Suppose that a rotor of moment of inertia $I_r$ has initial angular speed $\omega_0$ at time $t = 0$ when it begins to slow down during the interval $[0, t_1]$ according to $\omega(t) = \omega_0 - \alpha t$ and comes to a stop at time $t = t_1$ and therefore $\alpha = \omega_0 / t_1$. Use work-energy techniques to find the frictional torque acting on the rotor.

**Solution:** The angular speed at $t = t_1$ is $\omega(t_1) = 0$ and therefore

$$\Delta K = 0 - (1/2)I_r\omega_0^2.$$

The rotational work is

$$W = -\int_{\theta_t}^{\theta_f} \tau_f \, d\theta = -\int_{t=0}^{t=t_1} \tau_f \omega(t') \, dt' = -\tau_f \int_{t'=0}^{t'=t_1} (\omega_0 - \alpha t') \, dt'$$

$$W = -\tau_f \left( \omega_0 t_1 - (1/2)\alpha t_1^2 \right) = -\tau_f \frac{\omega_0 t_1}{2}$$

Therefore the work energy law $W = \Delta K$ becomes

$$\frac{I_r \omega_0^2}{2} = -\frac{\tau_f \omega_0 t_1}{2}$$

and so we can solve for the frictional torque

$$\tau_f = \frac{I_r \omega_0}{t_1}.$$