A satellite of mass $m_s$ is in an elliptical orbit around a planet of mass $m_p$, which is located at one focus of the ellipse. The satellite has a speed $v_a$ at the distance $r_a$ (apoapsis) when it is furthest from the planet. The distance of closest approach is $r_p$ (periapsis). Let $G$ be the universal constant of gravity. The goal of this problem is to find the ratio $r_a / r_p$ in terms of $G$, $m_p$, $v_a$, and $r_a$.

a) Apply conservation of energy to the two states periapsis and apoapsis to determine a relationship between $r_p$ and $v_p$.

b) The gravitational torque on the satellite about the center of the planet is zero throughout the elliptic orbit of the satellite. At periapsis and apoapsis, the velocity of the satellite is perpendicular to the vector from the center of the planet to the satellite. Use the fact that the angular momentum is constant about the center of the planet to determine an expression for $v_p$ in terms of $r_p$, $v_a$ and $r_a$.

c) Use your results from parts a) and b) to determine an expression for $r_p$ in terms of $G$, $m_p$, $v_a$ and $r_a$.

d) See if you can determine the ratio $r_a / r_p$ in terms of $G$, $m_p$, $v_a$, and $r_a$. Hint: this will require some algebra manipulations of your result from part c).