Two point-like objects are located at the points A, and B, of respective masses $M_A = 2M$, and $M_B = M$, as shown in the figure below. The two objects are initially oriented along the $y$-axis and connected by a rod of negligible mass of length $D$, forming a rigid body. A force of magnitude $F = |\vec{F}|$ along the $x$ direction is applied to the object at A at $t = 0$ for a short time interval $\Delta t$. Neglect gravity. Give all your answers in terms of $M$, $D$, $F$ and $\Delta t$ as needed.

a) Describe qualitatively in words how the system moves after the force is applied: direction, translation and rotation.

b) How far is the center of mass of the system from the object at point B?

c) What is the direction and magnitude of the linear velocity of the center-of-mass after the collision?

d) What is the magnitude of the angular velocity of the system after the collision?

e) Is it possible to apply another force of magnitude $F$ along the positive $x$-direction to prevent the system from rotating? Does it matter where the force is applied?

f) Is it possible to apply another force of magnitude $F$ in some direction to prevent the center of mass from translating? Does it matter where the force is applied?
a) After the impulse is applied, the system will translate to the right and rotate clockwise about its center of mass.

b) If $X$ is the distance of the center of mass from $B$, then

$$3mX = 2mD \Rightarrow X = \frac{2}{3}D.$$ 

And so the distance of the center of mass from $A$ is

$$D - \frac{2}{3}D = \frac{1}{3}D.$$ 

(It makes sense that the center of mass is closer to the heavier object).

c) Remember: the center of mass moves as if (1) all the mass of the system is concentrated there, and (2) every force on the system is applied at the center of mass. The linear impulse on the system is given by

$$I = \int F \, dt = F \Delta t.$$ 

We can then use the “impulse = change in momentum” principle. The original linear momentum is 0, and the total mass of the system is $3m$, and so the impulse-momentum principle yields

$$3mv = F \Delta t \Rightarrow v = \frac{F \Delta t}{3m}.$$ 

d) Because $\Delta t$ is small, we can assume that the system does not move very much while the impulse is being applied. To find the change in angular momentum about the center of mass, we need to find the change in angular impulse about the center of mass.

$$\mathbf{J} = \int \mathbf{r}_{cm,F} \times \mathbf{F} \, dt = \frac{1}{3}D \cdot F \cdot \Delta t.$$ 

We can then use “angular impulse = change in angular momentum”. The angular momentum is originally 0, and so the final angular momentum is

$$I\omega = \frac{1}{3}DF\Delta t.$$ 

The moment of inertia $I$ of the rod about its center of mass is given by

$$I = 2m \cdot \left(\frac{1}{3}D\right)^2 + m \cdot \left(\frac{2}{3}D\right)^2 = \frac{2}{3}mD^2.$$ 

And so
\[
\frac{2}{3}mD^2\omega = \frac{1}{3}DF\Delta t
\]
\[
\omega = \frac{DF\Delta t}{2mD^2}
\]
\[
\omega = \frac{F\Delta t}{2mD}.
\]

e) Clearly, if we apply a force below the center of mass, it will cause an angular impulse in the opposite direction, and will tend to cancel out the angular impulse from \(F\). If we manage to make the angular impulses cancel out exactly, the body will not rotate. Let’s imagine that the new force \(F\) is applied at a distance \(X\) below the center of mass. The new angular impulse is then
\[
J = \frac{1}{3}DF\Delta t - XF\Delta t.
\]

If we want this to be equal zero then
\[
\frac{1}{3}DF\Delta t = XF\Delta t \Rightarrow \frac{1}{3}D = X.
\]

Therefore the new force must be applied at a distance \(D/3\) below the center of mass for the body not to rotate.

f) Yes – as long as the total linear impulse is 0, the body will not translate. To do this, we simply need to apply a new force \(F\) in the opposite direction. It does not matter where this force is applied.