IC-W10D3-5 Inelastic Angular Collision Solution

A rigid hoop of radius $R$ and mass $m$ is lying on a horizontal frictionless table and pivot at the point $P$ (shown in the figure). A point-like object of the same mass $m$ is moving to the right (see figure) with speed $v_0$. It collides and sticks to the hoop at the midpoint of the hoop. The moment of inertia of a hoop for an axis passing through the center of mass and perpendicular to the plane of the hoop is $I_{cm} = mR^2$. After the collision, the hoop rotates counterclockwise about its pivot point with angular speed $\omega_f$.

The goal of this problem is to find the change in mechanical energy of the hoop and object due to this completely inelastic collision.

a) Calculate the angular momentum about the pivot point $P$ immediately before and after the inelastic collision of the system consisting of the object and the hoop.

b) Consider the hoop and the object as the system. Are there any external torques about the pivot point $P$ on the system consisting of the object and the hoop due to the collision between the object and the hoop? Why or why not? Hint: Is there a force at the pivot? Does that pivot force contribute a torque about the pivot? What about the collision forces?

c) Based on your responses to part a) and b), determine an expression for the angular speed $\omega_f$ of the system immediately after the collision in terms of $v_0$, $m$, and $R$ as needed.

d) What is the change in mechanical energy of the hoop and object due to this completely inelastic collision? Express your answer only in terms of $v_0$ and $m$. 
Angular momentum is constant about pivot point P

\[ \vec{L}_{p,i} = \vec{r}_{p,i} \times m \vec{v}_0 = Rm \vec{v}_0 \]

\[ \vec{L}_{p,f} = \vec{r}_{p,f} \times m \vec{v}_f + I_p \vec{\omega}_f \]

\[ = \sqrt{2} R m \vec{v}_f \vec{k} + I_p \vec{\omega}_f \vec{k} \]

\[ \vec{v}_f = \sqrt{2} R \vec{w}_f, \quad I_p = mR^2 + mR^2 = 2mR^2 \]

Note: particle of mass m is rotating in a circle about P of radius \( \sqrt{2} R \)

\[ \vec{L}_{p,f} = (2mR^2 \vec{w}_f + 2mR^2 \vec{w}_f) \vec{k} = 4mR^2 \vec{w}_f \]

\[ \vec{L}_{p,i} = \vec{L}_{p,f} \Rightarrow \]

\[ Rm \vec{v}_0 = 4mR^2 \vec{w}_f \Rightarrow \]

\[ \vec{w}_f = \frac{1}{4} \frac{\vec{v}_0}{R} \]
\[ K_i = \frac{1}{2} mv_i^2 \]

\[ K_f = \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2 \]

\[ = \frac{1}{2} m 2R^2 \omega_f^2 + \frac{1}{2} 2mR^2 \omega_f^2 \]

\[ = 2mR^2 \omega_f^2 = 2mR^2 \left( \frac{1}{4} \frac{v_0}{R} \right)^2 = \frac{1}{8} m v_0^2 \]

\[ \Delta K = K_f - K_i = 2mR^2 \omega_f^2 - \frac{1}{2} m v_0^2 \]

\[ \Delta K = \frac{1}{8} m v_0^2 - \frac{1}{2} m v_0^2 = -\frac{3}{8} m v_0^2 \]