Physics 8.01 L Pset # 1 - Solutions

Q 1-1) 2.10

Goal: Identify velocity from displacement graph

$v = \frac{dx}{dt}$, so slope of graph is velocity

(a) zero slope at $\boxed{IV}$

(b) constant velocity where slope is straight line (does not change) and positive : $\boxed{I}$

(c) constant and negative : $\boxed{V}$

(d) increasing where slope jumps from less steep to more steep : $\boxed{II}$

(e) decreasing where slope goes from more steep to less steep : $\boxed{III}$

Q1-2) 2.29

Goal: Read kinematic information from velocity graph.

(a) $v(t) = u + at$ (Constant acceleration)

acceleration is slope of velocity graph,

$a = \frac{dv}{dt}$, slope $= \frac{\Delta y}{\Delta x}$

$= \frac{8-0}{0-6} = -\frac{8}{6}$

$= -\frac{4}{3} cm/s^2$

- Notice graph is a straight line with points (0, 8), (6, 0), so we can write equation for $v(t)$

so $v(t) = 8 - \frac{4}{3}t$

At $t = 4.0 s$, $v(t) = 8 - \frac{4}{3} \times 4$

$= 8 - \frac{16}{3}$

$= 8 - 5\frac{1}{3}$ $\boxed{= 2\frac{2}{3} cm/s}$

At $t = 7.0 s$, $v(t) = 8 - \frac{4}{3} \times 7$

$= 8 - \frac{28}{3}$

$= 8 - 9\frac{1}{3}$ $\boxed{= -\frac{4}{3} cm/s}$

(b) $a(t) = a(3.0 s)$

$= a(6.0 s)$

$= a(7.0 s)$

$\boxed{= -\frac{4}{3} cm/s^2}$ for all values of t

Acceleration is slope of graph and as noted, it is constant.

(c) $s = ut + \frac{1}{2}at^2$, $t = 4.5$

$= 8(4.5) - \frac{1}{2} \cdot \frac{4}{3}(4.5)^2$

$= \boxed{22.5 \text{ cm}}$

Since acceleration is constant, we can use

$s = ut + \frac{1}{2}at^2$

$a = \frac{-4}{3}$, $u = 8$

For $t = 7.5s$,

First calculate for $t = 6$

$s_1 = 8(6) - \frac{1}{2} \cdot \frac{4}{3}(6)^2$

$= 24$ cm

Now for $6s < t < 7.5s$,

$s_2 = |ut + \frac{1}{2}at^2|$, $u = 0$, $a = \frac{-4}{3}$

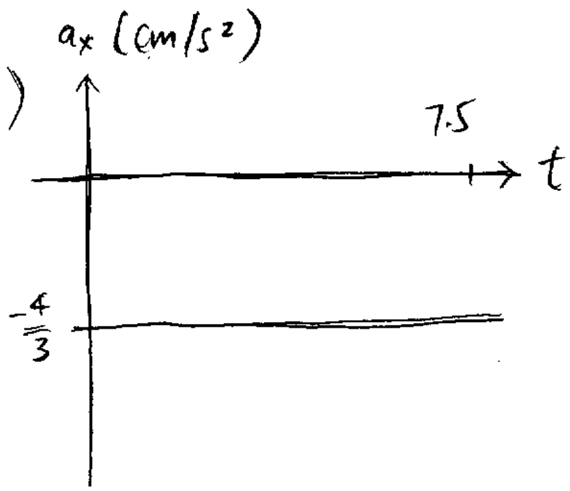
$= \left| -\frac{1}{2} \cdot \frac{4}{3} \times (1.5)^2 \right|$

$= 1.5$ cm

Total distance $= 2.4 + 1.5 = \boxed{25.5 \text{ cm}}$

Since they ask for distance and not displacement, we need to use the absolute value of the displacement..

