a) In a process that is virtually instantaneous, the gun shoots a projectile of mass \( m_p \) horizontally to the right at a velocity \( u \) relative to the gun and recoils to the left inside the tube. What is the initial recoil velocity of the gun \( v_0 \) relative to the ground? Express your answer in terms of some or all of the parameters \( m_g \), \( m_p \), and \( u \).

**Solution:** Use conservation of momentum in the \( x \) direction.

\[
\begin{align*}
P_{x_{\text{initial}}} & = P_{x_{\text{final}}} \\
0 & = m_g v_0 + m_p (u + v_0) \\
-(m_g + m_p) & = m_p u \\
v_0 & = -\frac{m_p}{m_g + m_p} u
\end{align*}
\]

b) What is the final position \( x_f \) of the gun when it comes to rest? You should use \( v_0 \) and some or all of the parameters \( m_g \), \( m_p \), \( x_0 \), and \( u \) in your answer.
Solution: Use the work-kinetic energy theorem.

\[
K_{\text{final}} - K_{\text{initial}} = \int_{x_0}^{x_f} F_x \, dx
\]

\[
0 - \left(\frac{1}{2}\right)mgv_0^2 = \alpha \int_{x_0}^{x_f} \frac{dx}{x} = \alpha \ln \frac{x_f}{x_0}
\]

\[
ln \frac{x_f}{x_0} = -\frac{mgv_0^2}{2\alpha}
\]

\[
x_f = x_0 \exp\left[\frac{-mgv_0^2}{2\alpha}\right]
\]