Concept Questions with Answers

8.01
W13D2

Concept Q.: SHO and the Pendulum
Suppose the point-like object of a simple pendulum is pulled out at by an angle $\theta_0 << 1$ rad. Is the angular speed of the point-like object

1. always greater than
2. always less than
3. always equal to
4. only equal at bottom of the swing to the angular frequency of the pendulum?

Solution 2:
The angular frequency is a constant of the motion and by definition is $\omega_0 = \frac{2\pi}{T}$. For small angle the pendulum approximates a simple harmonic oscillator with $\omega_0 = \left(\frac{g}{l}\right)^{1/2}$. The angular speed $\omega$ by definition is the magnitude of the component of the angular velocity, $\omega = d\theta/dt$. Note that sometimes the symbol $\omega$ may be used for both quantities. Because we are just considering small angles, when can set $t = 0$ for the maximum angle of displacement $\theta_0$ then

$$\omega_z(t) = \frac{d\theta}{dt} = -\omega_0 \theta \sin(\omega_0 t)$$

Because $\theta \sin(\omega_0 t) << 1$ The angular speed is always less than the angular frequency $|\omega_z(t)| < \omega_0$

Concept Q. Ans.: SHO and the Pendulum

Concept Question: Physical Pendulum
A physical pendulum consists of a uniform rod of length $d$ and mass $m$ pivoted at one end. A disk of mass $m_1$ and radius $a$ is fixed to the other end. Suppose the disk is now mounted to the rod by a frictionless bearing so that is perfectly free to spin. Does the period of the pendulum

1. increase?
2. stay the same?
3. decrease?

Answer 3. When the disk is fixed to the rod, an internal torque will cause the disk to rotate about its center of mass. When the pendulum reaches the bottom of its swing, the decrease in potential energy will be result in an increase in the rotational kinetic energy of both the rod and the disk and the center of mass translation kinetic energy of the rod-disk system. When the disk is mounted on the frictionless bearing there is no internal torque that will make the disk start to rotate about its center of mass when the pendulum is released. Therefore when the pendulum reaches the bottom of its swing, the same decrease in potential energy will be transferred into a larger rotational kinetic energy of just the rod since the disc is not rotating and a greater increase in the center of mass translation kinetic energy of the rod-disk system.

So when the disk bearings are frictionless, the center of mass of the rod-disk system is traveling faster at the bottom of its arc hence will take less time to complete one cycle and so the period is shorter compared to the fixed disk.
Concept Question: Energy Diagram 3
A particle with total mechanical energy $E$ has position $x > 0$ at $t = 0$

1) escapes to infinity
2) approximates simple harmonic motion
3) oscillates around a
4) oscillates around b
5) periodically revisits a and b
6) two of the above

Solution 3. The range of motion for the particle is limited to the regions in which the kinetic energy is either zero or positive, so the particle is confined to move around the region surrounding a. The motion will be periodic but not simple harmonic motion because the potential energy function is not a quadratic function and only for quadratic potential energy functions will the motion be simple harmonic. Hence the particle oscillates around a.

Concept Question: Energy Diagram 4
A particle with total mechanical energy $E$ has position $x > 0$ at $t = 0$

1) escapes to infinity
2) approximates simple harmonic motion
3) oscillates around a
4) oscillates around b
5) periodically revisits a and b
6) two of the above

Solution 6. Now the particle oscillates around the region surrounding a. Since the energy is so close to the minimum of the potential energy, we can approximate the potential energy as a quadratic function and hence the particle motion approximates simple harmonic motion.