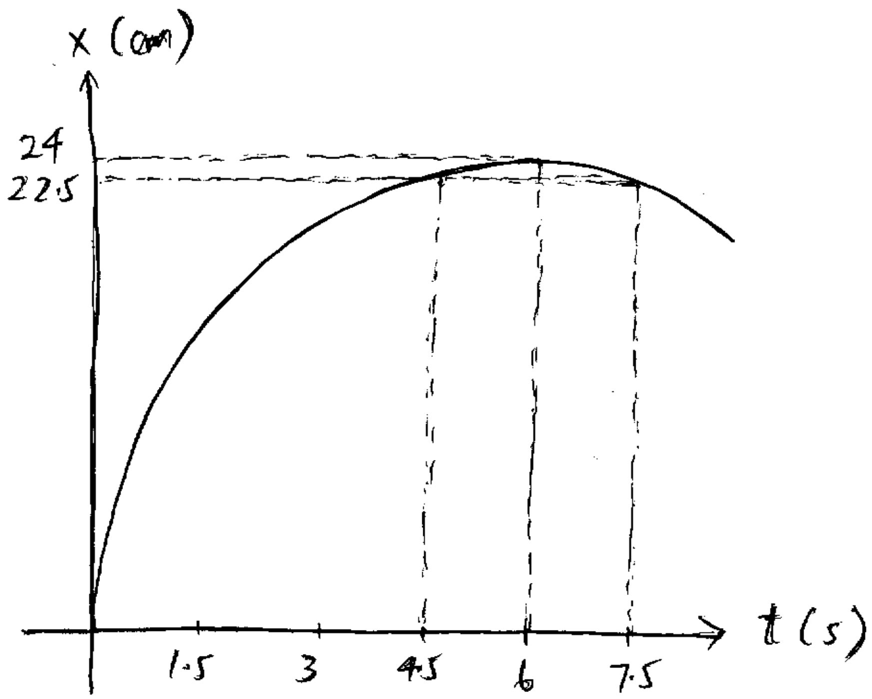
(d)



As noted, a_x is constant, and negative.

Displacement graph is a parabola as acceleration is constant.

The slope (velocity) is greatest initially, goes to zero at $t = 6$ s, and decreases.



We can use information from part (c) to draw this graph:

At $t = 6$, x is the average of the distance travelled between 4.5 s and 7.5 s due to symmetry of parabola. Thus

$$x = \frac{22.5 + 25.5}{2}$$

$$= 24 \text{ cm}$$

Q 1-3) 2.31

Vertices of graph : $(0, 20)$
$(5, 20)$
$(9, 45)$
$(13, 0)$

There are 3 segments of constant acceleration. Thus the instantaneous acceleration is equal to the average acceleration for that segment.

(a) At $t = 3$,

$$a_x = \frac{\Delta V}{\Delta t}$$
$$\boxed{= 0 \ ms^{-2}}$$

$t = 7$, $a_x = \frac{\Delta V}{\Delta t}$

$$= \frac{45 - 20}{9 - 5}$$
$$= \frac{25}{4} \quad \boxed{= 6\frac{1}{4} \ ms^{-2}}$$

$t = 11s$, $a_x = \frac{0 - 45}{13 - 9}$

$$= -45/4 \quad \boxed{= -11\frac{1}{4} \ ms^{-2}}$$

(b) For $0 < t < 5s$,

$s = 20 \times 5 = \boxed{100 \, m}$

$5s < t < 9s$

$s_2 = 20 \times 4 + \frac{1}{2} \cdot \frac{25}{4} (4)^2 = 130 \, m$

For constant acceleration, we can use

$s = ut + \frac{1}{2}at^2$

Total distance is the sum of distance travelled in each segment

Distance in first 9s

$= s + s_2$

$= 100 + 130 \text{ m} = \boxed{230 \text{ m}}$

For $9 \text{ s} < t < 13 \text{ s}$,

$s_3 = 45 \times 4 - \frac{1}{2}\left(\frac{45}{4}\right)(4)^2$

$= 90 \text{ m}$

Total distance in 13 s

$= s + s_2 + s_3$

$\boxed{= 320 \text{ m}}$

Distance travelled is sum of total area travelled under graph.

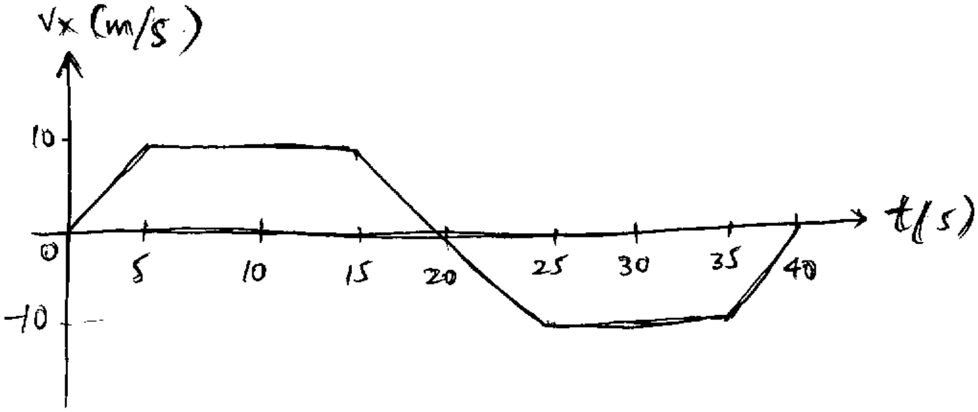
Since velocity is always positive, displacement keeps increasing ✓

