Problem 1. Young and Freedman 3.15

Inside a starship at rest on the earth, a ball rolls off the top of a horizontal table and lands a distance D from the foot of the table. The starship now lands on the unexplored Planet X. The commander, Captain Curious, rolls the same ball off the same table with the same initial speed as on earth and finds that it lands a distance 2.76D from the foot of the table. What is the acceleration due to gravity on Planet X?

Call the height of the table h and the initial horizontal velocity $v_{0,x}$. (See commentary at right.) As usual, we chose our coordinate system such that the motion in the two directions is only coupled through the time t that the ball hits the ground. Let's carry out the analysis on earth first. In the vertical direction, we have $\frac{1}{2}g_{earth}t_{earth}^2 = h$, which gives

$$t_{earth} = \sqrt{\frac{2h}{g_{earth}}} \qquad (1)$$

. Since there is no acceleration in the horizontal direction, the horizontal distance the ball travels is

$$D = v_{0,x}t_{earth} = v_{0,x}\sqrt{\frac{2h}{g_{earth}}} \qquad (2)$$

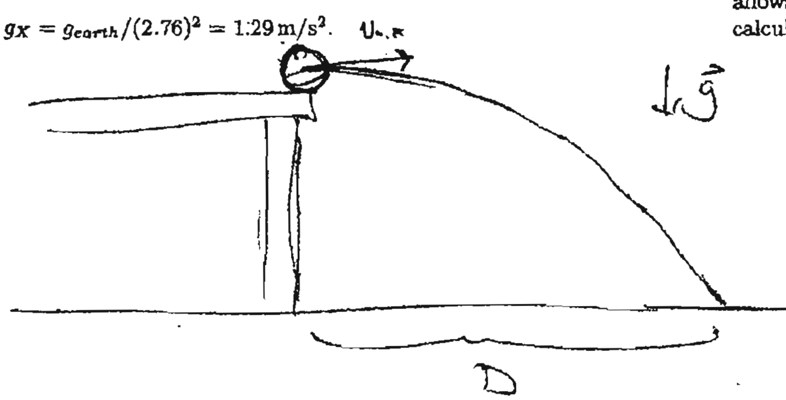
. However, a similar analysis holds for Planet X:

$$2.76D = v_{0,x}t_X = v_{0,x}\sqrt{\frac{2h}{g_X}} \qquad (3)$$

. Dividing these last two equations, we have

$$2.76 = \sqrt{\frac{g_{earth}}{g_X}} \qquad (4)$$

so $g_X = g_{earth}/(2.76)^2 = 1.29\,\mathrm{m/s^2}$.



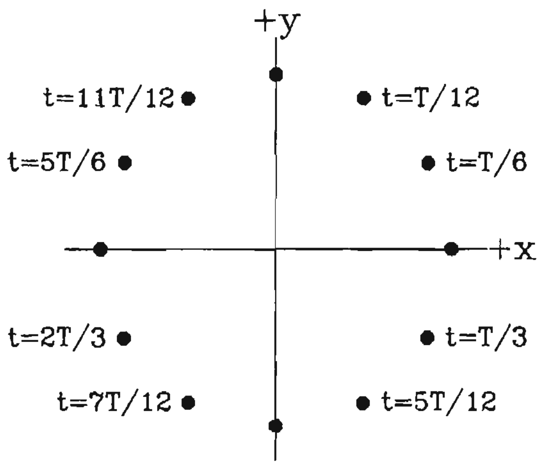
Commentary

Goal: To compare two experiments conducted in different gravitional fields to determine the value of acceleration due to gravity in the second location.

The first thing to note in this problem is what we're given... and what we're not. We are given a distance D, but not its numerical value, though the problem asks for a numerical answer for g_x. D, then, should drop out of our final answer in some fashion. We're also given $2.76D$ as the distance the ball rolls on Planet X. This suggests relating the dimensionless ratio $2.76D/D = 2.76$ to some dimensionless ratio comparing the strength of gravity on the two planets.

Likewise, we're told that the same table is used in both experiments, and the ball rolls off with the same initial horizontal velocity. Call the height of the table h, and the horizontal velocity $v_{0,x}$. These likewise should drop out of our final answer, but identifying and naming these parameters allows us to get a grip on our calculation.

Problem 2. An object travels in uniform circular motion along a path with circumference C, as shown in the figure below, and period T.



a) What is the object's instantaneous vector velocity $\vec{v}$ at time $t = 0$? (In this and all subsequent parts, comment to the right of the dashed line).

$|\vec{v}| = \frac{C}{T}$

$\vec{v} = \frac{C}{T}\hat{i}$

speed = $\left(\frac{\text{circumference}}{\text{period}}\right)$

velocity is tangent to circle

units: $\left[\frac{C}{T}\right] = \frac{m}{s}$

b) What is the object's instantaneous vector velocity $\vec{v}$ at time $t = T/2$?

$\vec{v} = -\frac{C}{T}\hat{i}$

@ $t = T/2$ directed opposite to $t = 0$

c) What is the object's average vector velocity $< \vec{v} >$ between $t = 0$ and $t = T/2$?

