Momentum and Impulse

8.01
W07D1
Consider two carts, of masses $m$ and $2m$, at rest on an air track. If you push first one cart for 3 s and then the other for the same length of time, exerting equal force on each, the momentum of the light cart is

1. four times
2. twice
3. equal to
4. one-half
5. one-quarter

the momentum of the heavy cart.
Momentum and Impulse

Obeys a conservation law
Simplifies complicated motions
Describes collisions
Basis of rocket propulsion & space travel
Concept Question: Pushing Identical carts

Identical constant forces push two identical objects A and B continuously from a starting line to a finish line. If A is initially at rest and B is initially moving to the right,

1. Object A has the larger change in momentum.
2. Object B has the larger change in momentum.
3. Both objects have the same change in momentum.
4. Not enough information is given to decide.
Concept Question: Pushing Non-identical Carts

Consider two carts, of masses \( m \) and \( 2m \), at rest on an air track. If you push one cart for 3 s and then the other for the same length of time, exerting equal force on each, the kinetic energy of the light cart is

(1) larger than
(2) equal to
(3) smaller than

the kinetic energy of the heavy car.
Suppose a ping-pong ball and a bowling ball are rolling toward you. Both have the same momentum, and you exert the same force to stop each. How do the distances needed to stop them compare?

1. It takes a shorter distance to stop the ping-pong ball.

2. Both take the same distance.

3. It takes a longer distance to stop the ping-pong ball.
Strategy: Momentum of a System

1. Choose system

2. Identify initial and final states

3. Identify any external forces in order to determine whether any component of the momentum of the system is constant or not
Worked Example: Sliding on Slipping Block

A small cube of mass $m_1$ slides down a circular track of radius $R$ cut into a large block of mass $m_2$ as shown in the figure below. The large block rests on a table, and both blocks move without friction. The blocks are initially at rest, and the cube starts from the top of the path. Find the velocity of the cube as it leaves the block.
A simple way to measure the speed of a bullet is with a ballistic pendulum, which consists of a wooden block of mass $m_1$ into which a bullet of mass $m_2$ is shot. The block is suspended from two cables, each of length $L$. The impact of the bullet causes the block and embedded bullet to swing through a maximum angle $\phi$.

a) Find an expression for the initial speed of the bullet as a function of $m_1$, $m_2$, $L$, $g$, and $\phi$.

b) Determine the ratio of the lost mechanical energy (due to the collision of the bullet with the block) to the initial kinetic energy of the bullet.
Demo : Center of Mass trajectory B78


Odd-shaped objects with their centers of mass marked are thrown. The centers of mass travel in a smooth parabola. The objects consist of: a squash racket, a 16” diameter disk weighted at one point on its outer rim, and two balls connected with a rod. This demonstration is shown with UV light.
CM moves as though all external forces on the system act on the CM

so the jumper’s cm follows a parabolic trajectory of a point moving in a uniform gravitational field
Center of mass passes under the bar
Table Problem: Center of Mass of Earth-Moon System

The mean distance from the center of the earth to the center of the moon is $r_{em} = 3.84 \times 10^8$ m. The mass of the earth is $m_e = 5.98 \times 10^{24}$ kg and the mass of the moon is $m_m = 7.34 \times 10^{22}$ kg. The mean radius of the earth is $r_e = 6.38 \times 10^6$ m. The mean radius of the moon is $r_m = 1.74 \times 10^6$ m. Where is the location of the center of mass of the earth-moon system? Is it inside the earth’s radius or outside?
Table Problem: Center of Mass of Rod:

A thin rod has length $L$ and total mass $M$.

a) Suppose the rod is uniform. Find the position of the center of mass with respect to the left end of the rod. **One half does this part.**

b) Now suppose the rod is not uniform but has a linear mass density that varies with the distance from the left end according to

$$\lambda(x) = \frac{\lambda_0}{L^2} x^2$$

where $\lambda_0$ is a constant and has SI units [kg-m$^{-1}$]. Find $\lambda_0$ and the position of the center of mass with respect to the left end of the rod. **The other half does this part.**
Demo: Leaping on and off a Scale
A person jumps off the ground. Suppose the person pushes off the ground with a constant force of magnitude $F$ for a time interval $\Delta t$. What was the magnitude of the displacement of the center of mass of the person?
Demo Jumping: Non-Constant Force

- Plot of total external force vs. time for Andy jumping off the floor. Weight of Andy is 911 N.
Demo Jumping: Impulse

- Shaded area represents impulse of total force acting on Andy as he jumps off the floor

\[ \mathbf{F}_{\text{total}}(t) = \mathbf{N}(t) + \mathbf{F}_{\text{grav}} \]

\[ \mathbf{I}[t_i, t_f] = \int_{t_i}^{t_f} \mathbf{F}_{\text{total}}(t) \, dt = 199 \, \text{N} \cdot \text{s} \]
Demo Jumping

At the end of the time interval [0.11 s, 1.23 s], what was Andy’s center of mass velocity. Assume that at the beginning of the interval Andy was at rest. Andy’s weight is 911 N. The impulse is

\[
\vec{I}[t_i, t_f] = \int_{t_i}^{t_f} \vec{F}^{\text{total}}(t) \, dt = 199 \text{ N} \cdot \text{s}
\]
Demo Jumping: Height

• When Andy leaves the ground, the impulse is

\[ I_y[0.11 \text{ s}, 1.23 \text{ s}] = 199 \text{ N} \cdot \text{s} \]

• So the center of mass velocity is

\[ V_{cm,y}(t) = \frac{I_y[t_i,t]}{m} = \frac{(199 \text{ N} \cdot \text{s})(9.80 \text{ m} \cdot \text{s}^{-2})}{(911 \text{ N})} = 2.14 \text{ m} \cdot \text{s}^{-1} \]

What was the magnitude of the displacement of Andy’s center of mass after he left the floor?

\[ \Delta Y_{cm} = V_{cm,y}^2(t_f) / 2g = (2.14 \text{ m} \cdot \text{s}^{-1})^2 / (2)(9.80 \text{ m} \cdot \text{s}^{-2}) = 0.23 \text{ m} \]
Concept Question: Pushing a Baseball Bat Recall Issue with Phrasing

The greatest acceleration of the center of mass will be produced by pushing with a force $F$ at

1. Position 1
3. Position 2
5. Position 3
4. All the same