Q 2.31,

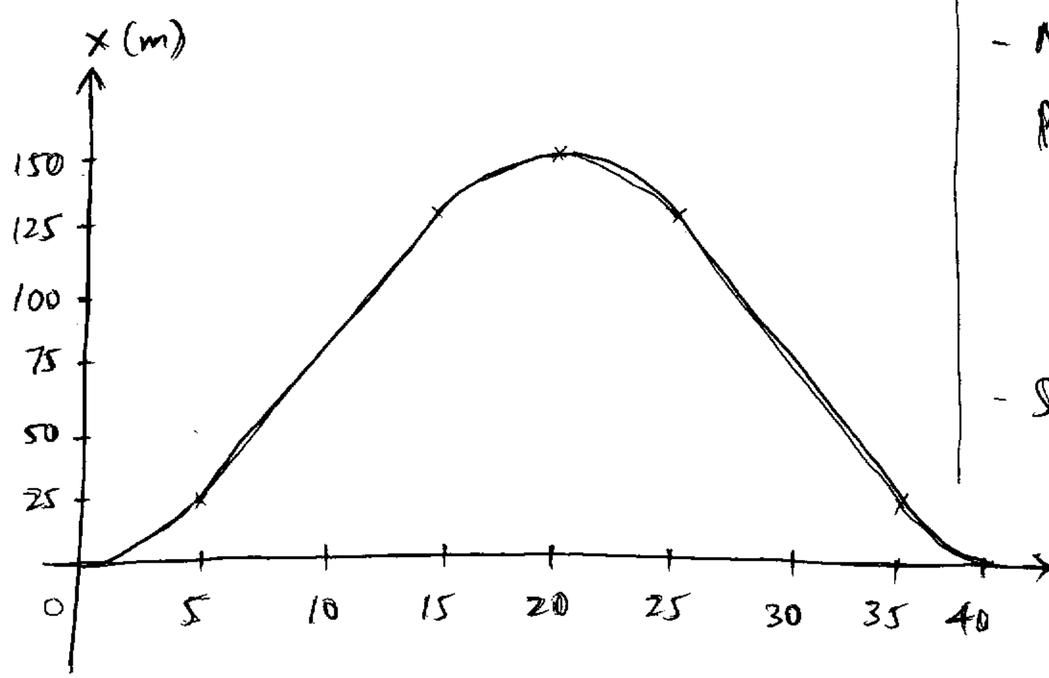
2 of 2.

Q 1-4) 2.32

At $t = 5$, $v_x = 2 \times 5 = 10\ ms^{-1}$



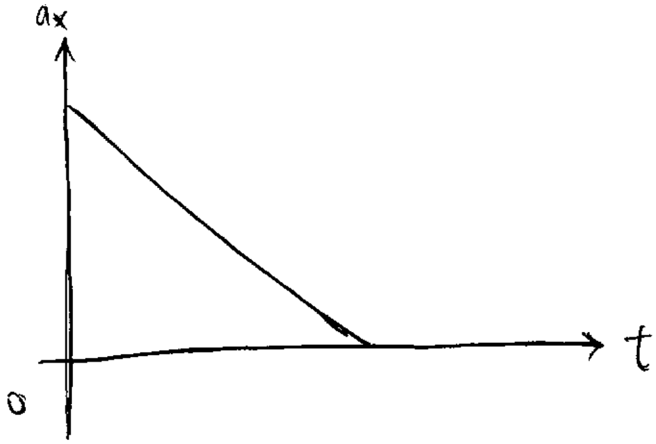
t	Δx	x
5	25	25
15	100	125
25	0	125
35	-100	25
40	-25	0



