A ball of radius $b$ and mass $M$ is rolling without slipping on the surface of a ring of radius $R > b$. The angular speed of the ball is $\omega$.

a) Suppose the ring is fixed. What is the rolling without slipping condition?

b) Suppose the ring is rotating with angular speed $\Omega$ as shown in the figure and the ball is rolling without slipping. What is the center of mass speed of the ball?

Solution

a) When the ring is fixed the rolling without slipping condition is $v_{cm} = b\omega$.

b) When the outer ring is rotating with angular speed $\Omega$, then the contact point on the ring is moving with speed $v = R\Omega$. A point on the rim of the ball when it is in contact with the surface of the ring has speed $v_{cm} - b\omega$. The rolling without slipping condition is that the point in contact with surface has zero relative velocity. Therefore $v_{cm} - b\omega = v$ or

$$v_{cm} = b\omega + R\Omega$$