Mechanical Energy and Simple Harmonic Oscillator

8.01
Week 06D2
One end of a spring with spring constant $k$ is attached to a block of mass $m$ that is resting on a frictionless surface. The other end of the spring is attached to a wall. The block is stretched a distance $x_0$ and given a hit in the positive $x$-direction resulting in an initial speed $v_0$. When the spring returns to it’s unstretched length, the mechanical energy of the block-spring system is

1. Zero
2. $\left(\frac{1}{2}\right)mv_0^2$
3. $\left(\frac{1}{2}\right)kx_0^2$
4. $\left(\frac{1}{2}\right)mv_0^2 + \left(\frac{1}{2}\right)kx_0^2$
5. None of the above
Next Reading Assignment: W07D1

Young and Freedman: 8.1-8.5
Concept Question: Solution to SHO Equation of Motion

Which of the following functions $x(t)$ has a second derivative which is proportional to the negative of the function

$$\frac{d^2 x}{dt^2} \propto -x?$$

1. $x(t) = \frac{1}{2}at^2$
2. $x(t) = Ae^{t/T}$
3. $x(t) = Ae^{-t/T}$
4. $x(t) = A\cos\left(\frac{2\pi}{T}t\right)$
Review: Block-Spring by Force Method
Concept Question: Simple Harmonic Motion

A block of mass $m$ is attached to a spring with spring constant $k$ is free to slide along a horizontal frictionless surface.

At $t = 0$ the block-spring system is stretched an amount $x_0 > 0$ from the equilibrium position and is released from rest. What is the $x$-component of the velocity of the block when it first comes back to the equilibrium?

1. $v_x = -x_0 \frac{T}{4}$
2. $v_x = x_0 \frac{T}{4}$
3. $v_x = -\sqrt{\frac{k}{m}}x_0$
4. $v_x = \sqrt{\frac{k}{m}}x_0$
Demo slide: spray paint oscillator C4

Illustrating choice of alternative representations for position as a function of time (amplitude and phase or sum of sin and cos)
Strategy:

1. Recognizing SHO equation
2. Remembering solutions
3. Using initial conditions
Worked Example: Block-Spring by Energy Method
Functional Relationships for a Mass-Spring Oscillator

- $x(t)/A$
- $v(t)/\omega A$
- $a(t)/\omega^2 A$
- Energy: $\frac{1}{2} k (x_{max})^2$
The position of a particle is given by

\[ x(t) = D \cos(\omega t) - D \sin(\omega t), \quad D > 0 \]

Where was the particle at \( t = 0 \)?

1) 1
2) 2
3) 3
4) 4
5) 5
6) 1 or 5
7) 2 or 4
Table Problem: Simple Pendulum by the Energy Method

a) Find an expression for the total energy in terms of the given quantities and their derivatives.

b) Find the equation of motion for $\theta(t)$.

c) Find the equation of motion if $\theta$ is always $<< 1$. 
Concept Question: SHO and the Pendulum

Suppose the point-like object of a simple pendulum is pulled out at an angle $\theta_0 << 1$ rad. Is the angular speed of the point-like object equal to the angular frequency of the pendulum?

1. Yes.

2. No.

3. Only at bottom of the swing.

Worked Example: fluid oscillations in a U-tube

A U-tube open at both ends to atmospheric pressure is filled with an incompressible fluid of density \( \rho \). The cross-sectional area \( A \) of the tube is uniform and the total length of the column of fluid is \( L \). A piston is used to depress the height of the liquid column on one side by a distance \( x \), and then is quickly removed. What is the frequency of the ensuing simple harmonic motion? Assume streamline flow and no drag at the walls of the U-tube. The gravitational constant is \( g \).
Demo slide: U-tube oscillations
A particle with total mechanical energy $E$ has position $x > 0$ at $t = 0$

1) escapes to infinity in the $-x$-direction
2) approximates simple harmonic motion
3) oscillates around $a$
4) oscillates around $b$
5) periodically revisits $a$ and $b$
6) not enough information
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Concept Question: Energy
Diagram 3

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Concept Question: Energy
Diagram 4

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