- Since the magnitude of the acceleration takes only two values, 2 and 0, we know that the slope of the velocity graph has only two values, and by symmetry, goes from $10\ ms^{-1}$ to $-10\ ms^{-1}$ only.
- Displacement is the area under the graph.
- Notice the question has perfect symmetry about $x = 20$, this helps simplify the work.
- Slope of x-t graph should be the v_x-t graph. ✓

Q 1-5) 2.72

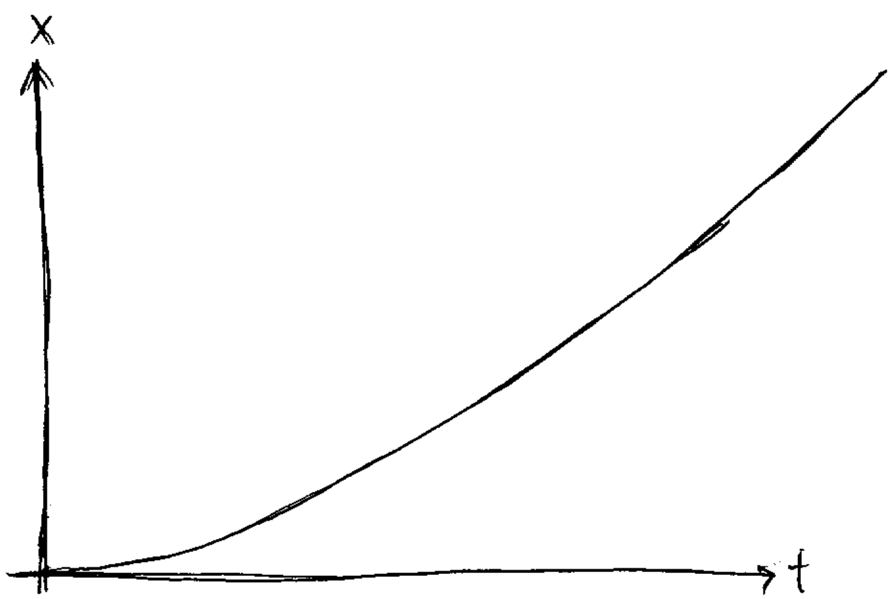
$a_x = \frac{dv}{dt}$ — slope of v_x-t graph



$x = \int v \, dt$

slope of x-t graph is v-t,

so x-t graph has to have gentle slope initially that increases to a constant value.



v_x-t appears like a parabola, with the slope decreasing at fairly constant rate.

- we can guess constant deceleration.

The graph of a_x is positive and decreasing, this makes sense as effect from air resistance or friction.

Q 1-6) 2.76

We need to match the time for the egg to fall, and the time for the professor's walk.

Height the egg falls $s = 46 - 1.8 = 44.2\,m$

$s = \frac{1}{2} g t_1^2$ since starting from rest and in free fall.

$\frac{2s}{g} = t_1^2$

$t_1 = \sqrt{\frac{2s}{g}}$ t_1 is time egg falls

to find t_2, time ~~from~~ for professor to walk,

$v = \frac{d}{t_2}$

$t_2 = \frac{d}{v}$

(d is distance of professor, v is walking speed)

$t_1 = t_2$ to be successfull

$\Rightarrow \frac{d}{v} = \sqrt{\frac{2s}{g}}$

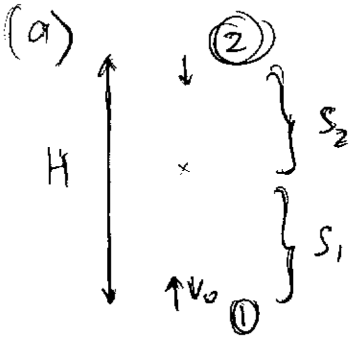
Solve for d: $d = v\sqrt{\frac{2s}{g}}$

$= 1.2 \times \sqrt{\frac{2 \times 44.2}{9.8}}$

$\boxed{= 3.6\,m}$

Q 1-7) 2.92

(a)



$$s_2 = \frac{1}{2}gt^2$$

$$s_1 = v_0 t - \frac{1}{2}gt^2$$

$$s_2 + s_1 = H$$

$$\Rightarrow \frac{1}{2}gt^2 + v_0 t - \frac{1}{2}gt^2 = H$$

$$v_0 t = H$$

$$\boxed{t = \frac{H}{v_0}}$$

To collide, the distance both balls travel (in opposite directions) has to add up to H.

