Block A of mass $m_A$ is moving horizontally with speed $V_A$ along a frictionless surface. It collides elastically with block B of mass $m_B$ that is initially at rest. After the collision block B enters a rough surface at $x = 0$ with a coefficient of kinetic friction that increases linearly with distance $\mu_k(x) = bx$ for $0 \leq x \leq d$, where $b$ is a positive constant. At $x = d$ block B collides with an unstretched spring with spring constant $k$ on a frictionless surface. The downward gravitational acceleration has magnitude $g$.

What is the distance the spring is compressed when block B first comes to rest? Express your answer in terms of $V_A$, $m_A$, $m_B$, $b$, $d$, $g$, and $k$. 
\[ F_{\text{ext}} = 0 \implies P_{\text{x},i} = P_{\text{x},f} \]

\[ m_A v_A = m_A v_{Axf} + m_B v_{Bxf} \]

\[ v_A = v_{Axf} + \frac{m_B}{m_A} v_{Bxf} \quad (1) \]

relative velocity:

\[ v_A = -(v_{Axf} - v_{Bxf}) \]

\[ v_A = v_{Bxf} - v_{Axf} \quad (2) \]

add (1) and (2):

\[ 2v_A = \left(1 + \frac{m_B}{m_A}\right) v_{Bxf} \]

\[ v_{Bxf} = \frac{2v_A}{1 + \frac{m_B}{m_A}} = \frac{2m_A v_A}{m_A + m_B} \quad (3) \]
\[ W_{nC} = E_f - E_i \]
\[ = \int_{0}^{d} bxmg \, dx = \frac{1}{2} kx_f^2 - \frac{1}{2} m_B v_{Bx1}^2 - \frac{1}{2} m_B g \frac{d^2}{2} \]

\[ = \frac{1}{2} k x_f^2 - \frac{1}{2} m_B v_{Bx1}^2 \]

\[ \Rightarrow x_f = \frac{m_B}{k} \left( v_{Bx1}^2 - \frac{1}{2} g d^2 \right) \quad \text{Use eq. (3)} \]

\[ x_f = \sqrt{\frac{m_B}{k} \left( \frac{2 m_A v_A^2}{m_A + m_B} - \frac{1}{2} g d^2 \right)} \]