IC_W08D3-2 Group Problem Helium-Lithium Collision: Solutions

This problem is on pset 7 and is a difficult problem so we will get you started today.

A thin target of lithium is bombarded by helium nuclei of energy $E_0$. The lithium nuclei are initially at rest in the target but are essentially unbound. When a helium nucleus enters a lithium nucleus, a nuclear reaction can occur in which the compound nucleus splits apart into a boron nucleus and a neutron. The collision is inelastic, and the final kinetic energy is less than $E_0$ by 2.8 MeV. (1 MeV = $10^6$ eV = $1.6 \times 10^{-13}$ J). The relative masses of the particles are: helium, mass 4; lithium, mass 7; boron mass 10; neutron, mass 1. The reaction can be symbolized

$$^7\text{Li} + ^4\text{He} \rightarrow ^{10}\text{B} + ^1\text{n} - 2.8 \text{ MeV}.$$ 

Let’s consider only the outgoing particles, boron and neutrons that are moving along the same line as the initial helium nucleus, (call this the $x$-direction).

a) Draw momentum diagrams for the initial and final states in a reference frame in which the target lithium nucleus is initially at rest.

b) What is the center of mass velocity of the system of lithium and helium? Does this velocity change due to the collision?

Answer:

$$\mathbf{\tilde{V}_{cm}} = \frac{(m_{\text{He}} \mathbf{\tilde{V}}_{\text{He}} + m_{\text{Li}} \mathbf{\tilde{V}}_{\text{Li}})}{m_{\text{He}} + m_{\text{Li}}} = \frac{m_{\text{He}} \mathbf{\tilde{V}}_{\text{He}}}{m_{\text{He}} + m_{\text{Li}}} = \frac{4m \mathbf{\tilde{V}}_{\text{He}}}{4m + 7m} = \frac{4}{11} \mathbf{\tilde{V}}_{\text{He}}$$

There are no external forces so the velocity of the center of mass remains constant.

e) Draw momentum diagrams for the initial and final states in a reference frame moving with the velocity of center of mass. What are the velocities of the helium
nucleus and the lithium in the center of mass reference frame? What is the momentum of the system in the center of mass reference frame?

Answer: The velocity of the helium in the center of mass reference frame is given by

\[
\vec{v}_{cm,He} = \vec{v}_{He} - \vec{v}_{cm} = \vec{v}_{He} - \frac{4}{11} \vec{v}_{He} = \frac{7}{11} \vec{v}_{He}.
\]

The velocity of the lithium in the center of mass reference frame is given by

\[
\vec{v}_{cm,Li} = \vec{v}_{Li} - \vec{v}_{cm} = -\frac{4}{11} \vec{v}_{He}.
\]

The momentum of the system in the center of mass reference frame is zero.

\[
\vec{p}_{cm,He} + \vec{p}_{cm,Li} = 4m \frac{7}{11} \vec{v}_{He} - 7m \frac{4}{11} \vec{v}_{He} = \vec{0}.
\]

Below threshold energy necessary to create the boron and neutron the helium and lithium just bounce off. The momentum diagrams for the initial and final states in a reference frame moving with the velocity of center of mass for that case are shown below.

c) The minimum initial kinetic energy necessary for the reaction to take place is called the threshold energy, \(E_{\text{th,threshold}}\). Draw a momentum diagram for the initial and final states in the center of mass reference frame when the initial kinetic energy of the incident particles is at the threshold energy (keep in mind the lithium target is also moving in the center of mass reference frame). Hint: What are the velocities of the outgoing particles at threshold?

Answer. At the threshold the energy is just enough to create the boron and the neutron so they are at rest in the center of mass reference frame.
d) Write down equations for conservation of energy and momentum in the center of mass frame when the initial kinetic energy of the incident particles is at the threshold energy. What is the threshold energy in the center of mass reference frame?

Answer: Momentum is zero.

\[ \mathbf{0} = \mathbf{p}_{cm,He} + \mathbf{p}_{cm,Li} = \mathbf{p}_{cm,B} + \mathbf{p}_{cm,n} = \mathbf{0}. \]

The boron and neutron are at rest at threshold energy. The incident kinetic energy is just equal to the energy necessary to create the boron and neutron which is 2.8 MeV.

\[ \frac{1}{2} (4m)v_{cm,He}^2 + \frac{1}{2} (7m)v_{cm,Li}^2 = E'_{0,\text{Threshold}} = 2.8 \text{ MeV}. \]

e) Draw momentum diagrams for the initial and final states in the center of mass reference frame when the initial kinetic energy of the incident particles lies in the range \( E'_{0,\text{threshold}} < E_0 < E'_{0,\text{threshold}} + 0.27 \text{ MeV} \). How many possible final states are there? (Hint: Which way is the neutron moving?)

Answer: The momentum diagrams for the initial and final states in the center of mass reference frame when the boron and neutron have non-zero kinetic energy above threshold are shown below. There are two possible final states corresponding to the two possible directions of the neutron and boron.
f) Draw momentum diagrams for the initial and final states in which the target is initially at rest for the case of part e). How many possible final states are there? (Hint: Which way is the neutron moving?)

Answer: There are two final states: state (a) corresponding to a slow neutron moving in the positive $x$-direction; and state (b) corresponding to a slow neutron moving in the positive $x$-direction.

g) What is the initial kinetic energy of the helium in this reference frame at threshold? How many possible final states are there? (Hint: Which way is the neutron moving?)

Answer: We have already determined that $v_{cm,He} = (7/11)v_{He}$ and $v_{cm,Li} = (4/11)v_{He}$.

Therefore the initial kinetic energy in the center of mass reference frame at threshold is

$$K_{cm,i} = \frac{1}{2} (4m)v_{cm,He}^2 + \frac{1}{2} (7m)v_{cm,Li}^2 = \frac{1}{2} (4m) \left( \frac{49}{121} \right) v_{He}^2 + \frac{1}{2} (7m) \frac{16}{121} v_{He}^2$$

$$= \frac{1}{2} (4m)v_{He}^2 \left( \frac{49}{121} + \frac{28}{121} \right) = \frac{1}{2} (4m) v_{He}^2 \frac{7}{11}$$
Note that the incident kinetic energy in lab frame is

\[ K_{\text{lab},i} = \frac{1}{2} (4m)v_{\text{He}}^2. \]

Therefore

\[ K_{\text{cm},i} = \frac{1}{2} (4m)v_{\text{He}}^2 \frac{7}{11} = \frac{7}{11} K_{\text{lab},i} = 2.8 \text{ MeV} \]

We set this equal to the threshold energy of 2.8 MeV in order to determine the incident kinetic energy at threshold in the lab reference frame

\[ \frac{7}{11} K_{\text{lab},i} = 2.8 \text{ MeV}. \]

Thus

\[ K_{\text{lab},i} = 4.4 \text{ MeV}; \text{ (at threshold)} \]