Consider an object undergoing circular motion. At time $t$, the particle is located at the point $P$. Choose polar coordinates as shown in the figure below left. Find a vector decomposition expression for the unit vectors $(\hat{r}(t), \hat{\theta}(t))$ in terms of the unit vectors $(\hat{i}, \hat{j})$ as shown in the figure below right.

**Solution:**

In figure above right, we see that a vector decomposition expression for $\hat{r}(t)$ and $\hat{\theta}(t)$ in terms of $\hat{i}$ and $\hat{j}$ is given by

$$\hat{r}(t) = \cos \theta(t) \hat{i} + \sin \theta(t) \hat{j} \quad (0.1)$$

$$\hat{\theta}(t) = -\sin \theta(t) \hat{i} + \cos \theta(t) \hat{j} \quad (0.2)$$