$\langle \vec{v} \rangle = \frac{\vec{r}_2 - \vec{r}_1}{\Delta t}$ $\vec{r}_1 = R\hat{j}$, $\vec{r}_2 = -R\hat{j}$

$\langle \vec{v} \rangle = \frac{-2R\hat{j}}{(T/2 - 0)}$; $R = \frac{C}{2\pi}$

$\langle \vec{v} \rangle = -\frac{2C}{\pi T}\hat{j}$

$\left(\frac{\text{average}}{\text{velocity}}\right) = \left(\frac{\text{displacement}}{\text{elapsed time}}\right)$

(definition!)

(cannot use constant acceleration formula)

$\langle \vec{v} \rangle \neq \frac{\vec{v}_1 + \vec{v}_2}{2}$

units: $\left[\frac{C}{T}\right] = \frac{m}{s}$

d) What is the object's average vector acceleration $<\vec{a}>$ between $t = 0$ and $t = T/2$?

$$<\vec{a}> = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

$$= \frac{-\frac{C}{T}\hat{i} - \frac{C}{T}\hat{i}}{T/2} = -\frac{4C}{T^2}\hat{i}$$

$$\left(\begin{matrix}\text{average}\\\text{acceleration}\end{matrix}\right) = \left(\frac{\text{change in velocity}}{\text{elapsed time}}\right)$$

use parts (a) & (b)

$$<\vec{a}> \neq \frac{\vec{a}_1 + \vec{a}_2}{2}$$

units: $\left[\frac{C}{T^2}\right] = \frac{m}{s^2}$

e) What is the object's instantaneous acceleration $\vec{a}$ at time $t = 0$?

$$|\vec{a}| = \frac{v^2}{R} = \frac{(C/T)^2}{C/2\pi} = \frac{2\pi C}{T^2}$$

inward or $-\hat{j}$

units $\left[\frac{C}{T^2}\right] = \frac{m}{s^2}$

Now consider a second "primed" coordinate frame x', y' moving with vector velocity $-\frac{C}{T}\hat{i}$ with respect to the frame shown.

f) What is the object's instantaneous vector velocity $\vec{v}'$, with respect to the primed, x', y' frame at $t = T/2$?

$$\vec{v}_{object,new} = \vec{v}_{object,old} + \vec{v}_{old,new}$$

$$= \vec{v}_{object,old} - \vec{v}_{new,old}$$

$$= -\frac{C}{T}\hat{i} - \left(-\frac{C}{T}\hat{i}\right) = 0$$

they cancel; imagine running alongside & seeing no velocity

g) What is the object's instantaneous vector acceleration $\vec{a}'$, with respect to the primed, x', y' frame at $t = T/2$?

$$\vec{a}_{object,new} = \vec{a}_{object,old} + \vec{a}_{old,new}$$

0, not accelerating

$$\vec{a}' = \frac{2\pi C}{T^2}\hat{j} \text{ or upward}$$

same as part (e) but opposite direction

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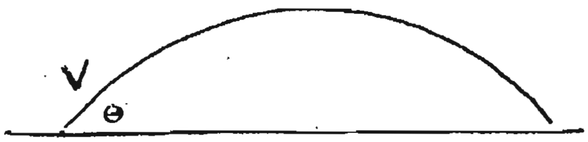
Problem 3. A projectile is launched toward the east at angle θ to the ground. At the same instant a rocket sled starting at rest at the same point accelerates along the ground toward the east with constant acceleration. Find the value of that accleration that causes the projectile to hit the sled when it returns to ground level.



projectile

$$X_1 = V\cos\theta\, t$$

$$y_1 = V\sin\theta\, t - \tfrac{1}{2}gt^2$$

$$0 = V\sin\theta\, t - \tfrac{1}{2}gt^2$$

$$V\sin\theta\, t = \tfrac{1}{2}gt^2$$

$$t = \frac{2V\sin\theta}{g}$$

$$X_1 = \frac{2V^2\sin\theta\cos\theta}{g}$$

sled

$$X_2 = \tfrac{1}{2}at^2$$

$$X_1 = X_2 \quad \text{when } y_1 = 0$$

$$\tfrac{1}{2}a\left(\frac{2V\sin\theta}{g}\right)^2 = \frac{2V^2\sin\theta\cos\theta}{g}$$

$X_1 = X_2$

$$\tfrac{1}{2}at^2 = V\cos\theta\, t \qquad a = \frac{2V\cos\theta}{t}$$

$$a = 2V\cos\theta\left(\frac{g}{2V\sin\theta}\right)$$

$$a = g\frac{\cos\theta}{\sin\theta} = g\cot\theta = \frac{g}{\tan\theta}$$

COMMENTS

projectile motion

Hits ground when $y = 0$

Find ~~t~~ t, plug in to find x

Constant acceleration

Want projectile & sled at same x when $y = 0$

simpler algebra

Solve for a

Checks: a has correct units

$\theta \to 90°$ $a \to 0$ because projectile goes straight up and down