Note that since s_1 and s_2 are measured in opposite directions, g takes on opposite signs in both equations.

(b) When ball 1 is at highest point of motion, its velocity is zero

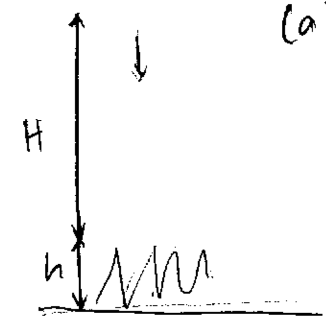
thus $u = v_0 - gt = 0$

$$\Rightarrow v_0 = gt$$

$$= g\left(\frac{H}{v_0}\right) \quad \text{(using } t \text{ from (a))}$$

$$\Rightarrow \boxed{H = \frac{v_0^2}{g}}$$

Q 1-8) 2.94



(a) Since starting from rest,

$u_1 = 0$

$v_1^2 = 2gs$

$\boxed{v_1 = \sqrt{2gH}}$

For constant g, we can use

$v_1^2 = u_1^2 + 2as$

(b) $u_2^2 = 2as$

$= 2ah$

since $u_2 = v_1$ from (a)

$2ah = 2gH$

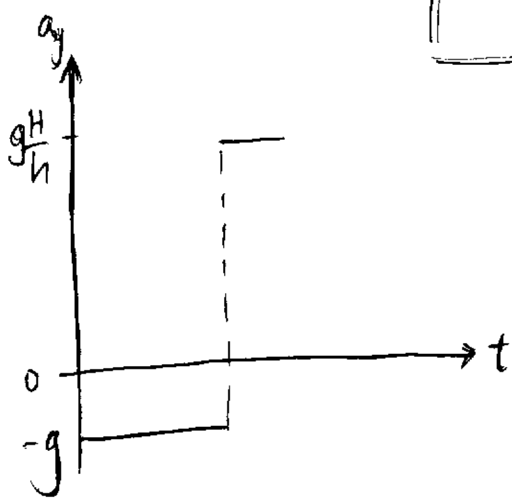
$\boxed{a = \frac{gH}{h}}$

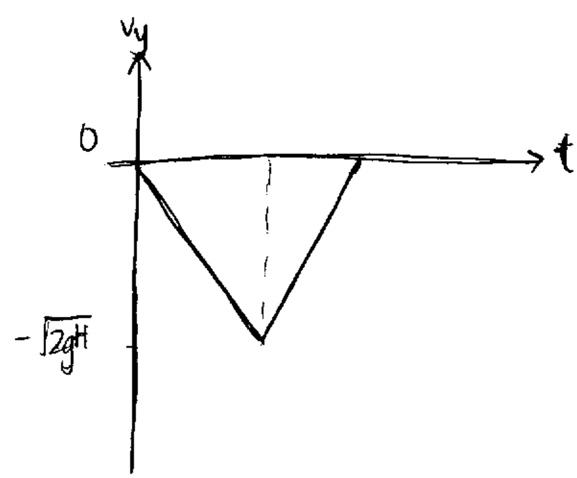
For constant deceleration,

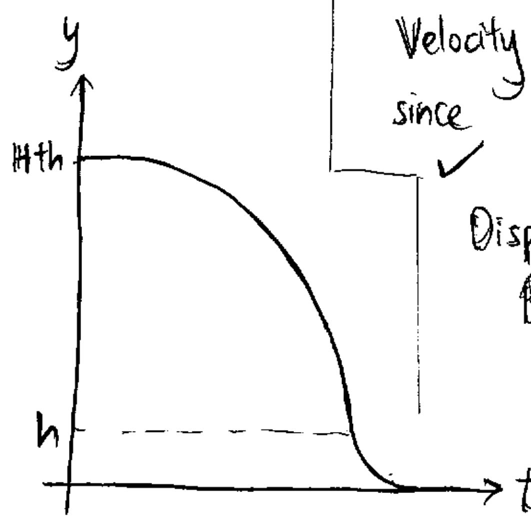
$v_2^2 = u_2^2 - 2as$,

now $v_2 = 0$

(c)







It is easier to sketch a_y-t graphs and work backwards, since we know constant acceleration in both parts of problem.

Velocity is always negative since apple always falling ✓

Displacement is always parabolic, with slope according to v-t graph. ✓