Problem 2 Sliding Along a Sphere

An object of mass $m$ initially sits on top of a large sphere of radius $R$ that is fixed to the ground as shown in the figure. The object gets a small push and begins to slide along the surface of the sphere. You may assume that the initial kinetic energy is negligible. There is a friction force between the object and the surface that varies with the angle $\theta$ according to $f = f_0 \sin\theta$ where $f_0 < mg$ is a constant. Let $g$ denote the magnitude of acceleration due to gravity.

The object just loses contact with the surface of the sphere at an angle $\theta_f$ with respect to the vertical when it has a speed $v_f$.

a) What is the work done by the friction force on the object as the object moves through the angle $\theta = 0$ to the angle $\theta = \theta_f$? Hint: the small displacement of the object on the surface of the sphere is given by $d\vec{r} = Rd\theta \hat{\theta}$.

b) What is the work done by the gravitational force on the object as the object moves through the angle $\theta = 0$ to the angle $\theta = \theta_f$?

c) Based on your results from parts a) and b), use the work-energy theorem to determine an expression for the kinetic energy of the object just before the object leaves the surface of the sphere in terms of $\theta_f$, $m$, $g$, $R$, $f_0$ and $v_f$ as needed.
d) When the object reaches the angle $\theta_f$, apply Newton’s Second Law to determine a second independent relationship between $\theta_f$, $m$, $g$, $R$, and $v_f$ as needed.

e) Using your results from parts c) and d), determine an expression for $v_f$ in terms of $m$, $g$, $R$, and $f_0$. Do not include $\theta_f$ in your answer.