A spherical billiard ball of uniform density has mass $m$ and radius $R$ and moment of inertia about the center of mass $I_{cm} = \frac{2}{5}mR^2$. The ball, initially at rest on a table, is given a sharp horizontal impulse by a cue stick that is held an unknown distance $h$ above the centerline (see diagram below). The coefficient of sliding friction between the ball and the table is $\mu_k$. You may ignore the friction during the impulse. The ball leaves the cue with a given speed $v_0$ and an unknown angular velocity $\omega_0$. Because of its initial rotation, the ball eventually acquires a maximum speed of $\left(\frac{9}{7}\right)v_0$. The point of the problem is to find the ratio $h / R$.

a) Write up your plan for solving this problem. You may find some of the following concepts useful: angular impulse is equal to the change in angular momentum; linear impulse is equal to the change in momentum; Newton’s Second Law; torque about the center-of-mass is proportional to the angular acceleration; if the torque about a point is zero, then the angular momentum about that point is constant; etc.

b) Find the ratio $h / R$. 

---

**Diagrams**

- Diagram of a spherical billiard ball with a cue stick applying an impulse from above the centerline, labeled with $h$ and $R$. 
- Diagram of a billiard ball with a given speed $v_0$ and angular velocity $\omega_0$. 

---

**W15D2-5 Table Problem Billiards Solution**
Solutions:

a) There are several ways to approach this problem. The method presented here avoids any calculation of the force or torque provided by friction, or the details of the force between the cue and the ball. This method will first consider the “collision” between the cue and the ball by taking the collision point as the origin for finding the angular momentum, as the force between the cue and the ball exerts no torque about this point, and we are given that the friction may be ignored during this interaction. After this collision, the angular momentum will be taken about the initial contact point between the ball and the felt. (It should be noted that this method anticipates the answer, which does not involve the coefficient of friction $\mu_k$ and also relies on having done Problem 2 above.) It will be helpful to infer, either from the figure and from the fact that $v_f > v_o$, that the ball is given overspin.

b) With respect to the point where the cue is in contact with the ball, note that the rotational angular momentum and the angular momentum due to the motion of the center of mass have different signs; the former is clockwise and the latter is counterclockwise. The sum of these contributions to the angular momenta must sum to zero, and hence have the same magnitude;

$$I_{cm} \alpha_0 = mv_o h. \quad (0.1)$$

While the ball is rolling and slipping, angular momentum is conserved about the contact between the ball and the felt. The initial and final angular momenta are

$$L_{initial} = mv_o R + I_{cm} \alpha_0$$
$$= mv_o (R + h)$$
$$L_{final} = mv_f R + I_{cm} \alpha_f$$
$$= mv_f R + (2/5)(mR^2)(v_f/R)$$
$$= (7/5)mv_f R$$
$$= (9/5)mv_o R, \quad (0.2)$$

where Equation (0.1) and the given relations $I_{cm} = (2/5)mR^2$ and $(9/7)v_o$ have been used. Setting the initial and final angular momenta equal and solving for $h/R$ gives

$$\frac{h}{R} = \frac{4}{5} \quad (0.3)$$

(note that the figure is not quite to scale).
As an alternative, taking the angular momentum after the collision about the center of the ball, note that the time $\Delta t$ between the moments the ball is struck and when it begins to roll without slipping is $\Delta v/(\mu_g)$. But, if the angular momentum is taken about the center of the ball, after the ball is struck the angular impulse delivered to the ball by the friction force is

$$
(\mu_m mg)R \Delta t = I_{cm} (\omega_f - \omega_i).
$$

Eliminating $\Delta t$ between these expressions leads to the same result obtained by equating the first and third expressions in (0.2).