A uniform cubical block of mass \( m \) and edge length \( s \) slides without friction with speed \( v \) on a horizontal surface. It collides inelastically with a low ridge on the surface. It subsequently pivots without friction about that ridge. The moment of inertia of a cube about an axis normal to and centered on one of its faces (and thus passing through the center of mass) is \( ms^2 / 6 \). [Note that parts a) and b) can be done independently.]

a) Find the initial rotation rate of the cube \( \omega_0 \) just after the collision. Express your answer in terms of some or all of the parameters \( m, s, v \) and \( g \).

**Solution:** Angular momentum about the pivot point is conserved at the instant of the collision.

\[
I_{before} = I_{after}
\]

\[
mv(s/2) = I_s \omega_0 = [(1/6)ms^2 + m(s/\sqrt{2})^2] \omega_0 \quad \text{using the parallel axis theorem}
\]

\[
= (2/3)ms^2 \omega_0
\]

\[
\omega_0 = \frac{(3/4)v}{s}
\]

b) Find the maximum value of \( \omega_0 \) could have without causing the cube to topple over and end up resting on other side of the ridge. Express you answer in terms of some or all of the parameters \( m, s, \) and \( g \).

**Solution:** Conserve energy after the collision. Let the potential energy of the block be zero when it is resting on its face. The block will topple over if it has any kinetic energy left when the center of mass is at its highest point, that is when the center of mass is directly over the pivot point.
\[ E_{\text{initial}} = E_{\text{top}} \]

\[ K_{\text{initial}} + U_{\text{initial}} = K_{\text{top}} + U_{\text{top}} \]

\[ K_{\text{initial}} + 0 = 0 + U_{\text{top}} \]

\[ \frac{1}{2} I_\omega \omega_0^2 = mg\left(s/\sqrt{2} - s/2\right) = mgs\left(\frac{\sqrt{2} - 1}{2}\right) \]

\[ \omega_0^2 = \left(\sqrt{2} - 1\right) \frac{mgs}{I_p} = \left(\sqrt{2} - 1\right) \frac{mgs}{(2/3)ms^2} = \frac{3}{2} \left(\sqrt{2} - 1\right) \frac{g}{s} \]

The block will not topple over if

\[ \omega_0 = \frac{3g}{\sqrt{2s}\left(\sqrt{2} - 1\right)} \]