A hockey player shoots a “trick” hockey puck along the ice towards the center of the goal from a position $d$ directly in front of the goal. The initial speed of the puck is $v_0$ and the puck has a mass $m$.

Half way to the goal the puck explodes into two fragments. One piece of mass $m_1 = \left(\frac{3}{5}\right)m$ comes back towards the player and passes $3d/8$ to the side of the spot it was initially shot from with a speed $v_{1,f} = \left(\frac{5}{12}\right)v_0$. The other piece of the puck with mass $m_2 = \left(\frac{2}{5}\right)m$ continues on towards the goal with a speed $v_{2,f}$.

Assume that there is no friction as the puck slides along the ice and that the mass of explosive in the puck is negligible. By what distance, $y$, does the piece that continues towards the goal miss the center of the goal? Express your answer in terms of $d$.

Solution.

Because the puck explodes the mechanical energy of the system (the puck) is not constant (it increases due to the explosion, converting chemical energy into kinetic energy). However there are no external forces acting on the puck or the fragments of the puck since we assume the ice is frictionless. From the geometry of the problem $\tan \theta_{2,f} = y/(d/2)$ and so we should be able to use the fact that the momentum is constant to determine $\tan \theta_{2,f}$ and hence find the distance $y$.

The equations for the constancy of the components of momentum are
\[ m v_0 = -m_1(v_{1,f}) \cos \theta_{1,f} + m_2(v_{2,f}) \cos \theta_{2,f}, \quad (1) \]
\[ 0 = m_1(v_{1,f}) \sin \theta_{1,f} - m_2(v_{2,f}) \sin \theta_{2,f}. \quad (2) \]

Substitute the masses \( m_1 = (3/5)m \), \( m_2 = (2/5)m \), and \( v_{1,f} = (5/12)v_0 \) into Eqs. (1) and (2), yielding

\[ v_0 = -\frac{1}{4} v_0 \cos \theta_{1,f} + \frac{2}{5} v_{2,f} \cos \theta_{2,f}, \quad (3) \]
\[ 0 = \frac{1}{4} v_0 \sin \theta_{1,f} - \frac{2}{5} v_{2,f} \sin \theta_{2,f}. \quad (4) \]

From the geometry of the collision,
\[ \cos \theta_{1,f} = \frac{4d/8}{5d/8} = \frac{4}{5}, \quad (5) \]
\[ \sin \theta_{1,f} = \frac{3d/8}{5d/8} = \frac{3}{5}. \quad (6) \]

The Eqs. (3) and (4) become

\[ v_0 = -\frac{1}{5} v_0 + \frac{2}{5} v_{2,f} \cos \theta_{2,f}, \quad (7) \]
\[ 0 = \frac{3}{20} v_0 - \frac{2}{5} v_{2,f} \sin \theta_{2,f}. \quad (8) \]

Eq. (7) becomes

\[ 3v_0 = v_{2,f} \cos \theta_{2,f}. \quad (9) \]

Eq. (8)

\[ \frac{3}{8} v_0 = v_{2,f} \sin \theta_{2,f}. \quad (10) \]

Now divide Eq. (10) by Eq. (9) yielding

\[ \tan \theta_{2,f} = \frac{1}{8}. \quad (11) \]

From the geometry of the problem

\[ \tan \theta_{2,f} = \frac{y}{d/2}. \quad (12) \]

Comparing Eqs. (11) and (12), we can solve for the distance \( y \),

\[ y = d/16. \quad (13) \]