Two-Dimensional Rotational Kinematics

8.01
W09D1
Reading Quiz

The figure shows a graph of $\omega_z$ and $\alpha_z$ versus time for a particular rotating body. During which time intervals is the rotation slowing down?

1. $0 < t < 2 \text{ s}$
2. $2 \text{ s} < t < 4 \text{ s}$
3. $4 \text{ s} < t < 6 \text{ s}$
4. None of the intervals.
5. Two of the intervals.
6. Three of the intervals.
Next Reading Assignment: W09D2

Young and Freedman: 1.10 (Vector Product) 10.1-10.2, 10.5-10.6 ; 11.1-11.3
Concept Question: Angular Speed

Object A sits at the outer edge (rim) of a merry-go-round, and object B sits halfway between the rim and the axis of rotation. The merry-go-round makes a complete revolution once every thirty seconds. The magnitude of the angular velocity of Object B is

1. half the magnitude of the angular velocity of Object A.
2. the same as the magnitude of the angular velocity of Object A.
3. twice the magnitude of the angular velocity of Object A.
4. impossible to determine.
Table Problem: Rotational Kinematics

A turntable is a uniform disc of mass $m$ and a radius $R$. The turntable is initially spinning clockwise when looked down on from above at a constant frequency $f$. The motor is turned off and the turntable slows to a stop in $t$ seconds with constant angular deceleration.

a) What is the direction and magnitude of the initial angular velocity of the turntable?

b) What is the direction and magnitude of the angular acceleration of the turntable?

c) What is the total angle in radians that the turntable spins while slowing down?
Concept Question

All of the objects below have the same mass. Which of the objects has the largest moment of inertia about the axis shown?

(1) Hollow Cylinder
(2) Solid Cylinder
(3) Thin-walled hollow cylinder
Concept Question

Which has the smallest $I$ about its center?

1) Ring (M,R)

2) Disk (M,R)

3) Sphere (M,R)

4) All have the same $I$
Concept Question

Which axis gives the largest $I$ for the disk?

1) 

2) 

3) 

4) All have the same $I$
Consider a thin uniform rod of length $L$ and mass $M$.

Odd Tables: Calculate the moment of inertia about an axis that passes perpendicular through the center of mass of the rod.

Even Tables: Calculate the moment of inertia about an axis that passes perpendicular through the end of the rod.
Worked Example: Moment of Inertia for Uniform Disc

Consider a thin uniform disc of radius $R$ and mass $m$. What is the moment of inertia about an axis that pass perpendicular through the center of the disc?
A disk with mass $M$ and radius $R$ is spinning with angular speed $\omega$ about an axis that passes through the rim of the disk perpendicular to its plane. Moment of inertia about cm is $(1/2)MR^2$. Its total kinetic energy is:

1. $(1/4)MR^2\omega^2$
2. $(1/2)MR^2\omega^2$
3. $(3/4)MR^2\omega^2$
4. $(1/4)MR^2\omega^2$
5. $(1/2)MR^2\omega^2$
6. $(1/4)MR\omega$
Using energy techniques, calculate the speed of block 2 as a function of distance that it moves down the inclined plane using energy techniques. Let $I_P$ denote the moment of inertia of the pulley about its center of mass. Assume there are no energy losses due to friction and that the rope does not slip around the pulley.
In the above situation where a particle is moving in the x-y plane with a constant velocity, the magnitude of the angular momentum $\vec{L}$ about the origin

1) decreases then increases

2) increases then decreases

3) is constant

4) is zero because this is not circular motion
Concept Question:

The diagram above shows six possible combinations of position and velocity for a particle of mass \( m \) and speed \( v \) moving in the x-y plane. How many distinct values of the vector \( \mathbf{L} \) relative to the origin does this represent?

1) 1
2) 2
3) 3
4) 4
5) 5
6) 6
A particle moves in a circle about the $z$ axis in a plane parallel to, but above, the $x$-$y$ plane. Relative to the origin:

1) $\vec{L}$ is constant

2) $|\vec{L}|$ is constant but $\vec{L}/|\vec{L}|$ is not

3) $\vec{L}/|\vec{L}|$ is constant but $|\vec{L}|$ is not

4) $\vec{L}$ has no $z$ component
A particle of mass \( m = 2 \text{ kg} \) moves with a uniform velocity
\[
\vec{v} = 3.0 \text{ m} \cdot \text{s}^{-1} \hat{i} + 3.0 \text{ m} \cdot \text{s}^{-1} \hat{j}
\]
At time \( t \), the position vector of the particle with respect to the point \( S \) is
\[
\vec{r}_S = 2.0 \text{ m} \hat{i} + 3.0 \text{ m} \hat{j}
\]
Find the direction and the magnitude of the angular momentum about the origin, (the point \( S \)) at time \( t \).
A particle of mass $m$ moves in a circle of radius $R$ at an angular speed $\omega$ about the $z$ axis in the $x$-$y$ plane.

Find the angular momentum $\vec{L}$ relative to the origin.