A small cube of mass $m_1$ slides down a circular track of radius $R$ cut into a large block of mass $m_2$ as shown in the figure below. The large block rests on a table, and both blocks move without friction. The blocks are initially at rest, and $m_1$ starts from the top of the path. Find the velocity $v_i$ of the cube as it leaves the block.

**Solution:** If we consider the earth-cube-block system, there are no external forces in the horizontal direction so the horizontal component of momentum is constant, $p_{x,i} = p_{x,f}$. We can ignore the horizontal motion of the earth and so the momentum of the block and cube is constant. Initially the system is at rest, so the final horizontal component of the momentum is zero. Also energy is constant $E_i = E_f$ since there is no external work (the gravitational force is an internal force and the work done is describable by a change in potential energy. The initial and final states are shown in the figure below.

The condition that momentum in the x-direction is constant becomes

$$0 = m_1 v_{i,f} - m_2 v_{z,f}.$$  \hspace{1cm} (1)

We can solve Eq. (1) for the final velocity of the block.
\[ v_{2,f} = \frac{m_1 v_{1,f}}{m_2}. \]  

(2)

The condition that the energy is constant becomes

\[ U_i = U_f + K_f. \]  

(3)

If we choose as the zero point for potential energy the height that the cube leaves the block, then \( U_f = 0 \), and Eq. (3) becomes

\[ m_1 g R = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2. \]  

(4)

We can now substitute Eq. (2) into Eq. (4) to find that

\[ m_1 g R = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 \left( \frac{m_1 v_{1,f}}{m_2} \right)^2 = \frac{1}{2} m_1 v_{1,f}^2 \left( 1 + \frac{m_1}{m_2} \right). \]  

(5)

Thus we can solve Eq. (5) for the velocity of the cube

\[ \dot{v}_{1,f} = \sqrt{\frac{2 g R m_2}{m_2 + m_1}} \hat{i}. \]  

(6)