A particle is moving in a circle of radius $R$. At $t = 0$, it is located on the $x$-axis. The angle the particle makes with the positive $x$-axis is given by $\theta(t) = At - Bt^3$, where $A$ and $B$ are positive constants. Determine (a) the angular velocity vector, and (b) the velocity vector. Express your answer in polar coordinates. (c) At what time, $t = t_1$, is the angular velocity zero? (d) What is the direction of the angular velocity for (i) $t < t_1$, and (ii) $t > t_1$?

**Solution:** The derivative of $\theta(t) = At - Bt^3$ is

$$\frac{d\theta(t)}{dt} = A - 3Bt^2.$$ 

Therefore the angular velocity vector is given by

$$\mathbf{\omega}(t) = \frac{d\theta(t)}{dt} \mathbf{\hat{k}} = (A - 3Bt^2)\mathbf{\hat{k}}.$$ 

The velocity is given by

$$\mathbf{v}(t) = R \frac{d\theta(t)}{dt} \mathbf{\hat{\theta}}(t) = R(A - 3Bt^2)\mathbf{\hat{\theta}}(t).$$

The angular velocity is zero at time $t = t_1$ when

$$A - 3Bt_1^2 = 0 \Rightarrow t_1 = \sqrt{A / 3B}.$$ 

For $t < t_1$, $\frac{d\theta(t)}{dt} = A - 3Bt_1^2 > 0$ hence $\mathbf{\omega}(t)$ points in the positive $\mathbf{\hat{k}}$-direction.

For $t > t_1$, $\frac{d\theta(t)}{dt} = A - 3Bt_1^2 < 0$ hence $\mathbf{\omega}(t)$ points in the negative $\mathbf{\hat{k}}$-direction.