m_2 moves downhill, friction opposes motion.

if m_1 moves uphill, friction opposes motion

limiting cases:

$\mu \to 0$ $a_2 \propto (m_2 - m_1)$;

$m_1 = m_2$ a_2 negative.

and friction slows it down;

$\mu \to 0$ & $m_1 \to 0$

$a_2 = g\sin\theta$

b) Taking m_1 to be given, find the minimum value of m_2 required for the blocks to accelerate.

COMMENTS

$$0 = \frac{(m_2 - m_1)\sin\theta - \mu(m_2 - m_1)\cos\theta}{m_1 + m_2} g$$

gravity friction

$$(m_2 - m_1)\sin\theta = \mu(m_1 + m_2)\cos\theta$$

$$m_{2\,min}(\sin\theta - \mu\cos\theta) = m_1(\sin\theta + \mu\cos\theta)$$

$$m_{2\,min} = \underbrace{\frac{\sin\theta + \mu\cos\theta}{\sin\theta - \mu\cos\theta}}_{\text{dimensionless}} m_1$$

$$= \underbrace{\frac{\tan\theta + \mu}{\tan\theta - \mu}}_{>1 \text{ if } \mu \neq 0} m_1$$

$m_2 > m_1$

For blocks to accelerate down hill, $a_2 > 0$.

Minimum value of m_2 will give $a_2 = 0$

limiting cases

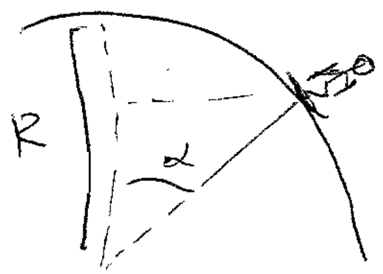
$\mu = 0$ $m_2 = m_1$; makes sense

$\tan\theta = \mu$ $m_2 \to \infty$

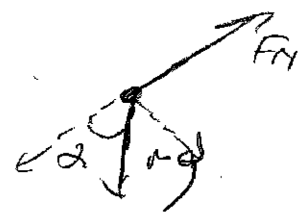
makes sense; gravity on m_2 never overcomes friction

7-7.) Y&F 7.63

Commentary



Losing contact means $F_N = 0$



$$mg\cos\alpha - F_N = \frac{mv^2}{R}$$

Because we're in circular motion

What is v? Use energy conservation

$$mg2R = mgR(1+\cos\alpha) + \tfrac{1}{2}mv^2$$

$$mgR = mgR\cos\alpha + \tfrac{1}{2}mv^2$$

$$gR(1-\cos\alpha) = \frac{v^2}{2}$$

$$\frac{mv^2}{R} = 2mg(1-\cos\alpha)$$

$$mg\cos\alpha - F_N = 2mg(1-\cos\alpha)$$

to lose contact, set $F_N = 0$

$$mg\cos\alpha = 2mg(1-\cos\alpha)$$

$$3\cos\alpha = 2$$

$$\cos\alpha = 2/3$$

$$\boxed{\alpha = \arccos(2/3)}$$

QQ!!

Goal: In a classic problem, find the angle α at which a mass sliding down a frictionless sphere loses contact with the sphere.

As usual, we set the sum of forces in the radial direction to the centripetal force.

Now, the skier loses contact when F_N (the normal... or "contact"... force) is zero.

We find the v in our eqn by energy conserv., setting the zero of grav PE to be the ground.

This is a cool result.

Problem 4. An object with mass m moves in one dimension, x under the influence of a consrvative force given by $F = -(4Px^3 - 2Qx)$ where both P and Q are positive.

a) What are the units of P and Q?

$$[P]\cdot[m^3] = [N] \Rightarrow [P] = \left[\frac{kg}{s^2\cdot m^2}\right]$$

$$[Q]\cdot[m] = [N] \Rightarrow [Q] = \left[\frac{kg}{s^2}\right]$$

COMMENTS

Each term should have units of Force $[N]$ or $\left[\frac{m\cdot kg}{s^2}\right]$

b) Find the potential energy of the mass as a function of its position.

$$U(x) = \underbrace{U(x_0)}_{\text{constant}} - \int_{x_0}^{x} F(x')\,dx'$$

Simplify by choosing $x_0 = 0$

$$U(x) = U(0) - \int_0^x -(4Px'^3 - 2Qx')\,dx'$$

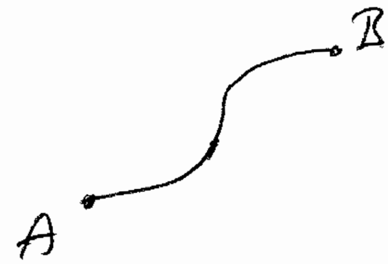
$$U(x) = U(0) + Px^4 - Qx^2$$

or

$$U(x) = Px^4 - Qx^2 + C$$

↑ constant

COMMENTS



$U(B) = U(A) - \left(\text{work done by force}\right)$ by definition

defined up to a constant

c) Find a position at which the mass is in stable equilibrium.

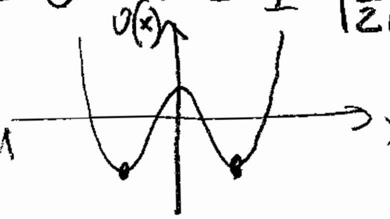
$-(4Px^3-2Qx)=0$

$-2x(2Px^2-Q)=0$

three equilibrium positions:

$x=0 \quad x=\pm\sqrt{\frac{Q}{2P}}$

Plot potential



$\Rightarrow$ Stable equilibrium at $x=\pm\sqrt{\frac{Q}{2P}}$

COMMENTS

Equilibrium when $F(x)=0$

Stable when $U(x)$ has a minimum

Can also use $\frac{d^2U}{dx^2}$ and check for concave

d) Find the work done by the force on the object as it moves from $x = 0$ to $x = \sqrt{\frac{Q}{2P}}$.

$U(x)-U(0)=-\text{Work}$

$\text{Work}=U(0)-U(x)$

$W=0-\left(P\left(\sqrt{\frac{Q}{2P}}\right)^4-Q\cdot\left(\sqrt{\frac{Q}{2P}}\right)^2\right)$

$W=-\frac{PQ^2}{4P^2}+\frac{Q\cdot Q}{2P}=-\frac{Q^2}{4P}+\frac{Q^2}{2P}$

$W=+\frac{Q^2}{4P}$

COMMENTS

Work can be calculated easily if we know potential energy

$W=U(A)-U(B)$

Sign is OK we are moving from maximum to minimum force does work on the mass $W>0$!