W14D1-1 Torque on a Gyroscope Solution

A wheel is at one end of an axle of length $d$. The axle is horizontal. The wheel is set into motion so that it executes uniform precession; that is, the wheel's center of mass moves with uniform circular motion. The wheel has mass $M$ and moment of inertia $I_{cm}$ about its central axis, through its center of mass. Its spin angular velocity has magnitude $\omega$ and is directed as shown in the figure below. Find the magnitude and direction of the torque about the pivot point in terms of the other parameters.

Solution:

Taking torques about the support point, the only force supplying the net torque is the weight of the wheel.

$$\vec{\tau}_p = \vec{r}_{P,cm} \times Mg = d\hat{r} \times Mg(-\hat{k}) = dMg\hat{\theta}. \quad (1)$$

The direction of this torque, from the right-hand rule, is into the page in the above figure.